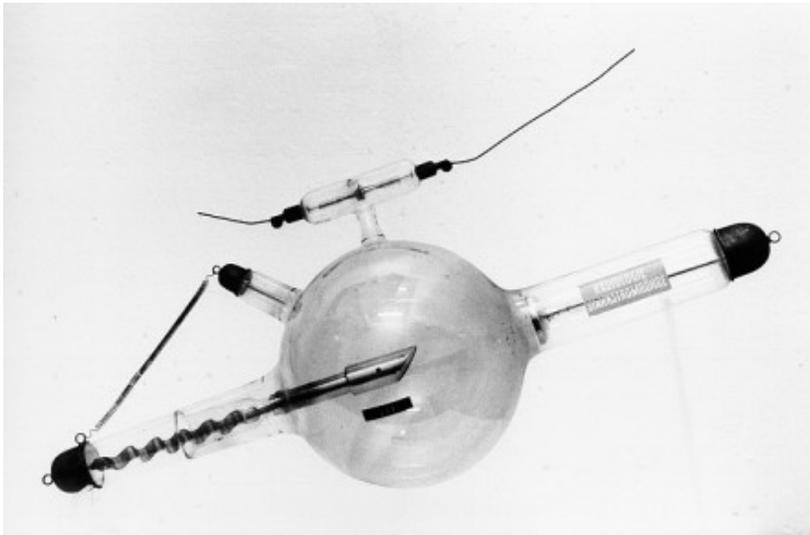
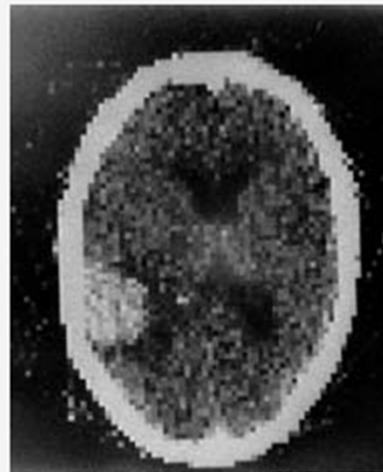


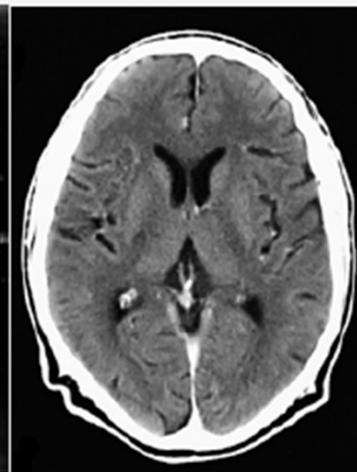
# Imaging with x-rays



Röntgen



CT



1974

1996

# *Imaging with x-rays*

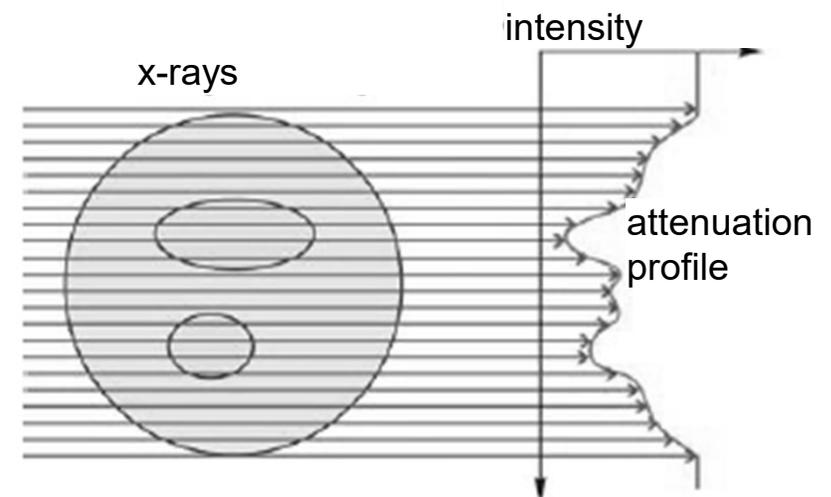
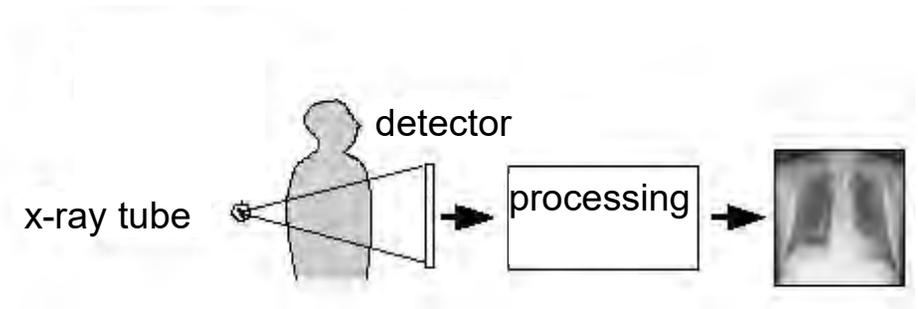
## **Contents:**

- historical overview
- physics of x-rays
- generating x-rays
- interactions with matter
- detectors
- imaging with x-rays
  - projection radiography
  - computed tomography

# Imaging with x-rays

## *principle*

- **active** imaging through exposure of energy
- **attenuation** of x-rays by different tissues



# Imaging with x-rays

## history

- 1895 Wilhelm Conrad Röntgen (27.3.1845 - 10.2.1923)  
discovery of x-rays on 8. November 1895  
imaging of Mrs. Röntgen's hand on 22. December 1895
- 1901 Nobel Physics prize awarded to Röntgen
- 1912 verification as e.m. wave with scattering experiments  
in crystals (Friedrich, Knipping, von Laue)
- 1917 Johann Radon: Radon-transform as  
mathematical principle for tomography  
(*Über die Bestimmung von Funktionen durch ihre Integralwerte  
längs gewisser Mannigfaltigkeiten*. Ber. vor Sächs. Akad. Wiss., 69, 262)  
Coolidge: x-ray tube with high vacuum



*Prof. Dr. W.C. Röntgen*



## *Imaging with x-rays*

*„Und läßt man der Phantasie weiter die Zügel schießen, stellt man sich vor, dass es gelingen würde, die neue Methode des photographischen Prozesses mit Hilfe von Strahlen aus den Crookeschen Röhren so zu vervollkommen, dass nur eine Partie der Weichteile des menschlichen Körpers durchsichtig bleibt, eine tiefer liegende Schicht aber auf der Platte fixiert werden kann, so wäre ein unschätzbare Behelf für die Diagnose zahlloser anderer Krankheitsgruppen als die Knochen gewonnen.“*

Anonym, Frankfurter Zeitung, 7. Januar 1896

(cited in: W. Kalender Computertomographie, Publicis MCD Verlag, 2000)

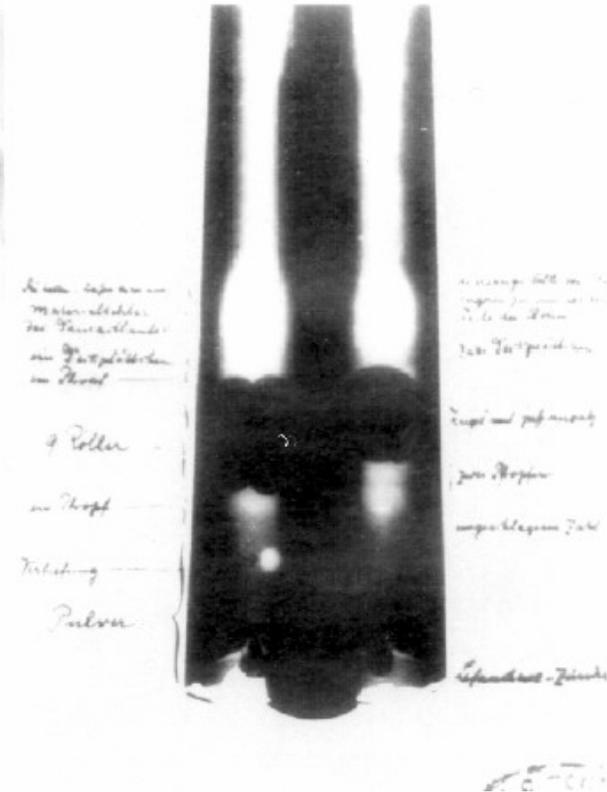
# Imaging with x-rays

early applications of x-rays (as of 1896)

x-raying of non-transparent objects



hand with ring



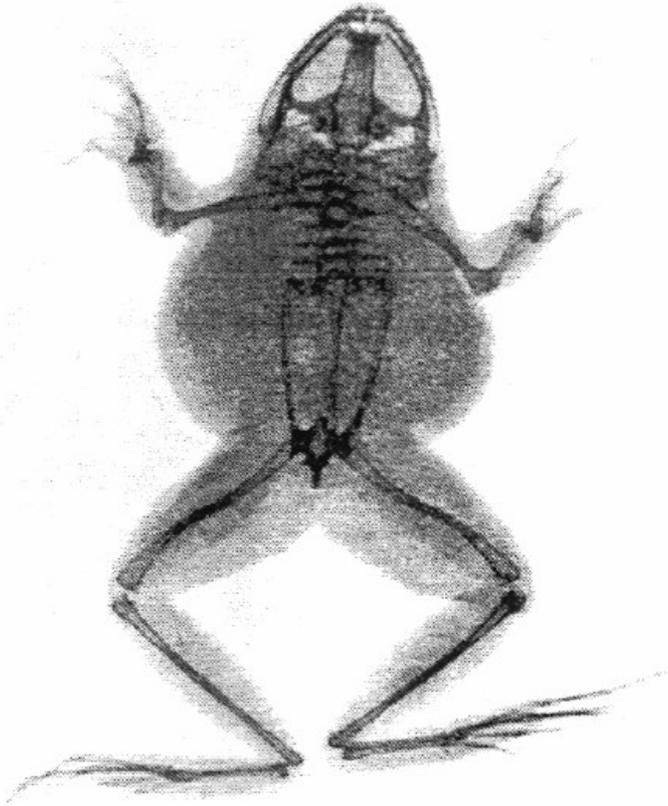
shotgun with bullets



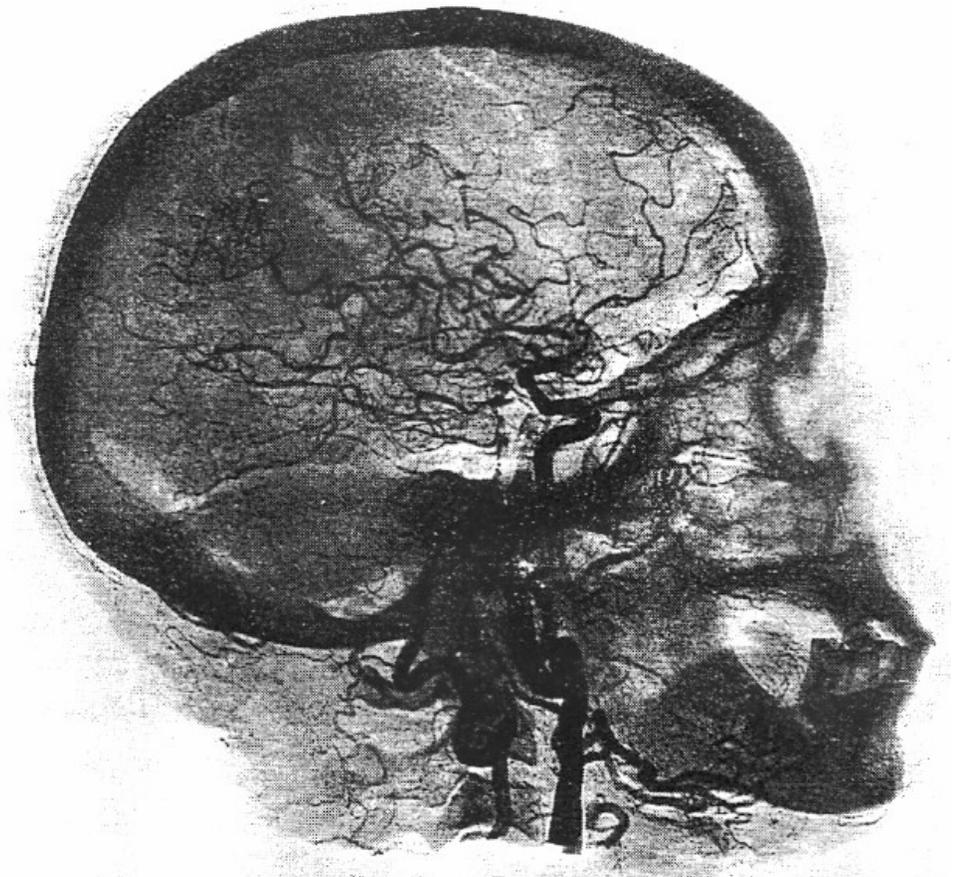
authenticity of jewelry<sub>6</sub>

## *Imaging with x-rays*

early applications of x-rays (as of 1896)



**radiogram of a frog  
(San Francisco 1896)**

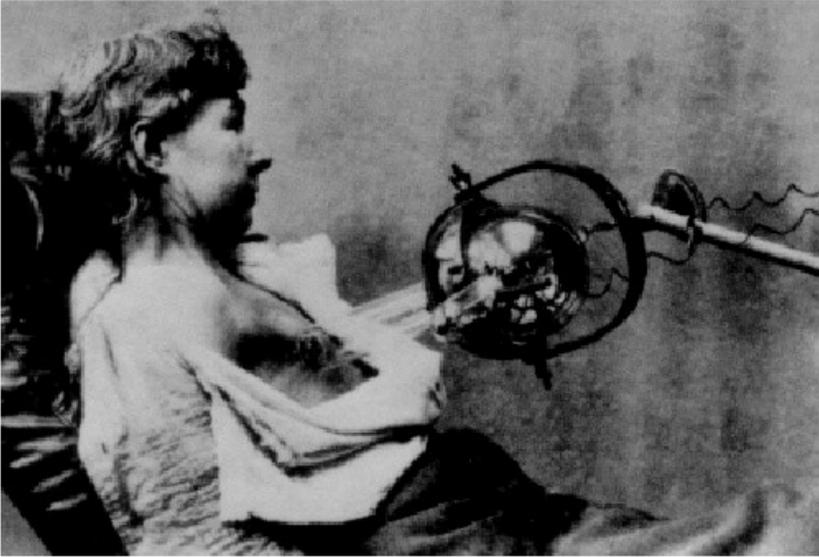


**arteriogram (1904)**

Mauld, R.F., *The Early History of X-Rays in Medicine*, Fig. 9 and Fig. 17 aus Michette, A.; Pfauntsch, S., *X-Rays: The First Hundred Years*; (Wiley 1996)

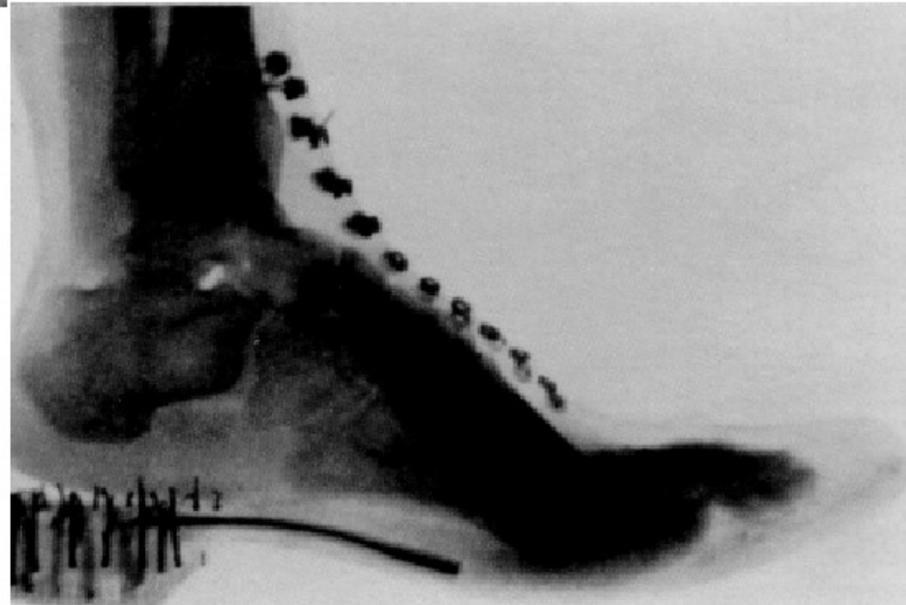
## *Imaging with x-rays*

early applications of x-rays (as of 1896)



treatment of breast  
cancer (1905)

adjustment of shoes  
controlled with x-rays



## *Imaging with x-rays*

### early applications of x-rays

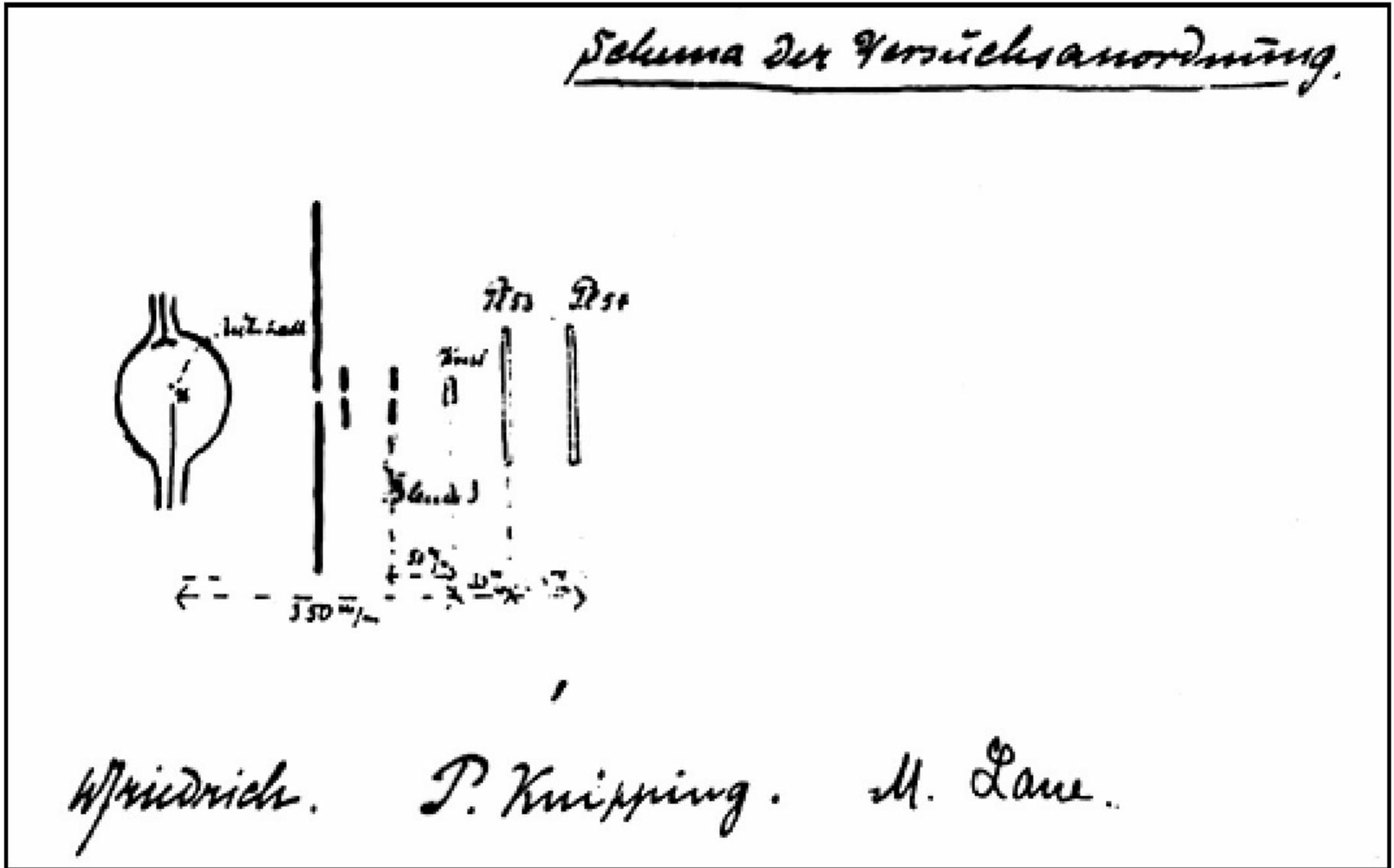


**Radiography in the Sudan campaign in the desert some 1200 miles from Kairo (1898). The X-ray tube was described as being suspended by means of an ingenious holder. The use of the inverse square law is to the advantage of the operator on the right but not of the soldier near the patient. It was this installation that the casualties from the battle of Omdurman were radiographed.**

Mauld, R.F., *The Early History of X-Rays in Medicine*, Fig. 11 aus Michette, A.; Pfauntsch, S., *X-Rays: The First Hundred Years*; (Wiley 1996)

# Imaging with x-rays

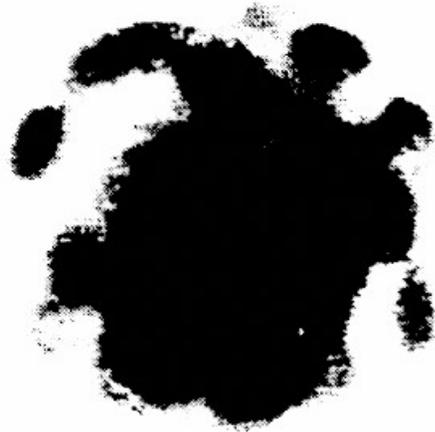
historical experiment by M. von Laue



## *Imaging with x-rays*

historical experiment by M. von Laue

Die erste Röntgen-  
Durchleuchtung eines  
Kristalls.



M. v. Laue.

## *Imaging with x-rays*

- 1938 Gabriel Frank: a method to generate cross-sectional images of the body with x-rays  
(German patent specification 1940)
- 1957/58 S.I. Tetel'baum, B.I. Korenblyum  
construction of one of the first CT-scanner at Politechnical Institute, Kiev, Russia
- 1961 William D. Oldendorf  
first x-ray CT images of a head phantom  
(idea: stationary detector; rotating probe)
- 1963/64 A.M. Cormack  
first description of an x-ray-based tomographic method (~CT)  
(*Representation of a function by its line integrals, with some radiological applications*. J Appl Phys, 34, 2722, 1963)

## *Imaging with x-rays*

1967 Godfrey N. Hounsfield (engineer)

EMI Lab., England; beginning of computed tomography

M.M. Ter-Pergossian: physical aspects of diagnostic radiology

1971/72 G.N. Hounsfield, J. Ambrose, J. Perry  
first CT for clinical applications



*Godfrey Hounsfield*

1973 transversal/axial CT

1979 A.M. Cormack u. G.N. Hounsfield:  
Nobel Physiology and Medicine prize

1980 digital radiography

2000 ca. 30.000 CT-installations worldwide

## *Imaging with x-rays*

### **basics of electromagnetic waves – x-rays:**

***light:***

#### **particle properties**

(photons, light quanta)

$$E = h f$$

$$[E] = 1\text{eV}$$

$$h = 4.136 \cdot 10^{-21} \text{ MeV s}$$

$$c = 2.997 \cdot 10^8 \text{ m s}^{-1}$$

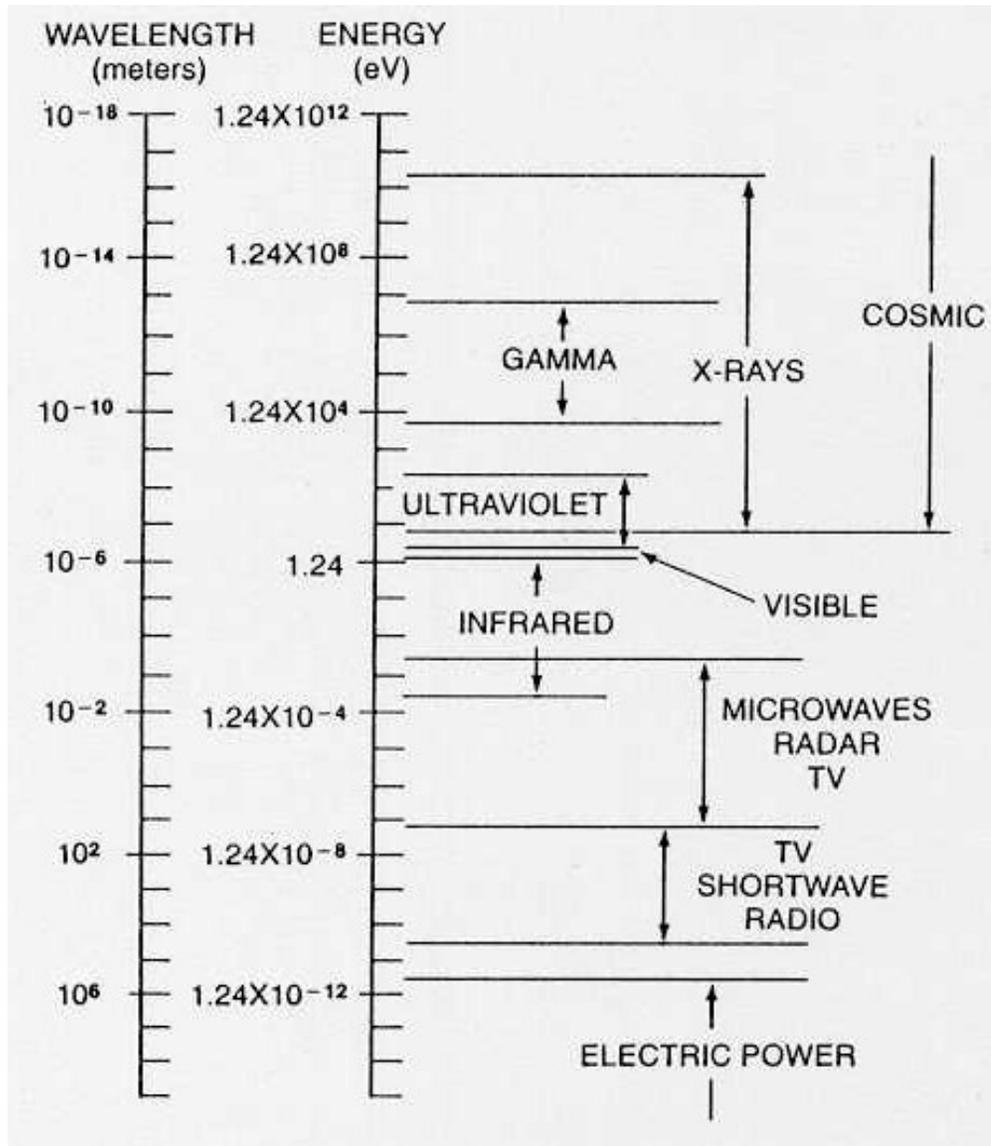
#### **wave properties**

e.m. wave with (mean) frequency  $f$  resp. wavelength  $\lambda$

$$c = \lambda f$$

# Imaging with x-rays

## basics of electromagnetic waves – x-rays:



range of wavelengths for  
x-rays

$$\lambda = 3 \cdot 10^{-8} \text{ m} - 3 \cdot 10^{-14} \text{ m}$$

$\Leftrightarrow$

$$f = 10^{16} \text{ Hz} - 10^{22} \text{ Hz}$$

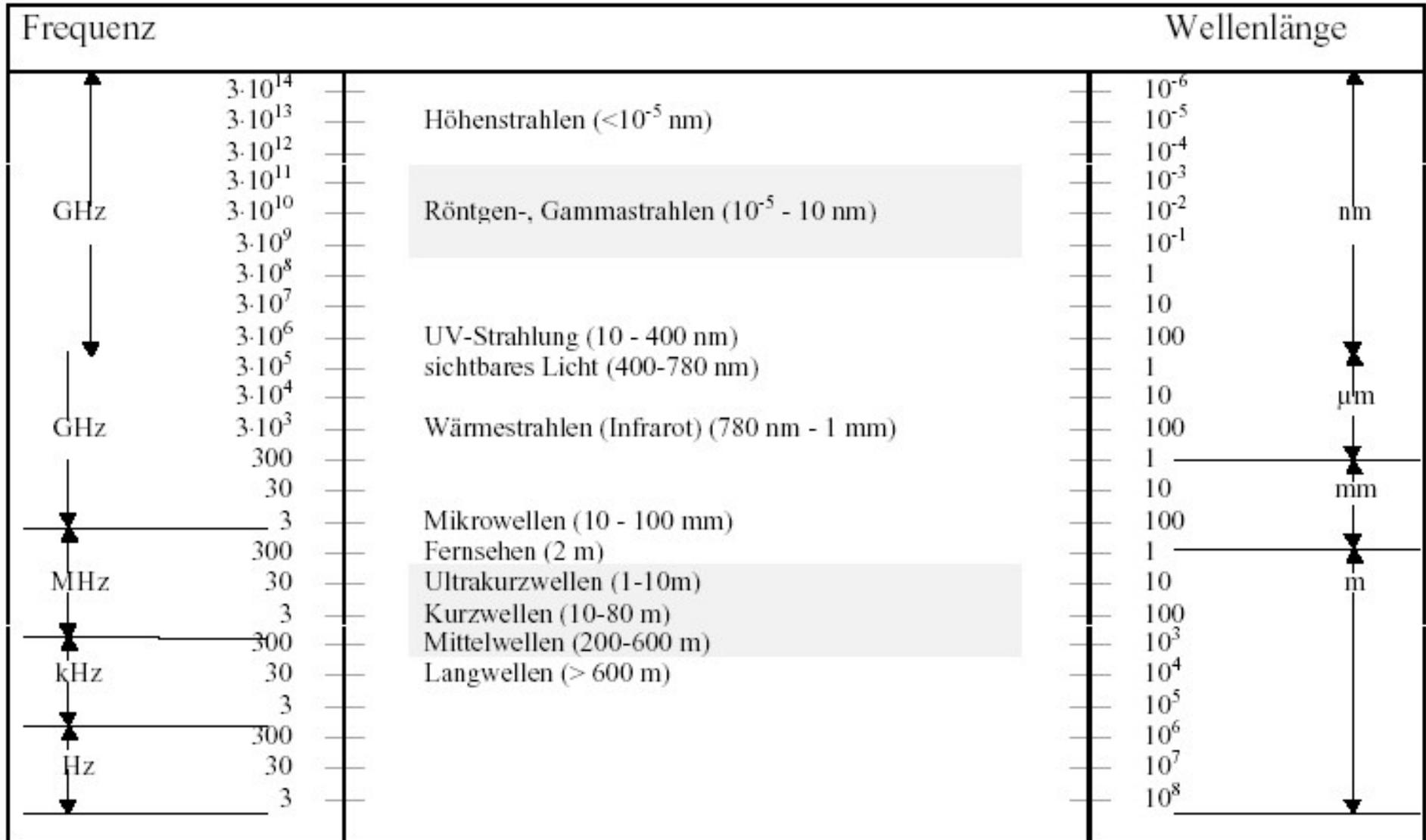
$$= 10^5 \text{ GHz} - 10^{13} \text{ GHz}$$

range of photon energies for  
x-rays:

$$E \approx 42 \text{ eV} - 41.36 \text{ MeV}$$

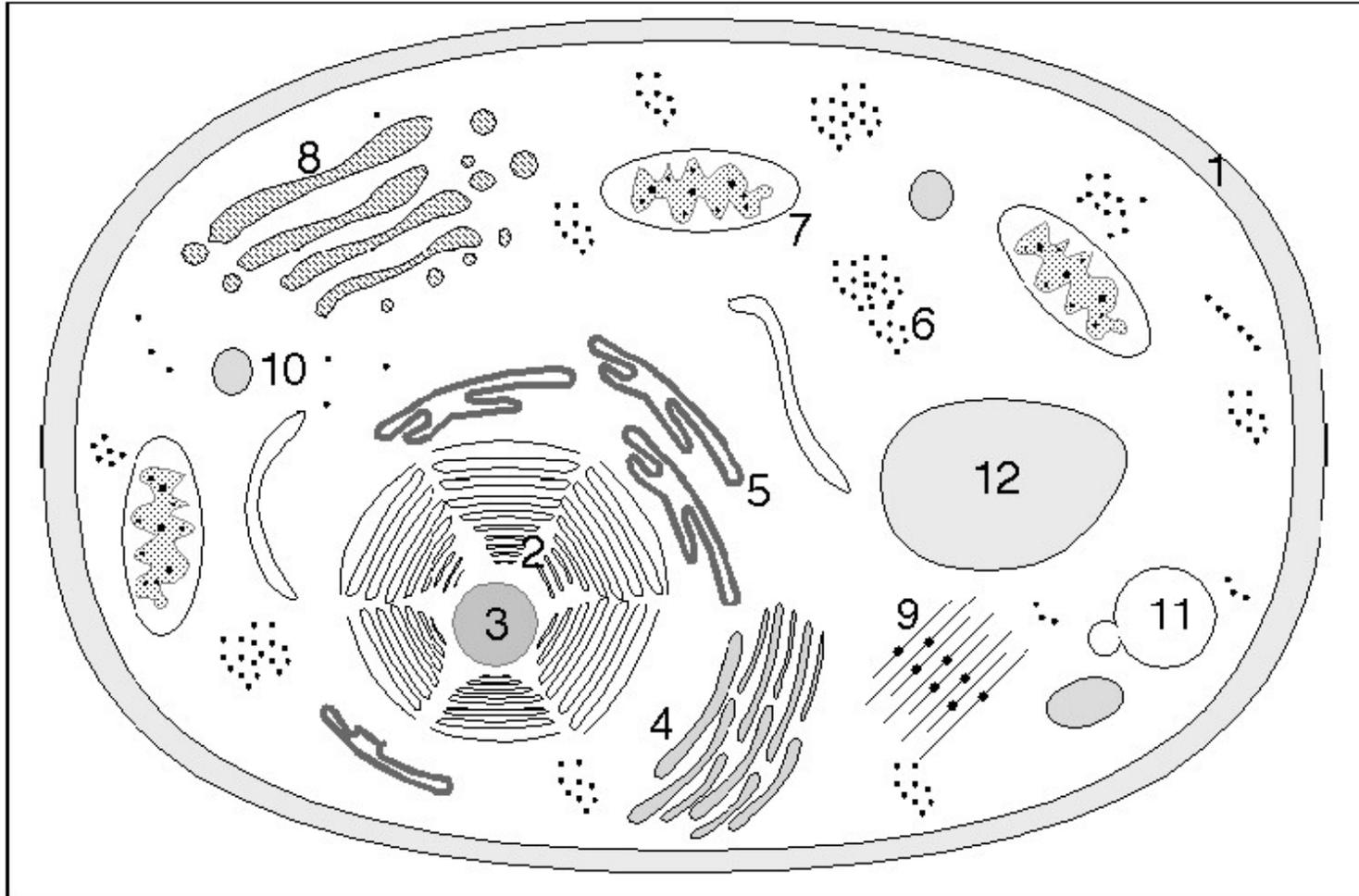
# Imaging with x-rays

## basics of electromagnetic waves – x-rays:



# Imaging with x-rays

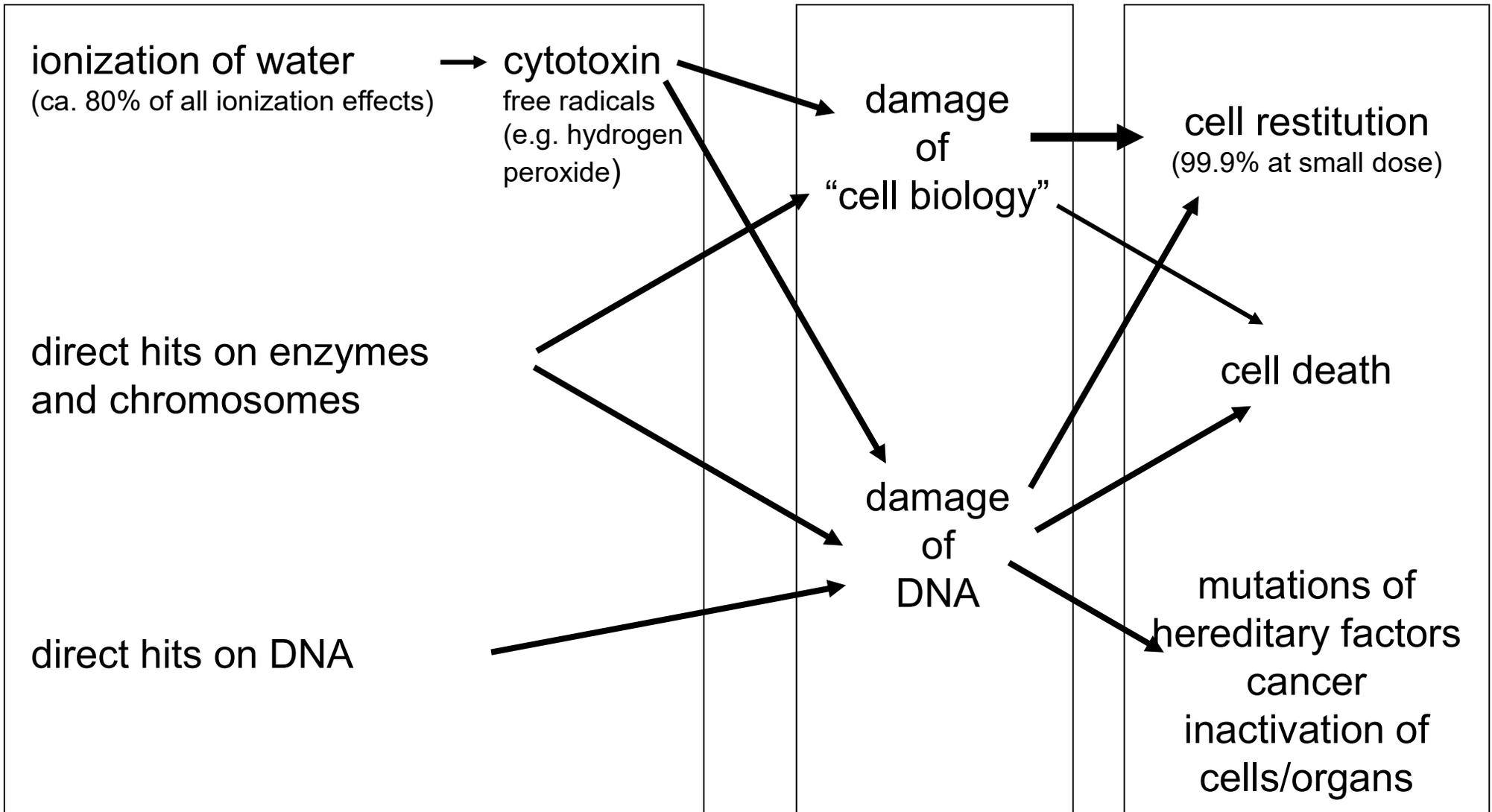
## biological impact of ionizing radiation:



- |                                  |                                 |                    |
|----------------------------------|---------------------------------|--------------------|
| 1) Zellmembran                   | 5) endoplasmat. Retikulum, rauh | 9) Zentrosom       |
| 2) Zellkern                      | 6) freie Ribosomen              | 10) Lipidtröpfchen |
| 3) Nukleolus                     | 7) Mitochondrien                | 11) Lysosomen      |
| 4) endoplasmat. Retikulum, glatt | 8) Golgi-Vesikel                | 12) Vakuolen       |

# Imaging with x-rays

## biological impact of ionizing radiation:



## *Imaging with x-rays*

### **biological impact of ionizing radiation:**

cancer mortality per 10 mSv (1 rem) and 1.000.000 persons

leukemia	20
breast cancer	25
lung cancer	20
bone cancer	5
thyroid cancer	5
other	50
<hr/>	
total	125

“natural” cancer mortality ~ 200.000 per 1.000.000 persons  
natural radiation exposure ~ 2.2 mSv per year

## *Imaging with x-rays*

### **biological impact of ionizing radiation:**

#### ***natural radiation exposure:***

<i>cosmic radiation</i>		<i>0.3 mSv/a</i>
<i>terrestrial radiation</i>		<i>0.5 mSv/a</i>
outdoors	0.43 mSv/a	
indoors	0.57 mSv/a	
<i>incorporated radioactive substances</i>		<i>0.3 mSv/a</i>
<i>inhalation of Radon reaction products</i>		<i>1.3 mSv/a</i>

## *Imaging with x-rays*

### **biological impact of ionizing radiation:**

#### ***artificial radiation exposure:***

<i>medical applications</i>		<i>1.4 mSv/a</i>
x-ray diagnosis	1.30 mSv/a	
nuclear medicine	0.07 mSv/a	
radiotherapy	0.03 mSv/a	
<i>fallout atomic bomb tests</i>		<i>0.01 mSv/a</i>
<i>consumer goods, research</i>		<i>0.03 mSv/a</i>
technical sources	0.01 mSv/a	
industrial products	0.01 mSv/a	
stray radiation emitters (TV)	0.01 mSv/a	
<i>job-related exposition</i>		<i>0.01 mSv/a</i>
<i>non-military use of nuclear power</i>		<i>0.01 mSv/a</i>
<b><i>total dose (natural + artificial)</i></b>		<b><i>1 - 4 mSv/a</i></b>

## **biological impact of ionizing radiation:**

### **radiation damage**

**deterministic:** above some threshold, severity of damage increases with dose

**stochastic:** damage probability increases with dose, no threshold

**somatic damage:** affects the whole body (e.g. malfunction of organs)

**genetic damage:** recessive mutations affecting succeeding generations (fusion of mutated genes and accumulation in population)

## Imaging with x-rays

### base items and units in dosimetry:

energy dose

$$D = \frac{\text{absorbed energy}}{\text{mass}} = \frac{dW}{dm}$$

unit: Gy (Gray)

$$1\text{Gy} = 1\text{J/kg}$$

Formerly: rd (Rad)

$$1\text{ rd} = 0.01\text{ Gy}$$

absorbed dose rate

$$\dot{D} = \frac{\text{energy dose}}{\text{time}} = \frac{dD}{dt}$$

unit: Gy/sec

(or /min, /h, /d, /a)

formerly: rd/sec

## Imaging with x-rays

### base items and units in dosimetry:

ion dose

$$J = \frac{\text{accumulated charge quantity}}{\text{mass}} = \frac{dQ}{dm}$$

unit: C/kg = As/kg

formerly: R (Röntgen)    1 R =  $2.58 \cdot 10^{-4}$  C/kg

ion dose rate

$$\dot{J} = \frac{dJ}{dt}$$

unit: A/kg

# Imaging with x-rays

## base items and units in dosimetry:

$$H = q \cdot D$$

$q$  = radiation - type - dependent factor

unit: Sv (Sievert)                      1 Sv = 1 J/kg

formerly: rem                              1 rem = 10 mSv

equivalent dose

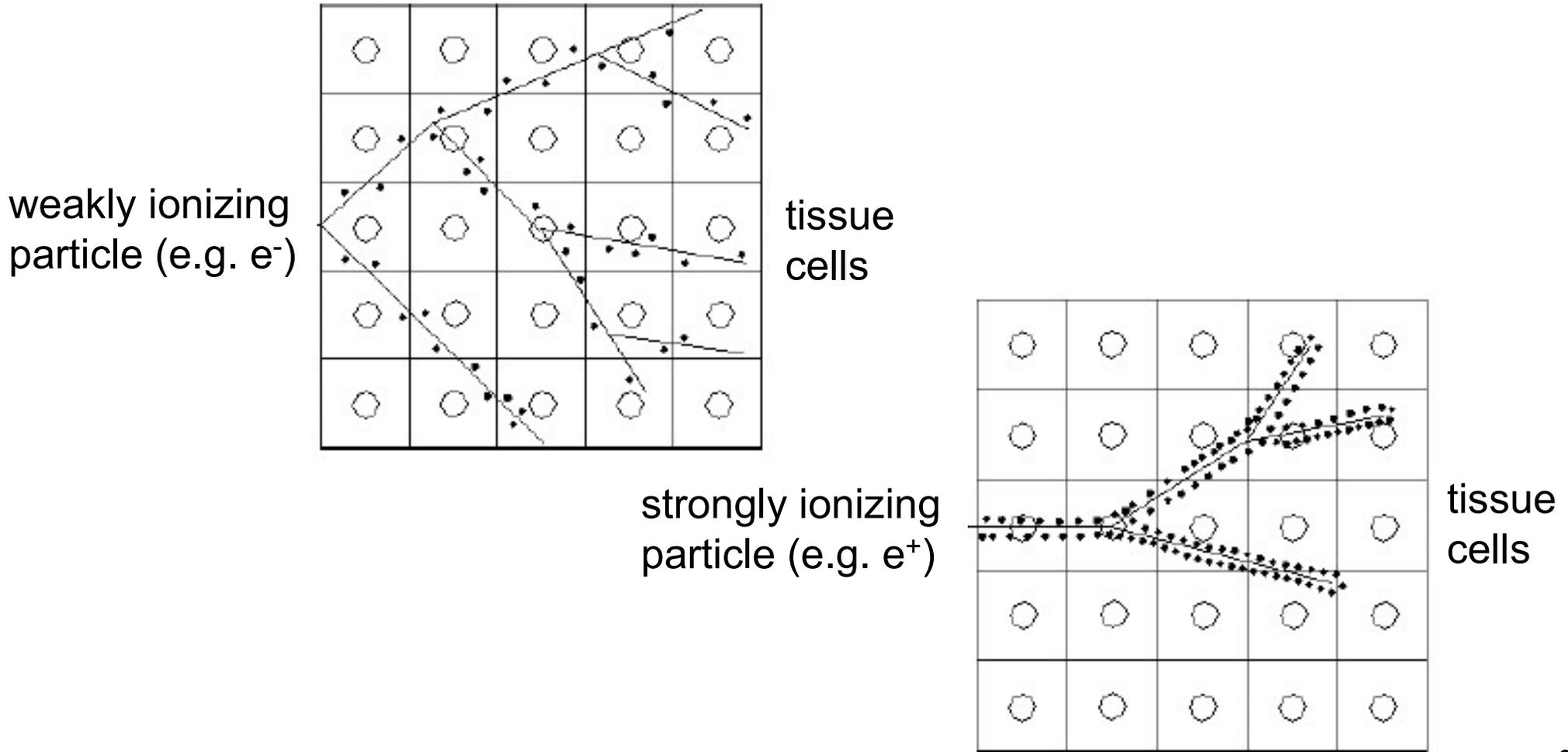
type of radiation	q
x-rays and gamma rays	1
beta rays	1
alpha rays	20
neutron rays	10

biological dose rate

$$\dot{H} = \frac{dH}{dt} \quad \text{unit: Sv/sec} \quad (\text{or /min, /h /d /a})$$

*Imaging with x-rays*

**microscopic distribution of deposited energy  
of various radiation types:**



## *Imaging with x-rays*

### **radiation protection:**

in general:

- avoid unnecessary exposition to ionizing radiation
- if not avoidable:  
**ALARA (As Low As Reasonably Achievable)**
- obligation to inform patients

dose  $D$  decreases with squared distance  $A$

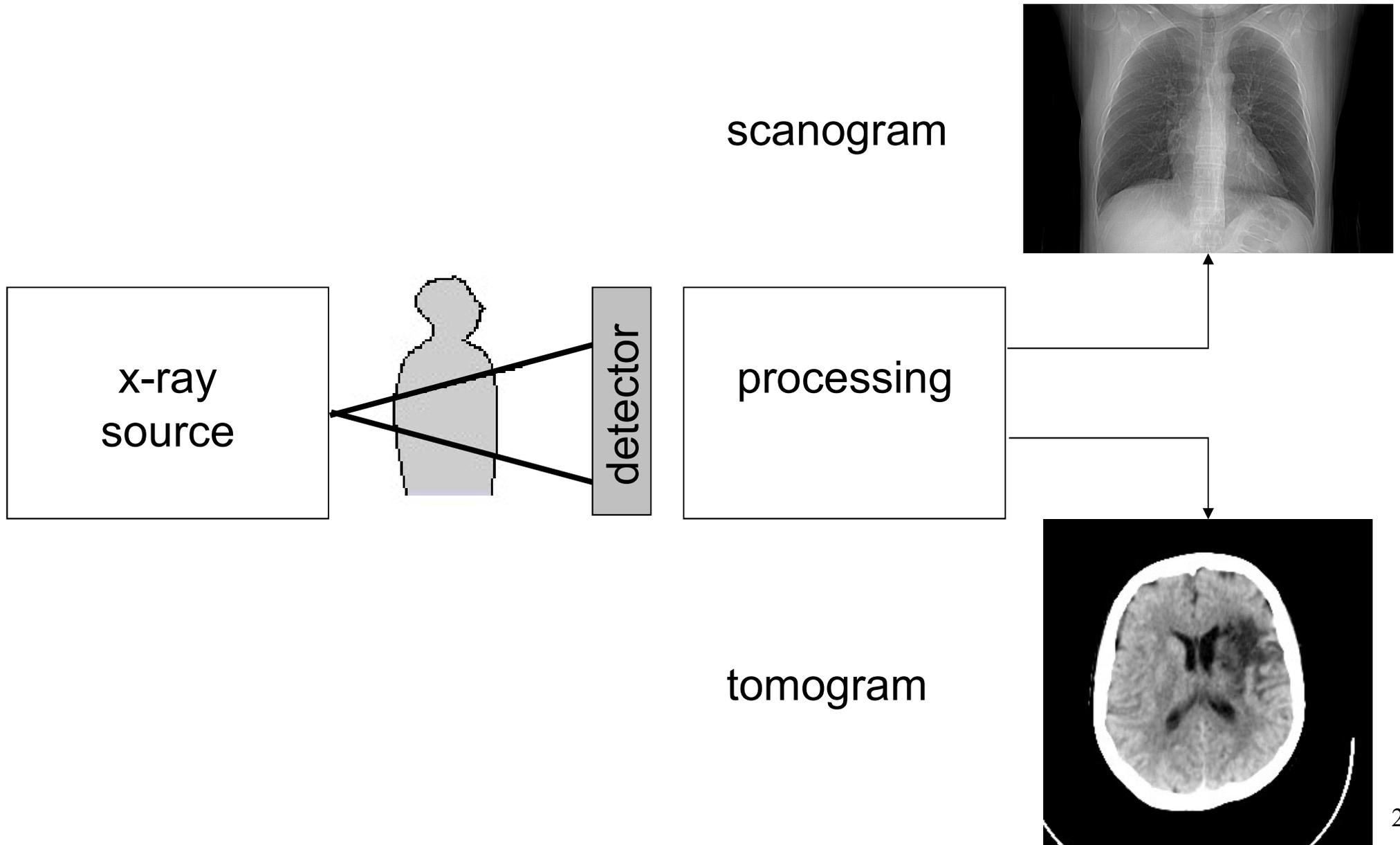
$$D \sim 1/A^2$$

dose  $D$  increases with exposition time  $t_{\text{exp}}$

$$D \sim t_{\text{exp}}$$

**generation  
of  
x-rays**

# Imaging with x-rays



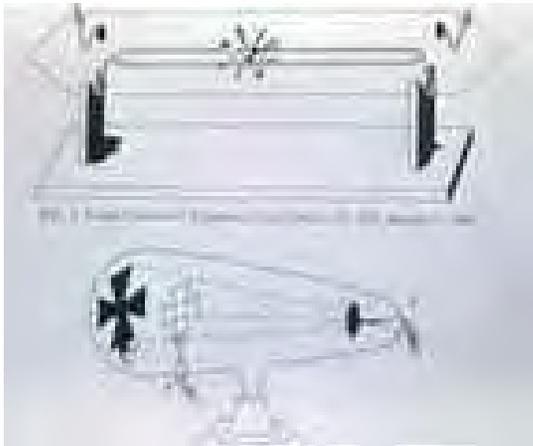
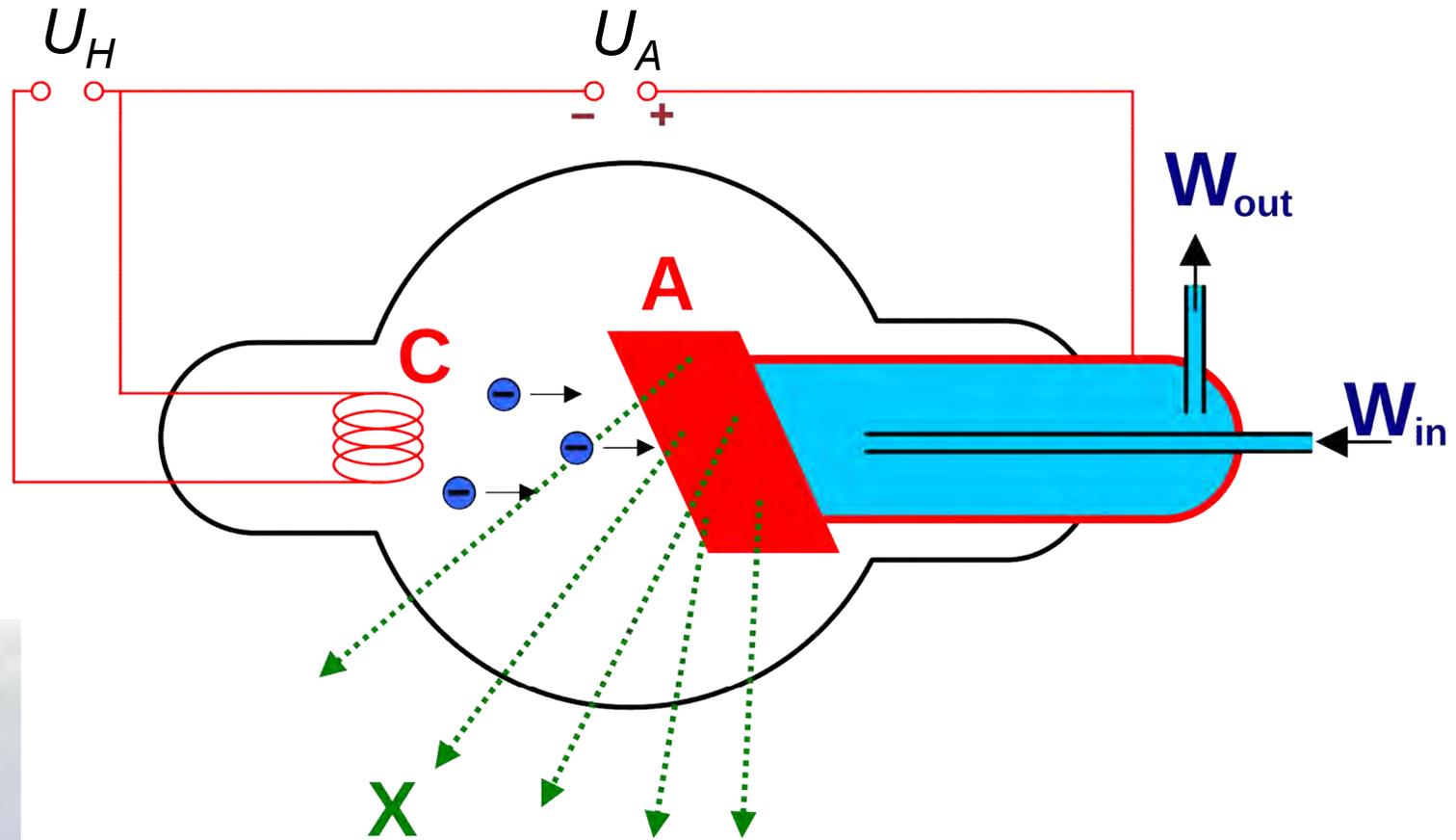
## **generation of x-rays:**

basic principle: ***photoelectric effect***

- heating of cathode ( $U_H$ )  $\rightarrow$  free electrons
- acceleration of electrons in electrical field ( $U_A$ : 100 – 150 kV)
- deceleration of electrons in anode  
(conversion: 99 %  $E_{\text{kin}} \rightarrow$  heat, 1 %  $\rightarrow$  x-rays)
- Bremsstrahlung, characteristic radiation
- vacuum ( $< 10^{-5}$  mbar); avoid interactions with molecules in air

# Imaging with x-rays

## generation of x-rays:



W. Crookes 1904

C: cathode (-)

A: anode (+)

$W_{in}$  and  $W_{out}$ : water inlet and outlet of the cooling device 31

## Imaging with x-rays

### x-ray frequency:

energy of accelerated electrons:

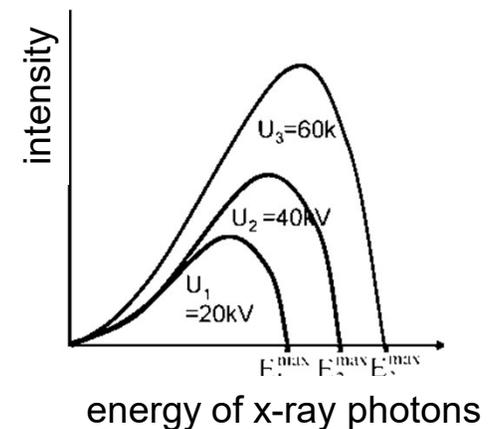
$$E_{kin} = e \cdot U_A$$

$$\text{with } E_{Photo} = h \cdot \nu$$

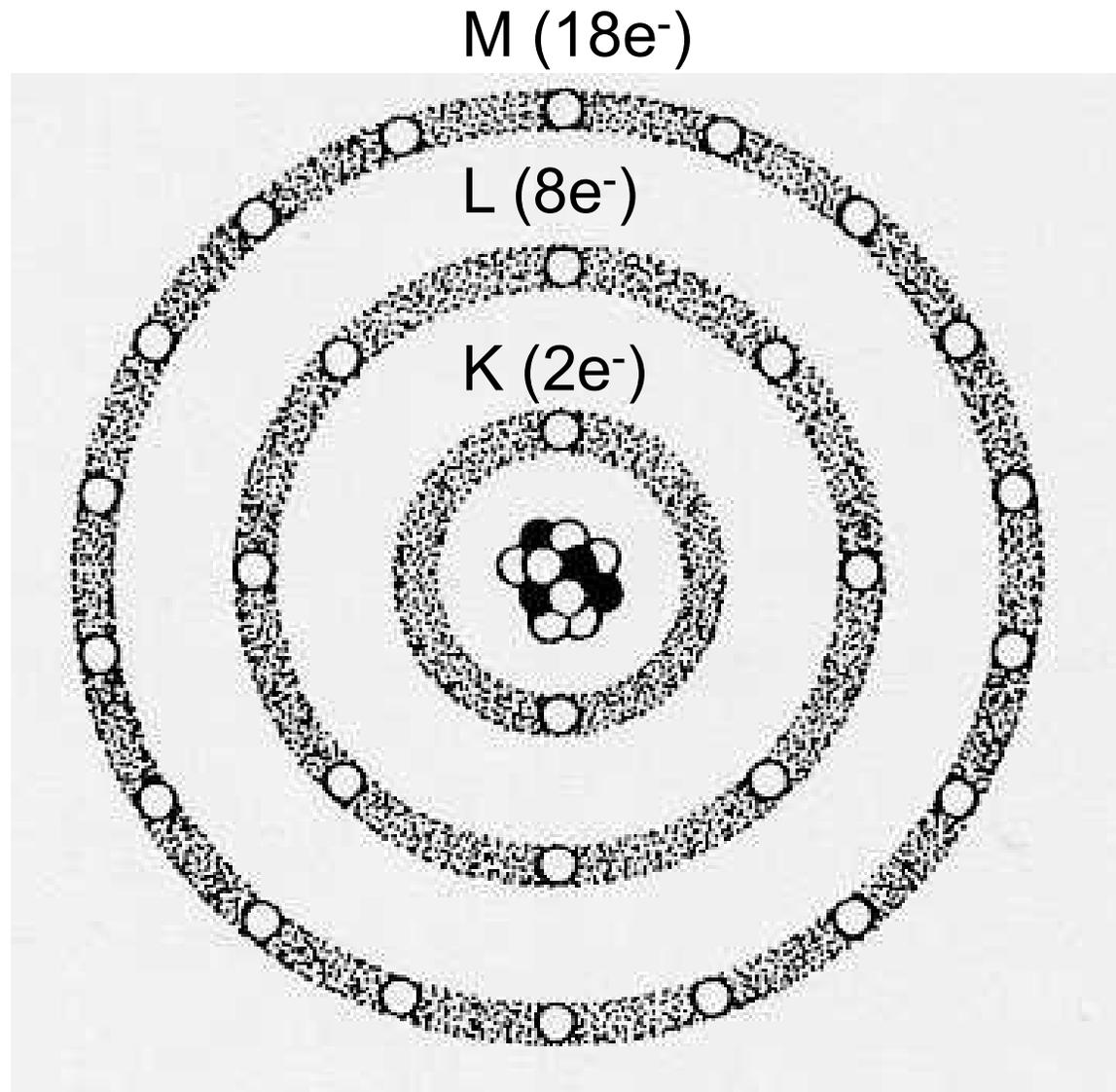
$$\Rightarrow \nu = \frac{e}{h} \cdot U_A$$

x-ray frequency depends linearly of acceleration voltage  $U_A$

$U_A$	$\lambda=1/\nu$	radiation strength
1 kV	1.242 nm	weak
10 kV	0.124 nm	medium
100 kV	0.012 nm	hard



**Bohr model**

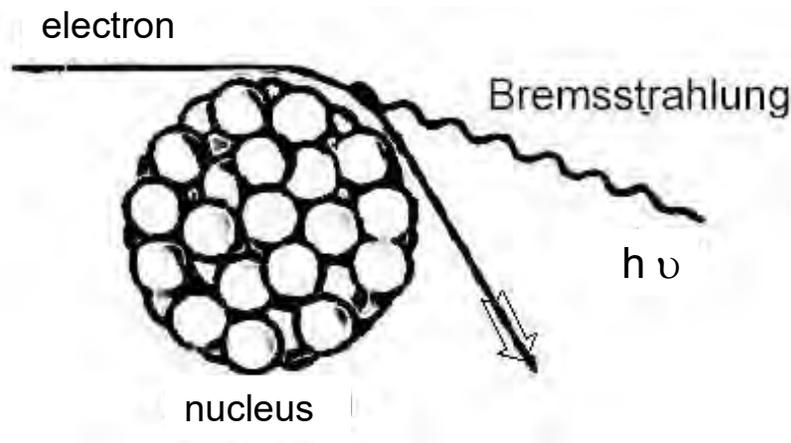


## x-ray energy

### 1. Bremsstrahlung:

accelerated electrons approach nucleus (between nucleus and K-shell)

- deflection (due to Coulomb potential of nucleus and shell electrons)
- deceleration ( $E_{\text{kin}}$  converted into e.m. energy)
- emission of energy in form of “Bremsstrahlung“



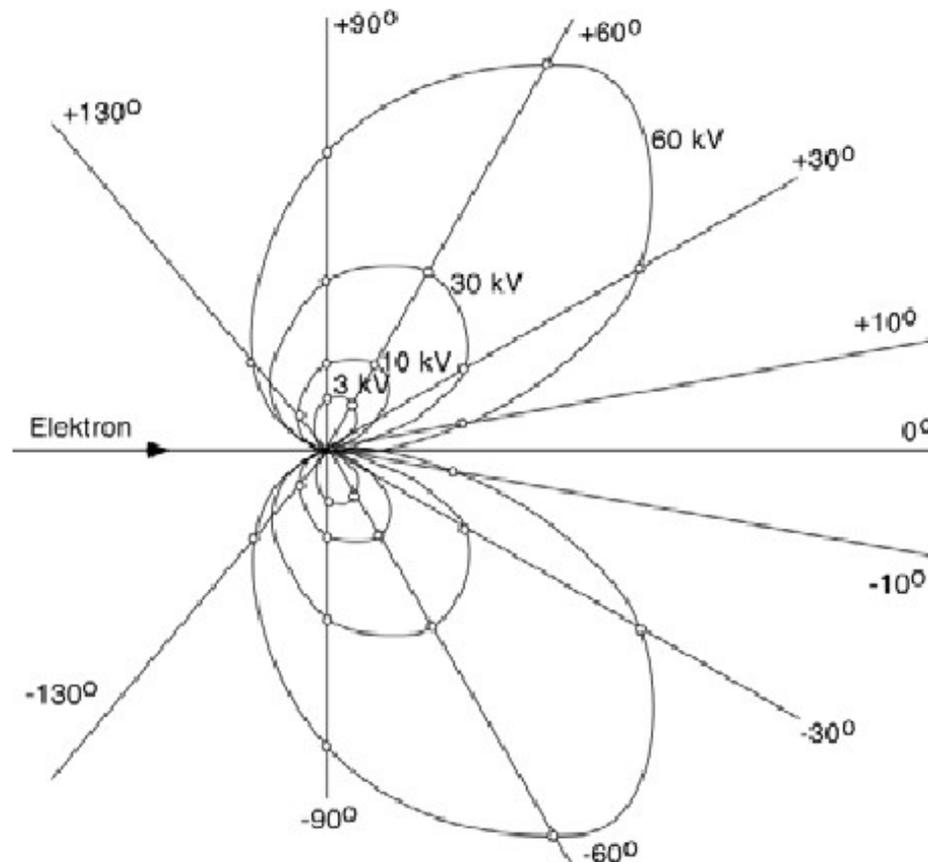
energy of Bremsstrahlung depends on trajectory of electrons

⇒ broad energy spectrum !

## x-ray energy

### 1. Bremsstrahlung:

spatial distribution of intensity of Bremsstrahlung (“radiation lobes”)



## x-ray energy

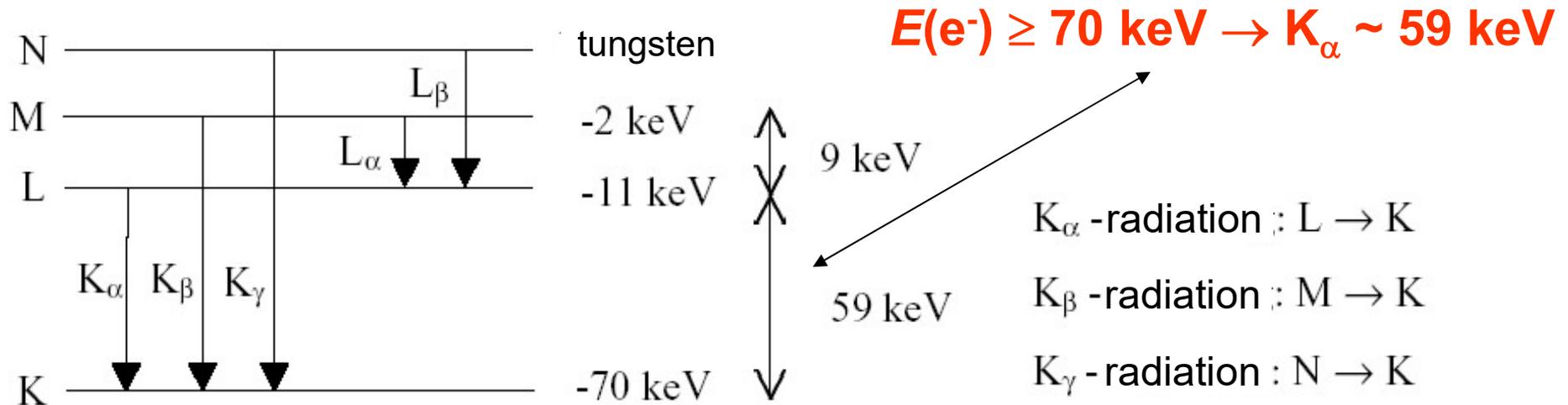
### 2. characteristic radiation:

accelerated electron strikes bound electron from K- (or L-)shell

⇒ ionization

vacant energy level (core hole) taken by electron from outer shell

emission of energy difference ( $h\nu = E_m - E_n$ ) as quantized photon with characteristic frequency  $\nu$



**energy of characteristic radiation solely material-dependent !**

## **Energie der Röntgenstrahlung:**

### **2. *characteristic radiation:***

energy of  $K_{\alpha}$ -radiation (Moseley's law):

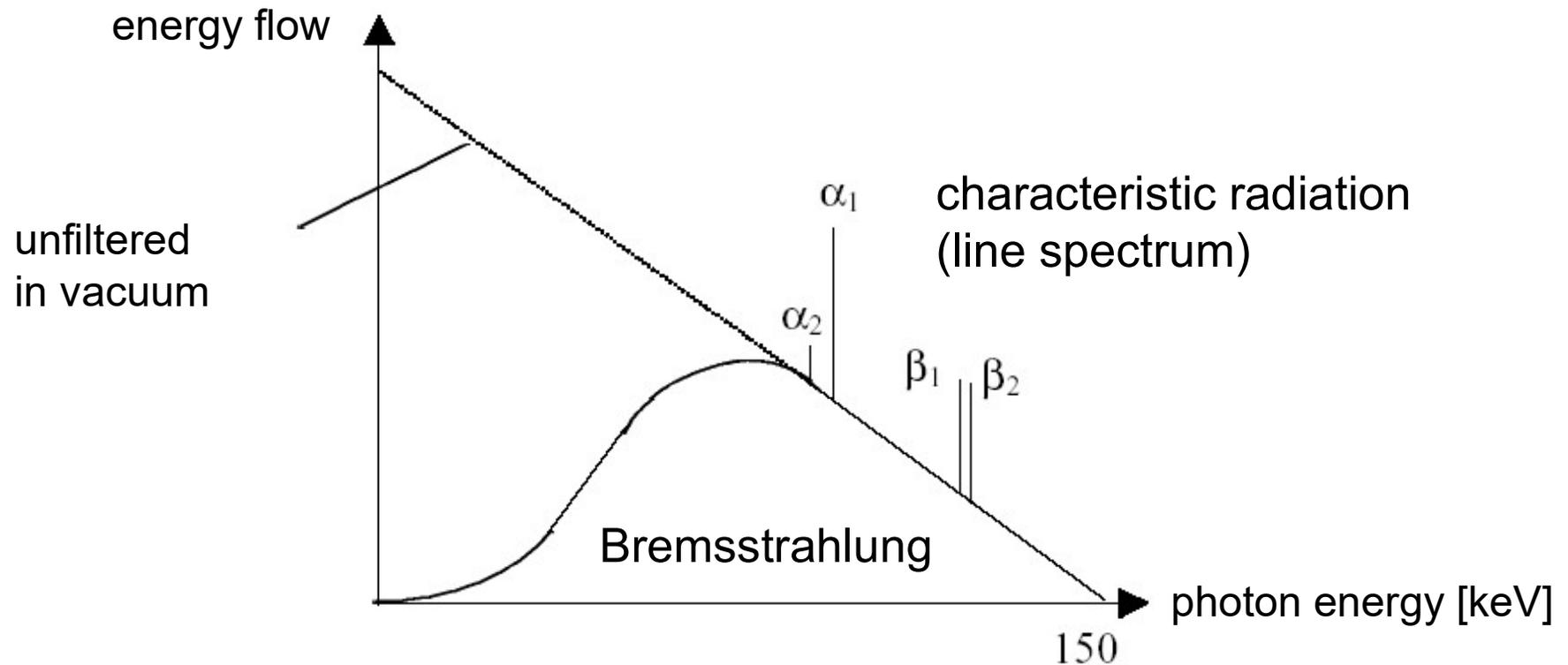
$$E_{K\alpha} = \frac{3}{4} \cdot R_{\infty} (Z - 1)^2$$

with  $R_{\infty}$  = Rydberg constant ( $3.29 \cdot 10^{15} \text{ s}^{-1}$ )

$Z$  = atomic number

## Energie der Röntgenstrahlung:

### 3. *full energy spectrum:*



## *Imaging with x-rays*

### **generation of x-rays:**

- frequency depends on acceleration voltage
- energy depends on material properties

⇒ requirements for anode material:

- high atomic number  $Z$  (yield increases with  $Z$ )
- high melting point  $T_{\max}$
- high heat conductivity  $\kappa$
  
- measure for quality =  $Z \cdot T_{\max} \cdot \kappa$

mostly used: tungsten or tungsten-rhenium

## *Imaging with x-rays*

### **generation of x-rays:**

quality criteria for x-ray sources in medical imaging

- high power  $\Rightarrow$  short exposition times
- small focus  $\Rightarrow$  acuity
- adjustable energy of quanta  $\Rightarrow$  contrast
- low production costs
- low maintenance, long lifetime

# Imaging with x-rays

## generation of x-rays:

quality criteria for x-ray sources in medical imaging

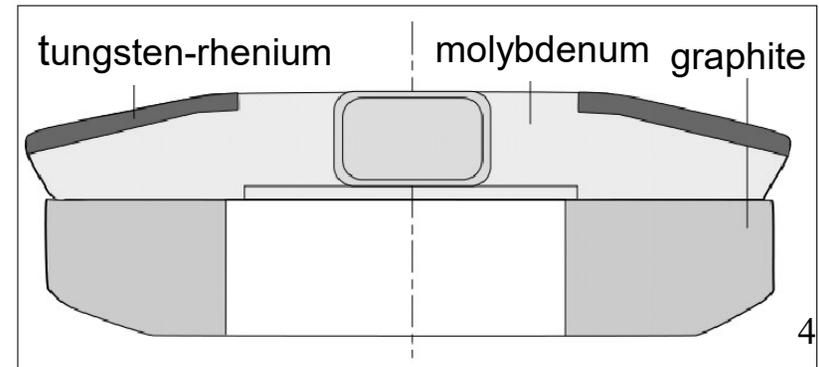
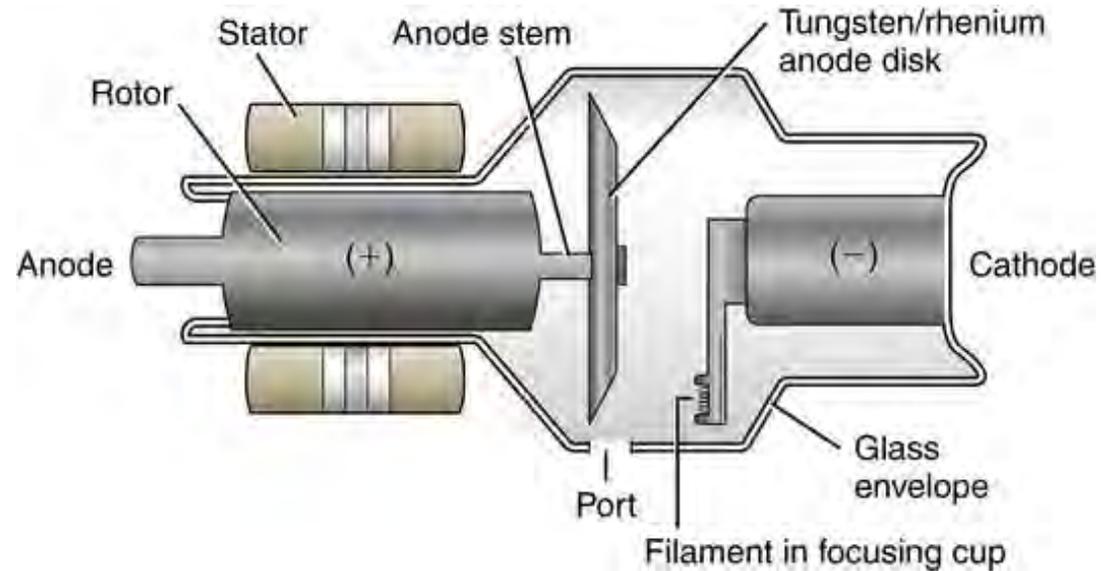
a high power and small focus

can be achieved with a

***tilted anode***

and with a

***rotating anode*** (heat dissipation)



# Imaging with x-rays

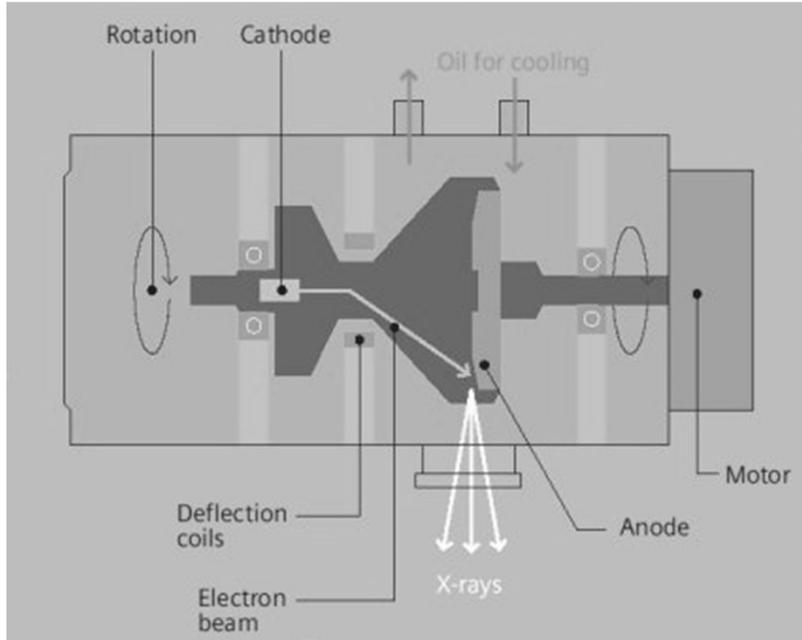
## anode material:

element	atomic number Z	max. temperature $T_{\max}$ [°C] @ $1.33 \cdot 10^{-2}$ Pa	heat conductance $\kappa$ [W cm <sup>-1</sup> K <sup>-1</sup> ]	stationary anodes		rotating anodes		
				$ZT_{\max}\kappa$	order	$\sqrt{\kappa\rho c}$	$ZT_{\max}\sqrt{\kappa\rho c}$	order
Cu	29	1032	3,98	119113	8	3,68	110135	10
Mo	42	2167	1,38	125599	7	1,88	171106	8
Ag	47	832	4,18	163450	4	3,18	124350	9
Ta	73	2587	0,55	103868	9	1,13	213402	6
W	74	2757	1,3	265223	1	1,81	369273	1
Re	75	2557	0,71	136160	6	1,38	264650	4
Os	76	2280	0,87	150754	5	1,77	306706	3
Ir	77	2220	1,46	249572	3	2,06	352136	2
Pt	78	1742	0,71	96472	10	1,41	191585	7
Au	79	(1063)	3,14	263687	2	2,81	235975	5
U	92	(1132)	0,25	26036	11	0,75	78108	11

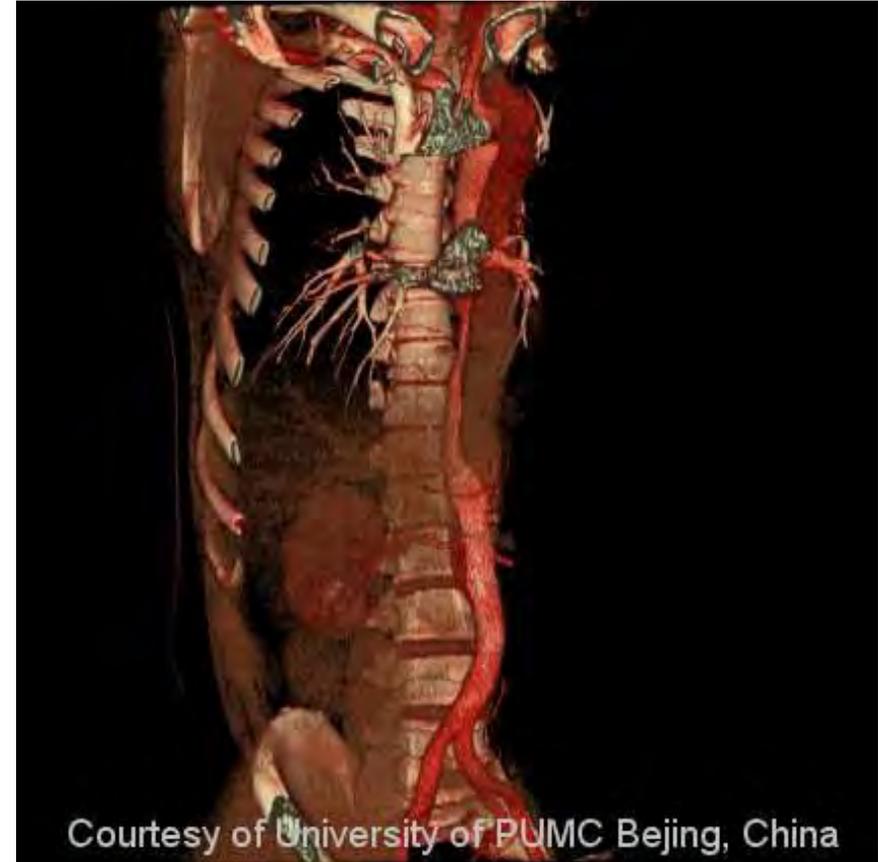
# Imaging with x-rays

## generation of x-rays:

example: Straton x-ray tube (Siemens, 2003)



- direct cooling of anode
- mechanics outside vacuum
- rotation time: 0.37 sec
- sub-mm volumen scans @ 500 mAs for 20 sec (64 mm/sec)
- dose reduction
- indep. of patient height and anatomy



## *Imaging with x-rays*

### **efficiency $\eta$ and radiation power $D$ :**

$$\eta \equiv \frac{\text{radiation power}}{\text{electrical power}} = k \cdot Z \cdot U_A \quad [\%]$$

where

$$k = 1.1 \cdot 10^{-9} \text{ [V}^{-1}\text{]}$$

$Z$  = atomic number of anode material

$U_A$  = acceleration voltage

example: tungsten anode,  $Z=74$ ,  $U_A=125$  kV  $\Rightarrow \eta = 1.02$  %  
(in praxis:  $< 1\%$  due to filtering and suppression; reminder: heat)

$$D \equiv Z \cdot I \cdot U_A^2$$

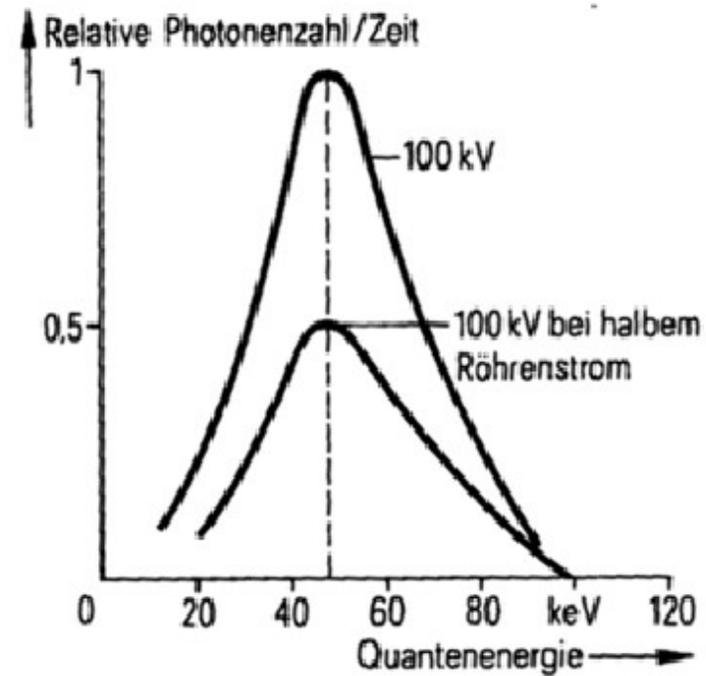
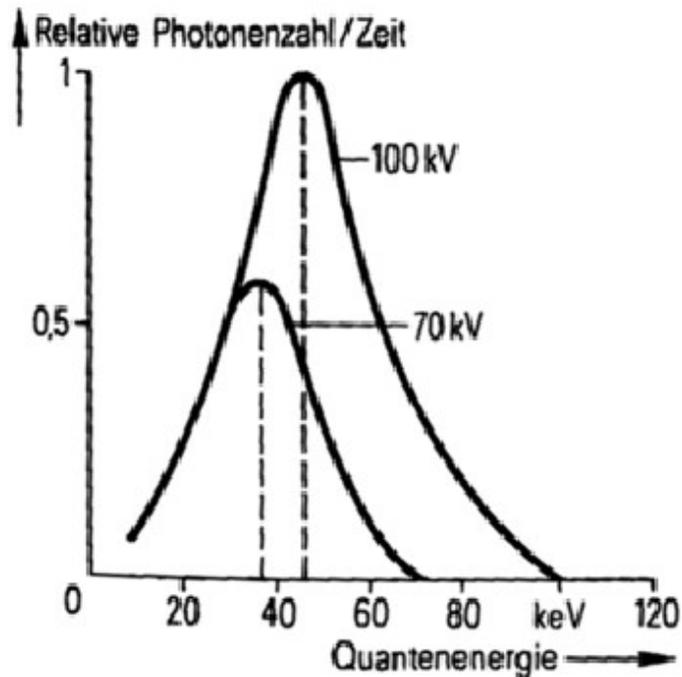
with

$I$  = current in tube (mostly fixed!)

## Imaging with x-rays

### Impact of acceleration voltage and tube current:

- flux density of x-ray radiation:  $\psi \sim Z \cdot I \cdot U_a^n$
- acc. voltage determines “strength“ of radiation potential impact (unfiltered:  $n=2$ ; with filter up to  $n=5$ )
- tube current determines number of photons/sec linear impact



# Imaging with x-rays

## x-rays:

### classification

radiation strength	$\lambda_{\min} - \lambda_{\max}$ [nm]	$f_{\min} - f_{\max}$ [GHz]	E [keV]
extra weak	0.25 - 0.06	$1.2 \cdot 10^9 - 3.3 \cdot 10^9$	5.0 - 13.6
weak	0.06 - 0.02	$3.3 \cdot 10^9 - 1.5 \cdot 10^{10}$	13.6 - 62
medium	0.02 - 0.01	$1.5 \cdot 10^{10} - 3.0 \cdot 10^{10}$	62 - 124
hard	0.01 - 0.005	$3.0 \cdot 10^{10} - 6.0 \cdot 10^{10}$	124 - 248
extra hard	< 0.005	$> 6.0 \cdot 10^{10}$	> 248

typical CT x-ray tube:

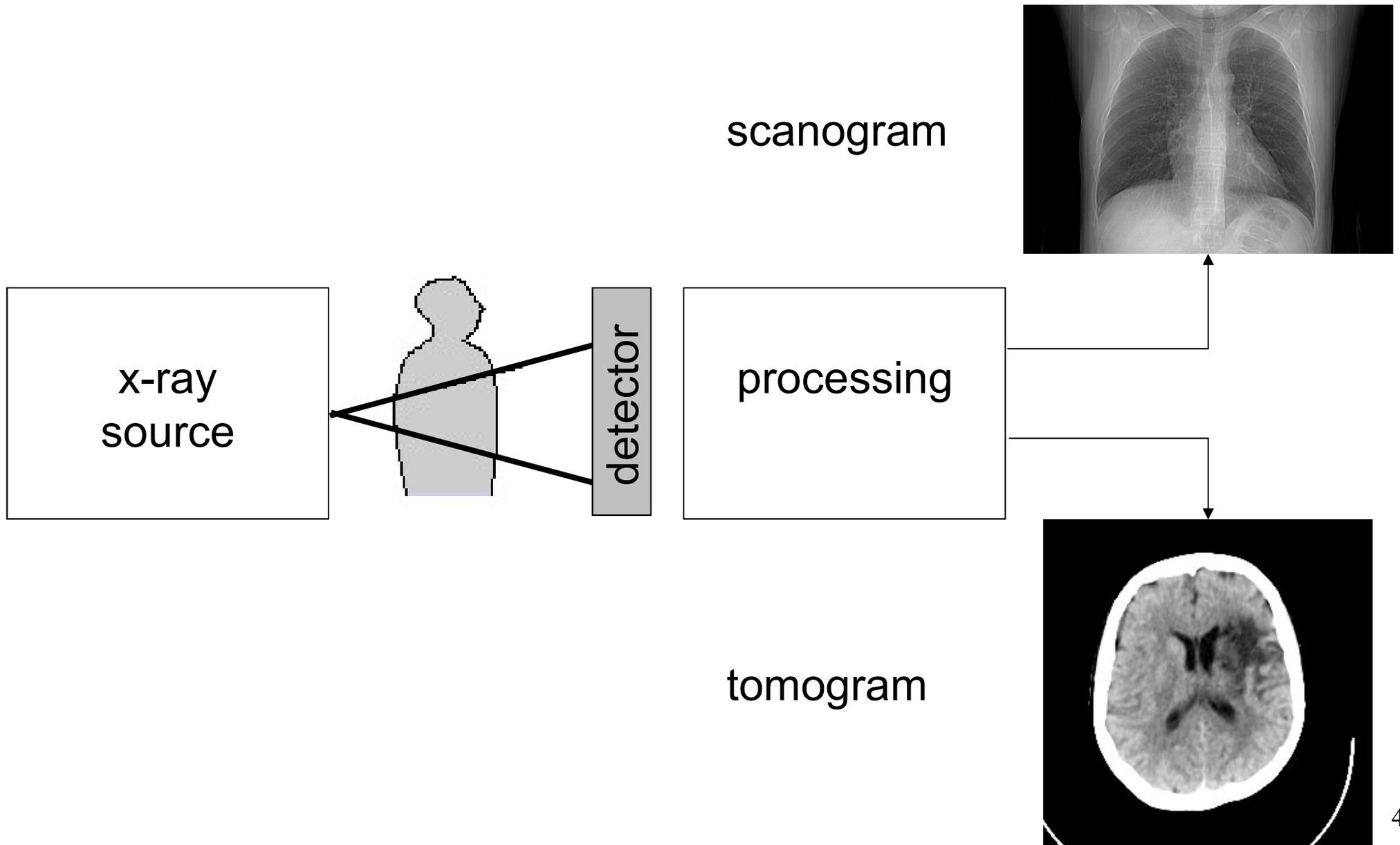
acceleration voltage 120 kV

tungsten anode: ~ 20 - 120keV

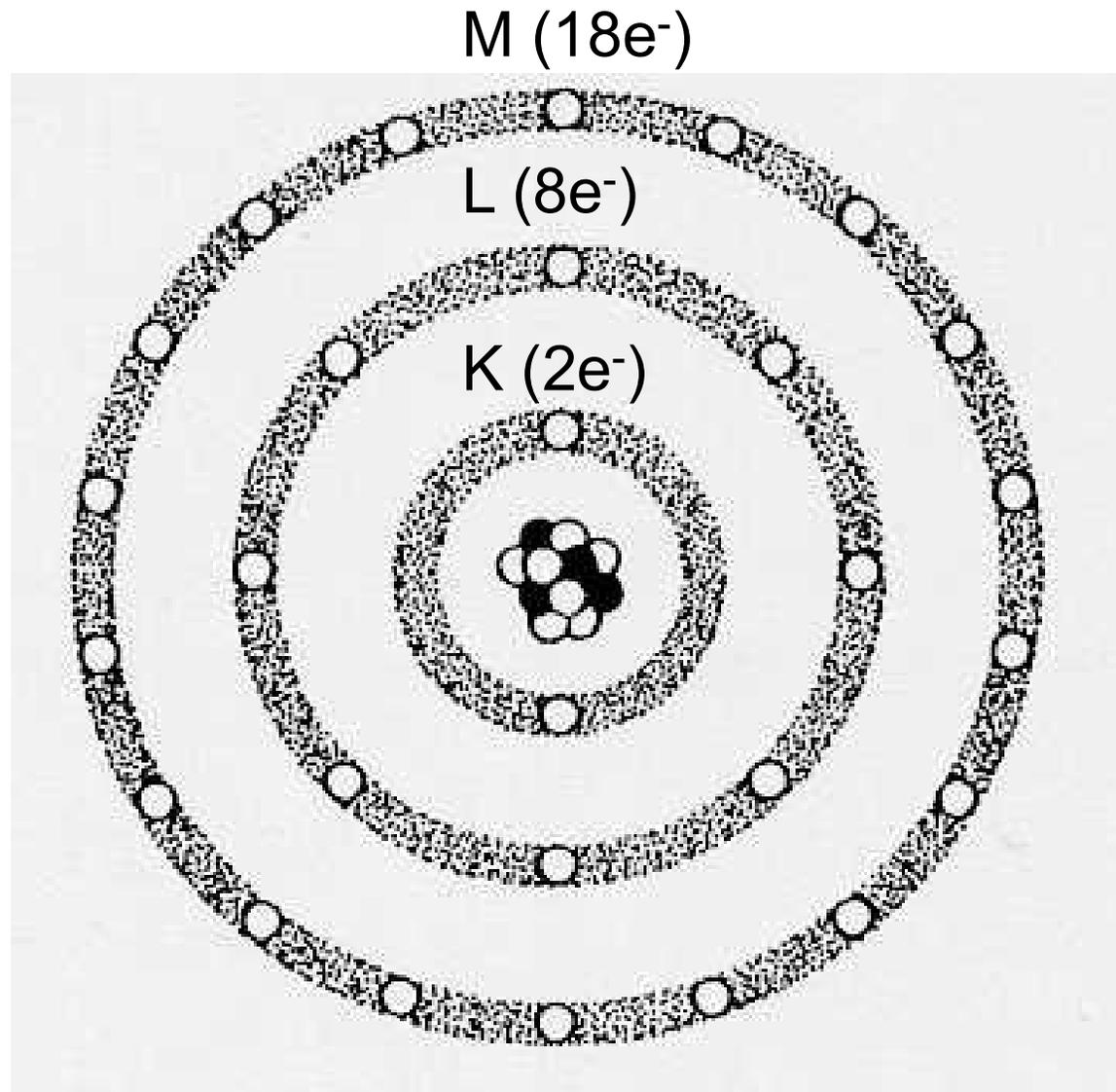
onset of ionization of living tissue at 15 eV !!

**interactions  
with  
matter**

# Imaging with x-rays



**Bohr model**

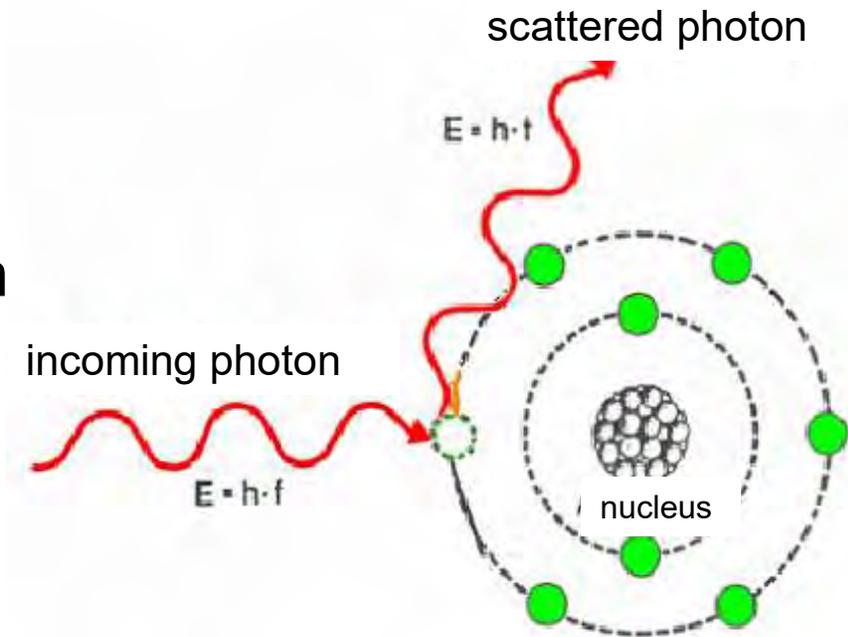


## Imaging with x-rays

### interaction with matter

- incoming photon interacts with object and changes its trajectory, but
  - no absorption
  - no change of energy of photon
- size of scattering body  $\ll$  wave length
- probability of occurrence:  
~ 5 % of applied x-rays
- disadvantageous for imaging: background noise (“*film fog*”)

### coherent scattering



# Imaging with x-rays

## interaction with matter

- photon transfers its full energy to shell electron (*K*- and *L*-shell)
- effect depends on photon energy
- effect size  $\sim 1/E^3$  (at high energies)

- energy balance:  $h \cdot f = 1/2 m_e v^2 + E_a$

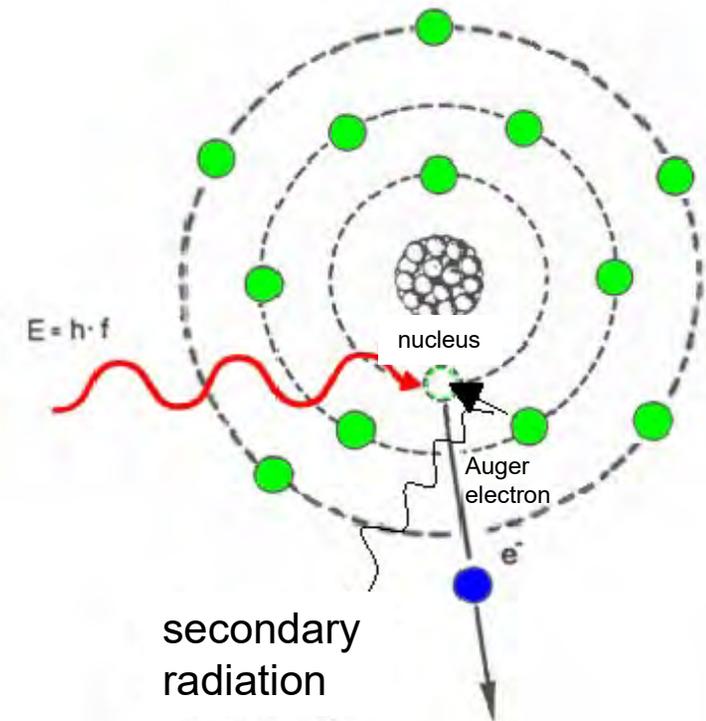
- secondary radiation when core hole is filled up with electron from outer shell (Auger electron)

probability of occurrence  $\sim Z^3$

$\Rightarrow$  amplifies absorption differences of different tissues !

$\Rightarrow$  important for diagnostic radiology !

## photo effect



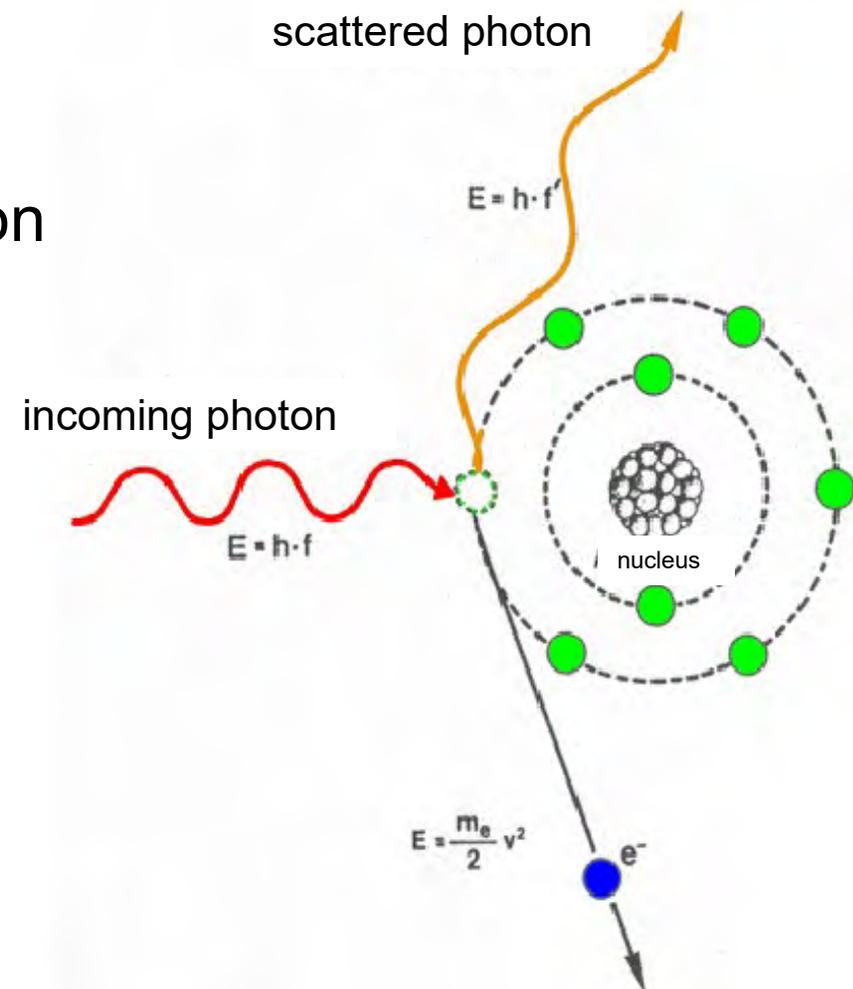
## Imaging with x-rays

### interaction with matter

- $E_\gamma < 1.022 \text{ MeV}$ :
- amount of energy transferred to electron depends on scattering angle  $\varphi$
- higher probability for electrons in outer shells (binding energy irrelevant)
- energy balance:

$$E_\gamma + E_{0e} = E_{\gamma'} + E_e$$

### Compton effect



## *Imaging with x-rays*

### **interaction with matter**

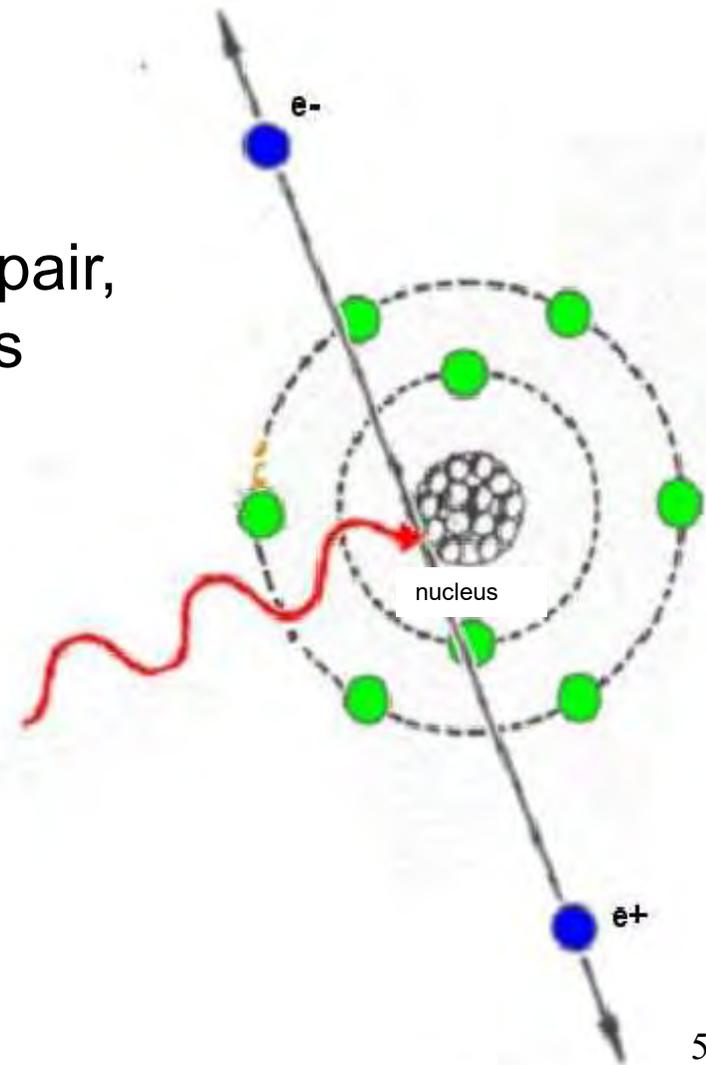
-  $E_\gamma \geq 1.022 \text{ MeV}$ :

- production of an electron ( $e^-$ ) / positron ( $e^+$ ) pair, if energy of incoming photon equals/exceeds twice the rest energy of an electron

- energy balance:

$$E_\gamma = E_e + E_p + 2m_e c^2$$

### ***pair production***



## *Imaging with x-rays*

### summary of types of interaction

#### **1. photo effect:**

$\gamma$ -quant transmits its full energy to a shell-electron

⇒ **absorption**

#### **2. Compton effect:**

scattering on electron; scattered radiation has lower energy and different trajectory

⇒ **scattering**

#### **3. pair production:**

radiation ( $E \geq 1.022$  MeV and if near nucleus) is being transformed into electron and positron

⇒ **transformation radiation → matter**

⇒

**attenuation = absorption + scattering**

## *Imaging with x-rays*

### **quantitative assessment of attenuation**

### **absorption law**

particle rate:  $N = \frac{\text{particles}}{\text{time}} = \frac{\Delta n}{\Delta t}$

intensity:  $I = \frac{\text{energy}}{\text{area} \cdot \text{time}} = \frac{E}{\Delta A \cdot \Delta t}$

with mono-energetic radiation:  $E = \Delta n \cdot E_\gamma$

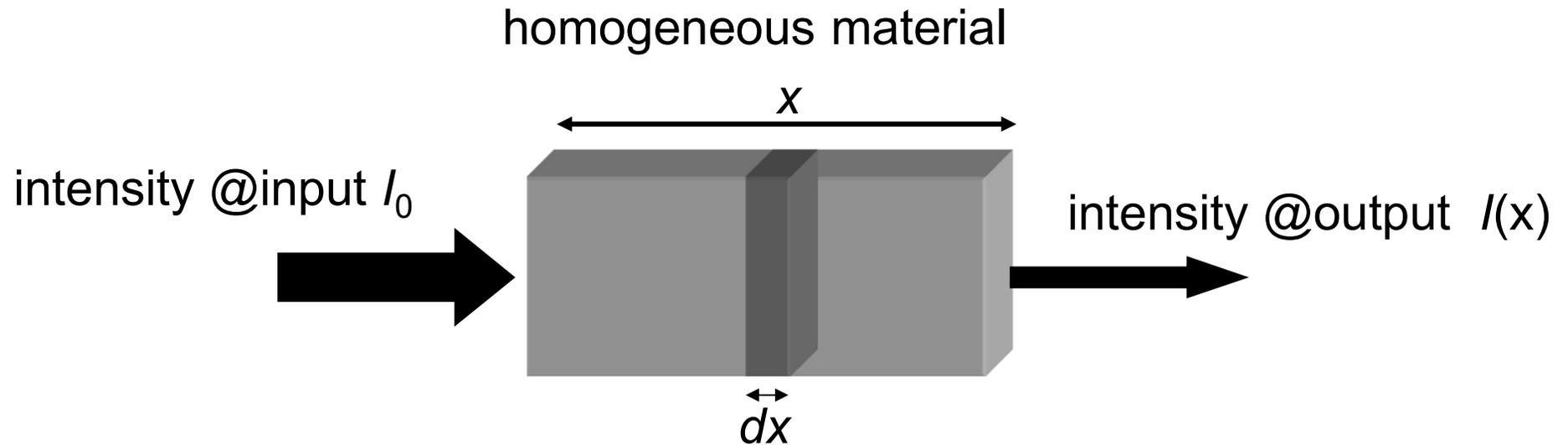
$$\Rightarrow I = \frac{E_\gamma \cdot \Delta n}{\Delta A \cdot \Delta t} = \frac{E_\gamma}{\Delta A} \cdot N$$

$$\Rightarrow I \propto N$$

# Imaging with x-rays

## quantitative assessment of attenuation

## absorption law (Lambert's law)



$$dN = -\mu \cdot N \cdot dx$$

$\mu$  = linear absorption coefficient

$$\Rightarrow N(x) = N_0 \cdot e^{-\mu \cdot x}$$

$$\Rightarrow I(x) = I_0 \cdot e^{-\mu \cdot x}$$

## Imaging with x-rays

### quantitative assessment of attenuation

### absorption law

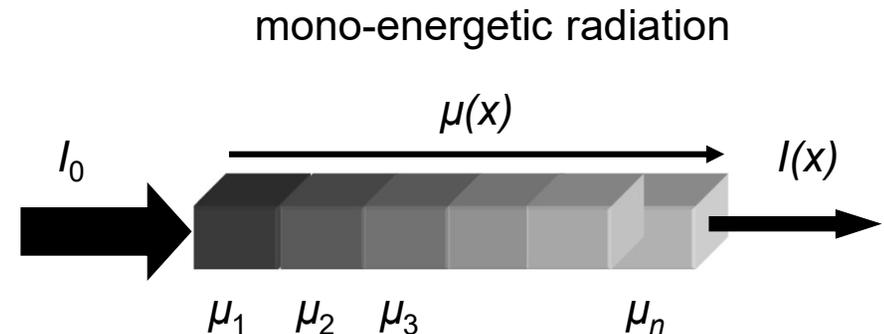
for **fixed**  $E_\gamma$ , we have in general:

$$dN = -\mu(x, y, E_\gamma, \rho, Z) \cdot N \cdot dx \Leftrightarrow \frac{dN}{N} = -\mu(x, y, E_\gamma, \rho, Z) \cdot dx$$

$$\Leftrightarrow \int_{N_0}^N \frac{1}{N} dN = -\int_0^x \mu(x, y, E_\gamma, \rho, Z) dx$$

$$\Leftrightarrow \ln\left(\frac{N}{N_0}\right) = -\int_0^x \mu dx \Leftrightarrow N = N_0 \cdot \exp\left(-\int_0^x \mu dx\right)$$

$$\Rightarrow I = I_0 \cdot \exp\left(-\int_0^x \mu dx\right)$$



# Imaging with x-rays

## attenuation coefficient $\mu$

## absorption law

in general, we have:

$$\mu = \tau + \sigma + (\chi)$$

photo effect      Compton effect      pair production

$$\mu = \frac{\rho}{A} \cdot N_A \cdot \mu' = \frac{\rho}{A} \cdot N_A \cdot (\tau' + \sigma')$$

$\mu'$  = atomic cross-section

where  $\tau' = \tau'(E_\gamma, Z) = Z^5 \cdot C(Z) \cdot \tau'_0(E_\gamma)$

and  $\sigma' = \sigma'(E_\gamma, Z) = Z \cdot \sigma'_0(E_\gamma)$

## Imaging with x-rays

### attenuation coefficient $\mu$

### absorption law

in general:  $\mu_{ges} = \mu_{photo} + \mu_{compt} + \mu_{pair}$  [cm<sup>-1</sup>]

$$\mu_{photo} = \frac{Z^{3.8}}{E_{\gamma}^3}$$

$$\mu_{compt} \approx \frac{Z}{E_{\gamma}}$$

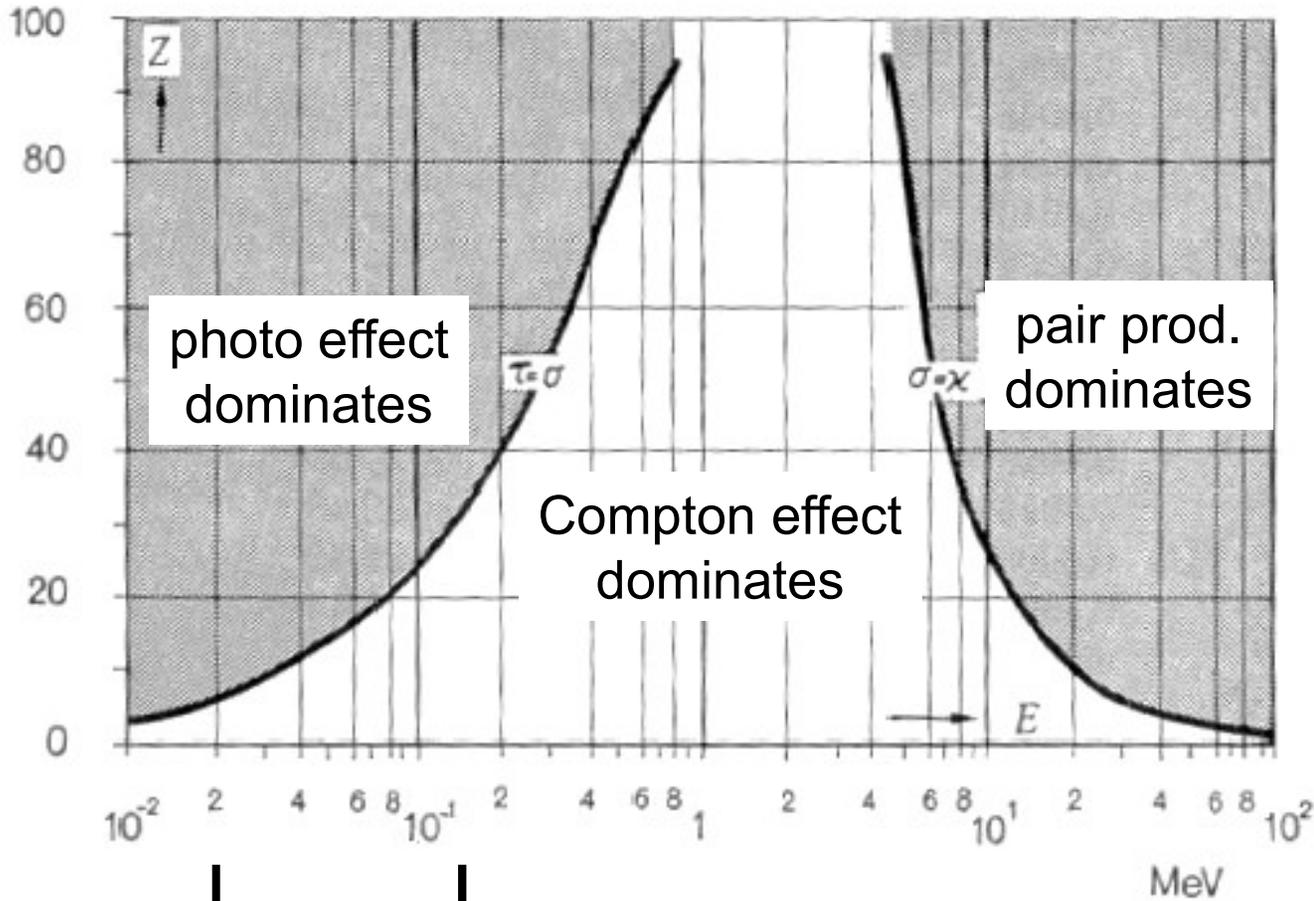
$$\mu_{pair} \approx Z^2 \ln E_{\gamma}$$

(alternatively: mass absorption coefficient  $\mu/\rho$  [cm<sup>2</sup>/g])

# Imaging with x-rays

attenuation coefficient  $\mu$

Z dependence



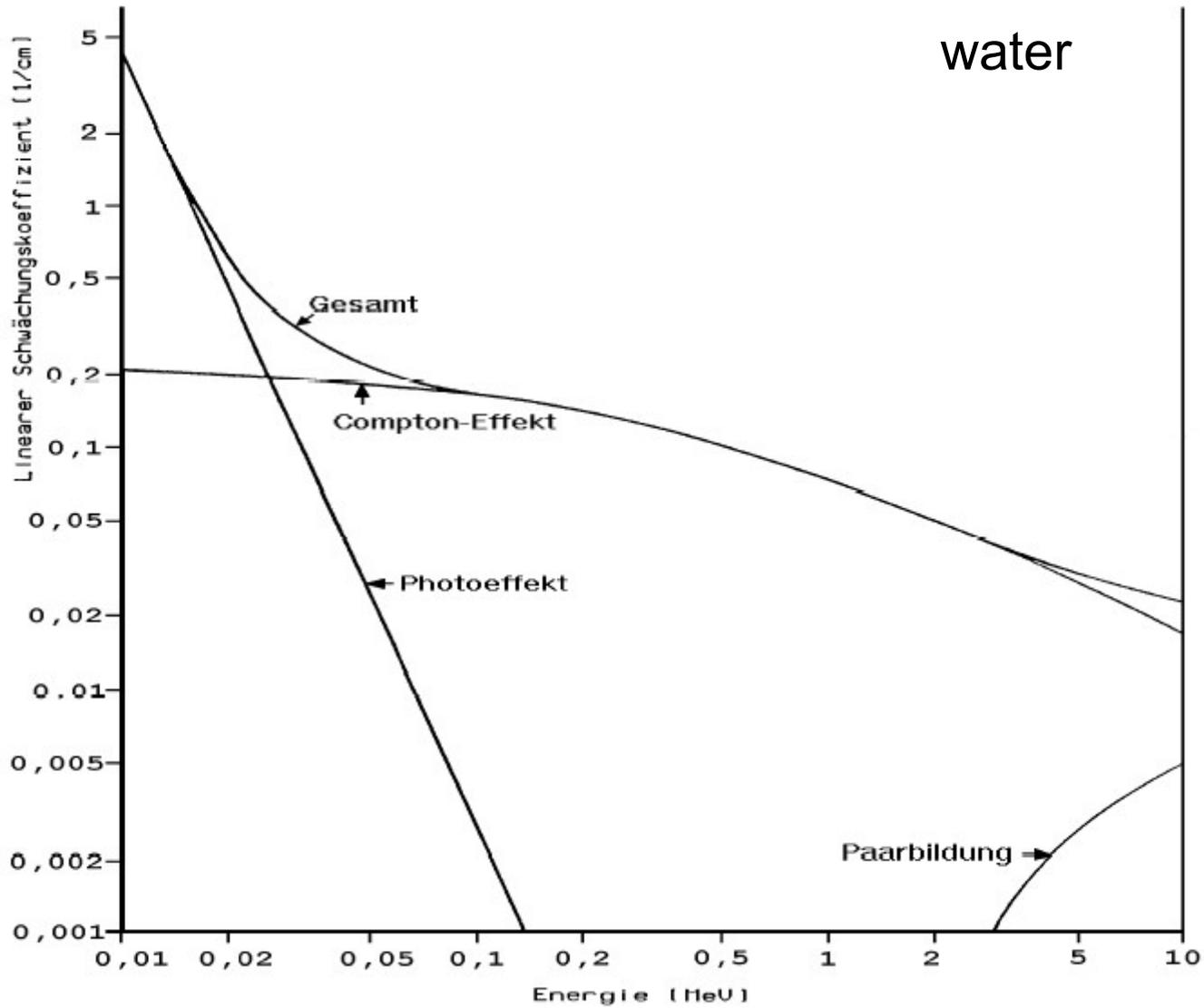
20 - 120keV

Abb. 6-15. Die Bereiche mit überwiegendem Photoeffekt, Compton-Effekt und Paarbildungseffekt werden durch die Kurven begrenzt, längs deren der Photoabsorptionskoeffizient  $\tau$  gleich dem Streukoeffizienten  $\sigma$  bzw. der Streukoeffizient  $\sigma$  gleich dem Paarbildungskoeffizienten  $\chi$  als Funktion der Ordnungszahl  $Z$  und der Photonenenergie  $E$  ist

# Imaging with x-rays

attenuation coefficient  $\mu$

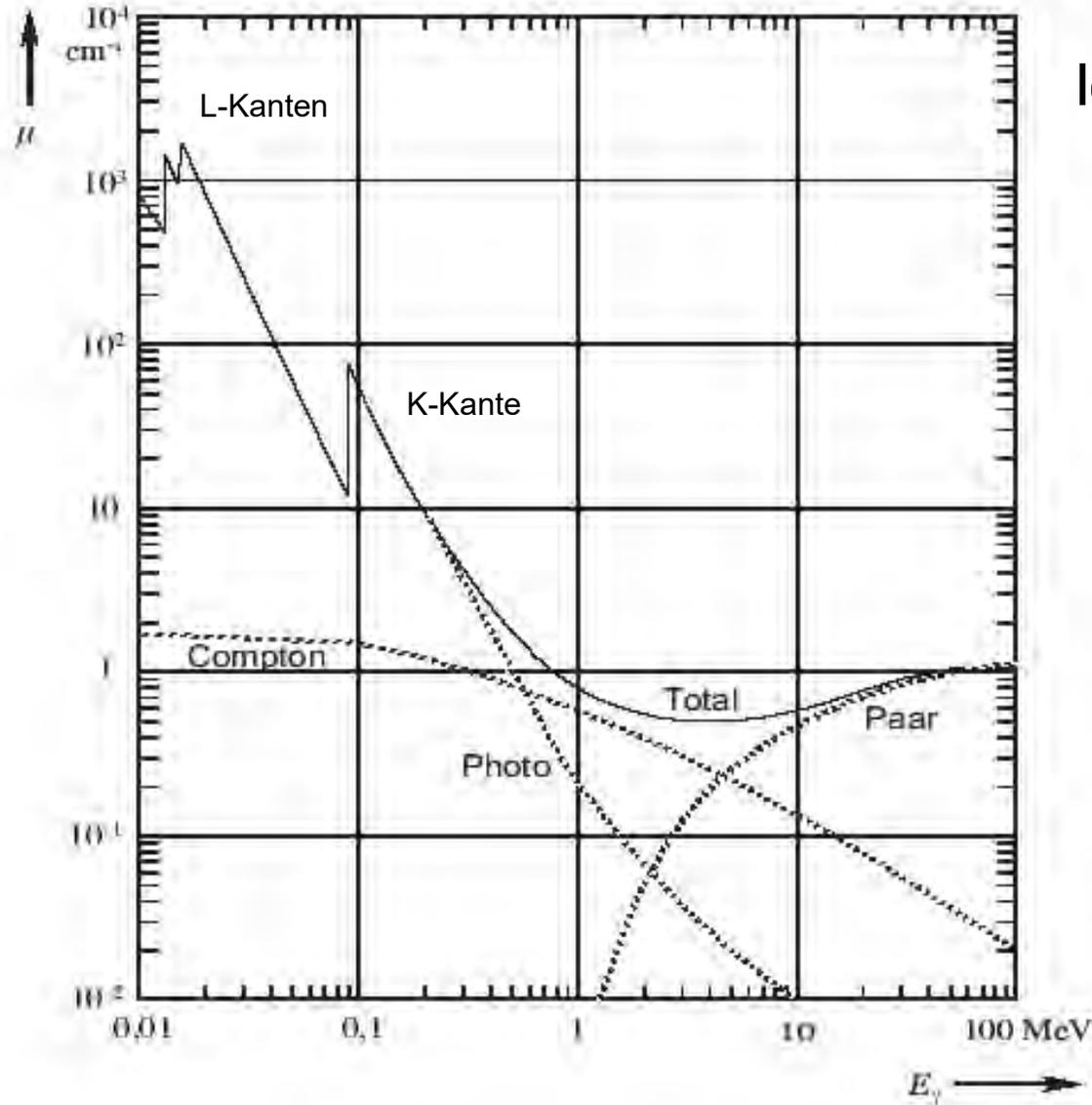
Z dependence



# Imaging with x-rays

attenuation coefficient  $\mu$

Z dependence



## *Imaging with x-rays*

### **mass absorption coefficient**

number of absorbed resp. scattered particles is proportional to the density of the absorber

mass absorption coefficient  $\mu' = \mu/\rho$  [cm<sup>2</sup>/g] equals absorption coefficient, if absorber has density  $\rho=1$

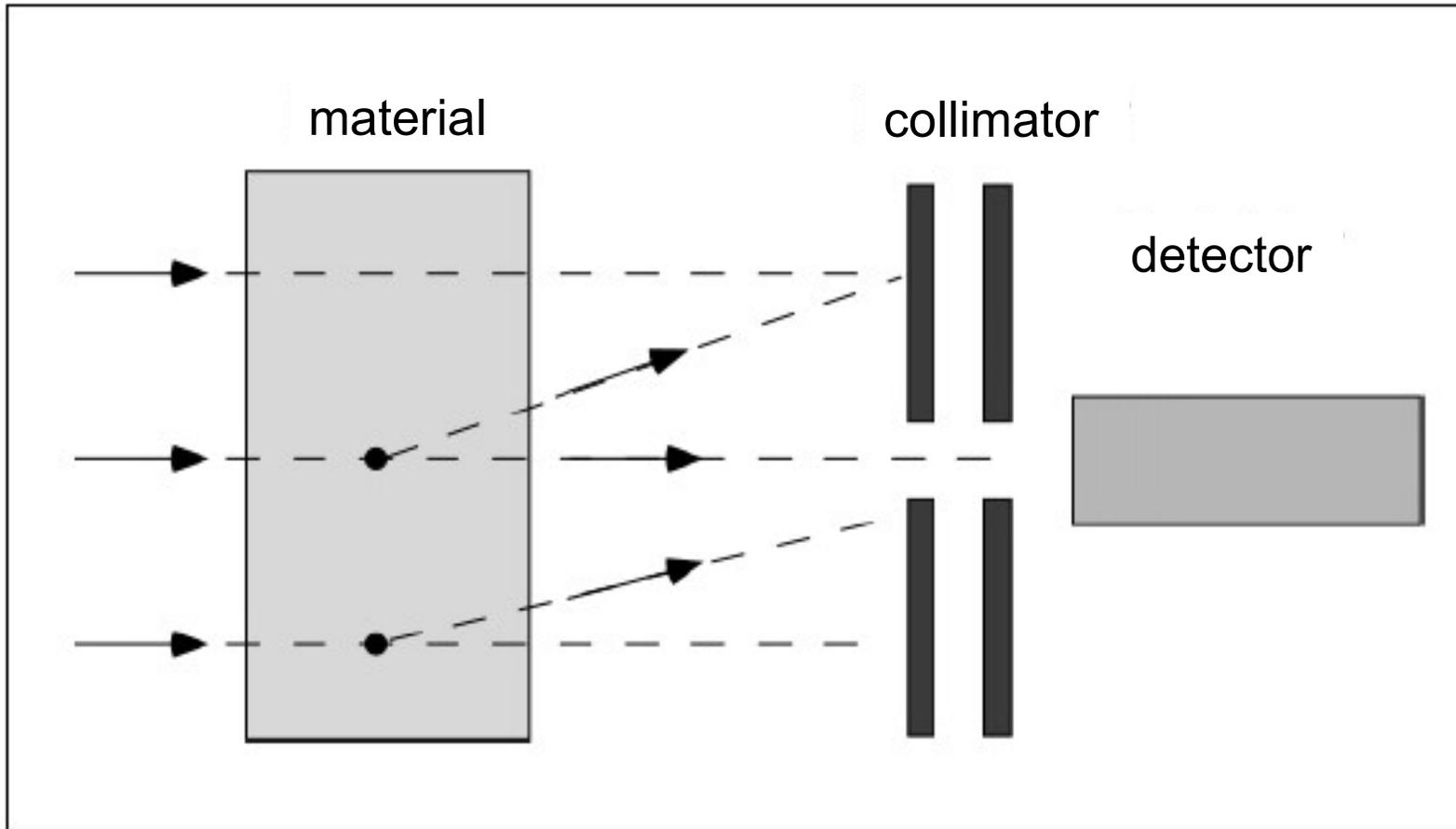
for mixed elements, we have:

$$\mu' = \sum_i \left( \frac{\mu}{\rho} \right)_i \cdot p_i \quad p_i = \text{mass contribution of } i\text{-th element}$$

$$\sum_i p_i = 1$$

# *Imaging with x-rays*

## **experimental set-up to measure attenuation coefficient $\mu$**



# Imaging with x-rays

## attenuation coefficient $\mu$

## absorption law

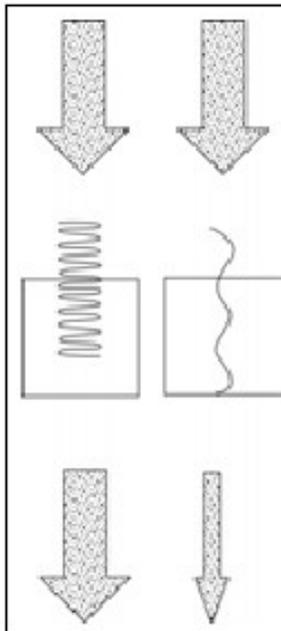
schematic model of factors that contribute to attenuation

wave length  $\lambda$

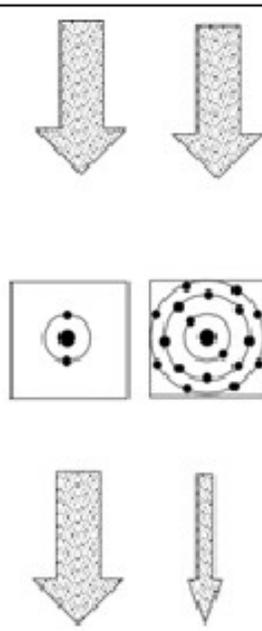
atomic number  $Z$

density  $\rho$  (spec. wght)

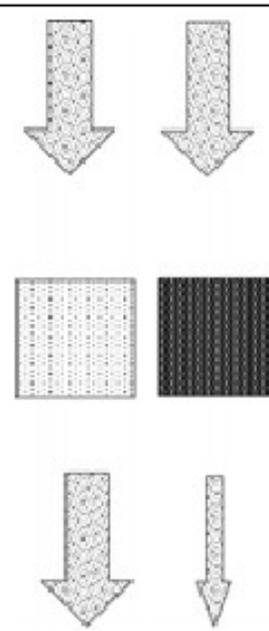
thickness  $d$



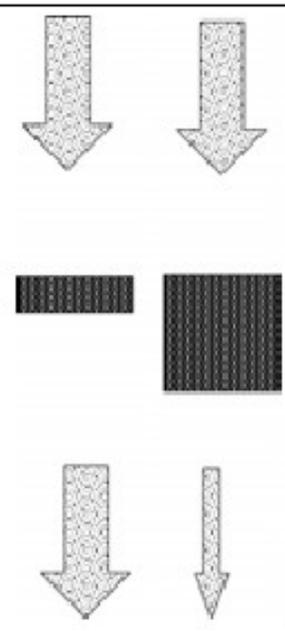
attenuation increases with  $\lambda^3$



attenuation increases with  $Z^3$



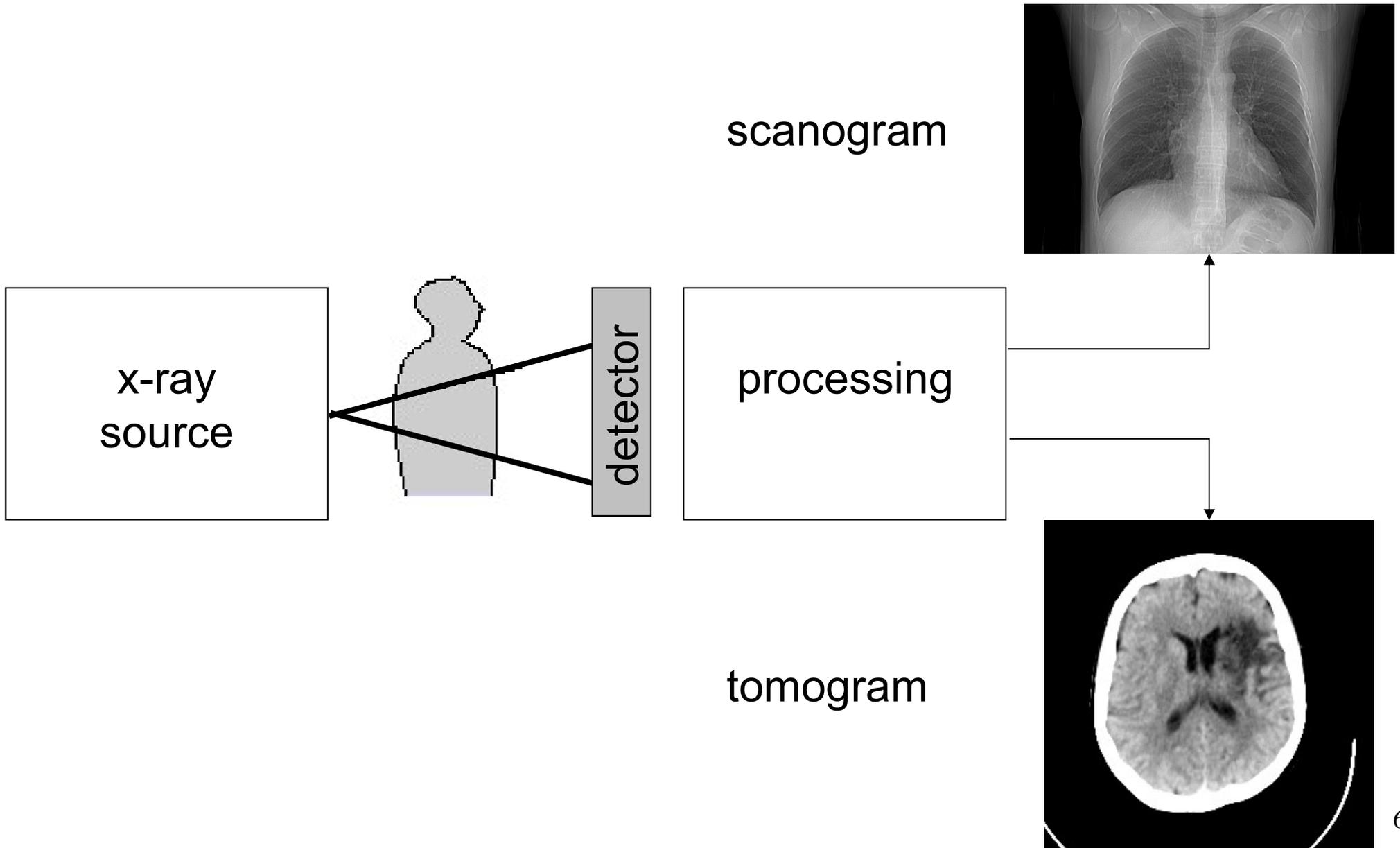
attenuation increases linearly with  $\rho$



attenuation increases linearly with  $d$

# **detectors**

# Imaging with x-rays



## *Imaging with x-rays*

### **detectors**

for projection radiography

- x-ray film
- intensifying screen (amplifying foil)
- storage phosphor plate (digital luminescence radiography; DLR)
- selenium film (xeroradiography)
- CCD camera
- x-ray image amplifier

for computed tomography (CT)

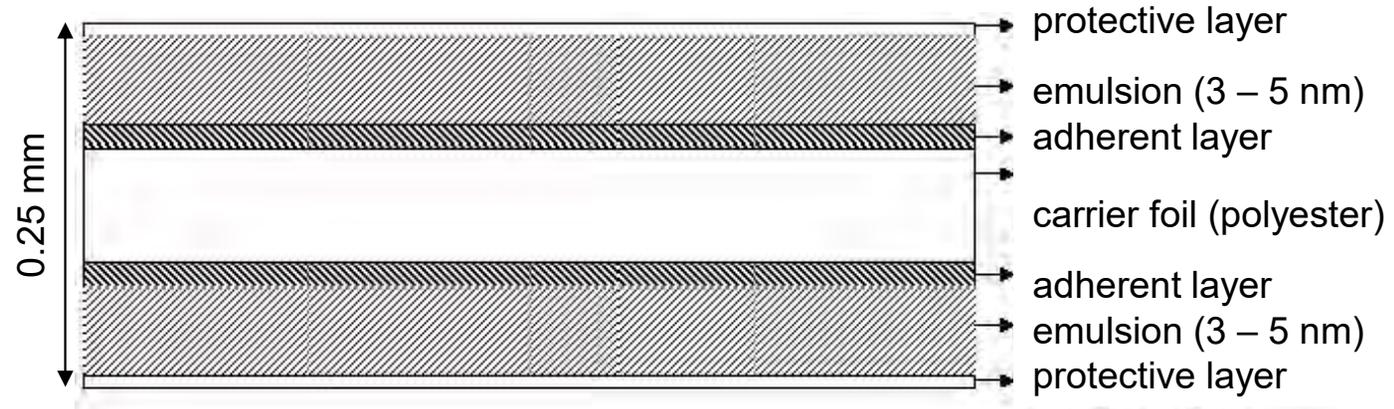
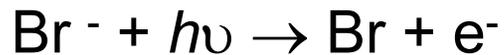
- gas detectors
- solid-state detectors

# Imaging with x-rays

## x-ray film

double layer film

emulsion:  
silver bromide crystals



released  $\text{Ag}^+$  ions nucleate at exposed regions

processing: reduction of nucleated ions to silver ( $\text{Ag}^+ + e^- \rightarrow \text{Ag}$ )

resolution:  $\geq 0.025$  mm

high  $\mu$  - low blackening

blackening depends on attenuation coefficient  $\mu$ , exposition time, dose

only 1% of quanta contribute to image !!

# Imaging with x-rays

## intensifying screen (amplifying foil)

improved usage of dose

convert x-rays into visible light

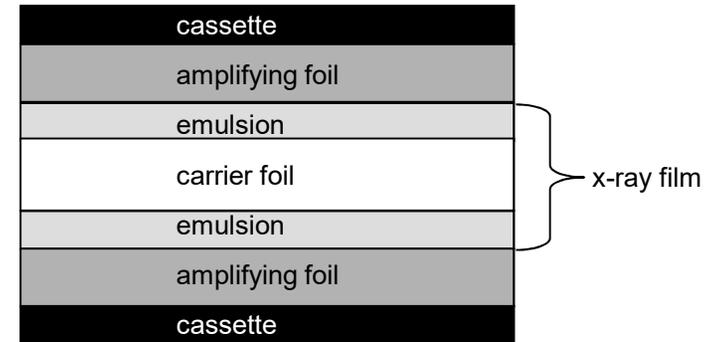
luminescence:

- $\gamma$ -quant generates free  $e^-$
- excited  $e^-$  relaxes to ground state by emission of light

amplification factor V: dose without foil/dose with foil (typical: 10-20)

requirements: high absorption, high yield of quanta,  
sufficient adjustment of spectrum to film sensitivity

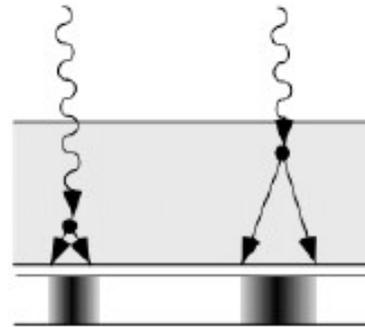
materials: calcium wolframite ( $\text{CaWO}_4$ ) cross-section (cs): 4%  
lanthanum oxybromide (doped w terbium)  $\text{LaOBr:Tb}$  cs 13 %  
gadolinium sulfide (doped w terbium)  $\text{Ga}_2\text{O}_2\text{S:Tb}$  cs 19 %  
70



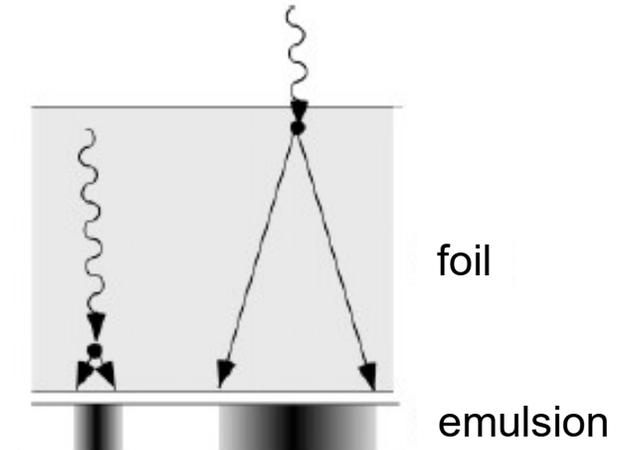
# Imaging with x-rays

## intensifying screen (amplifying foil)

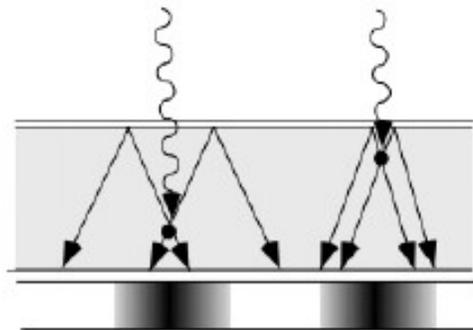
foil thickness  
and  
image fuzziness



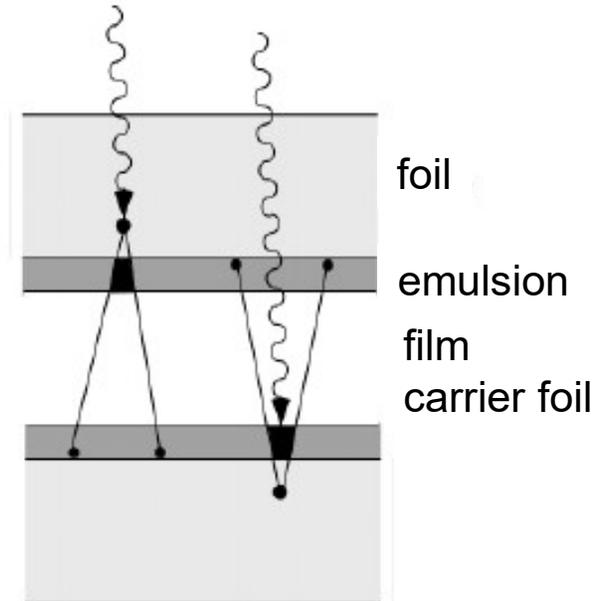
thin foil



thick foil



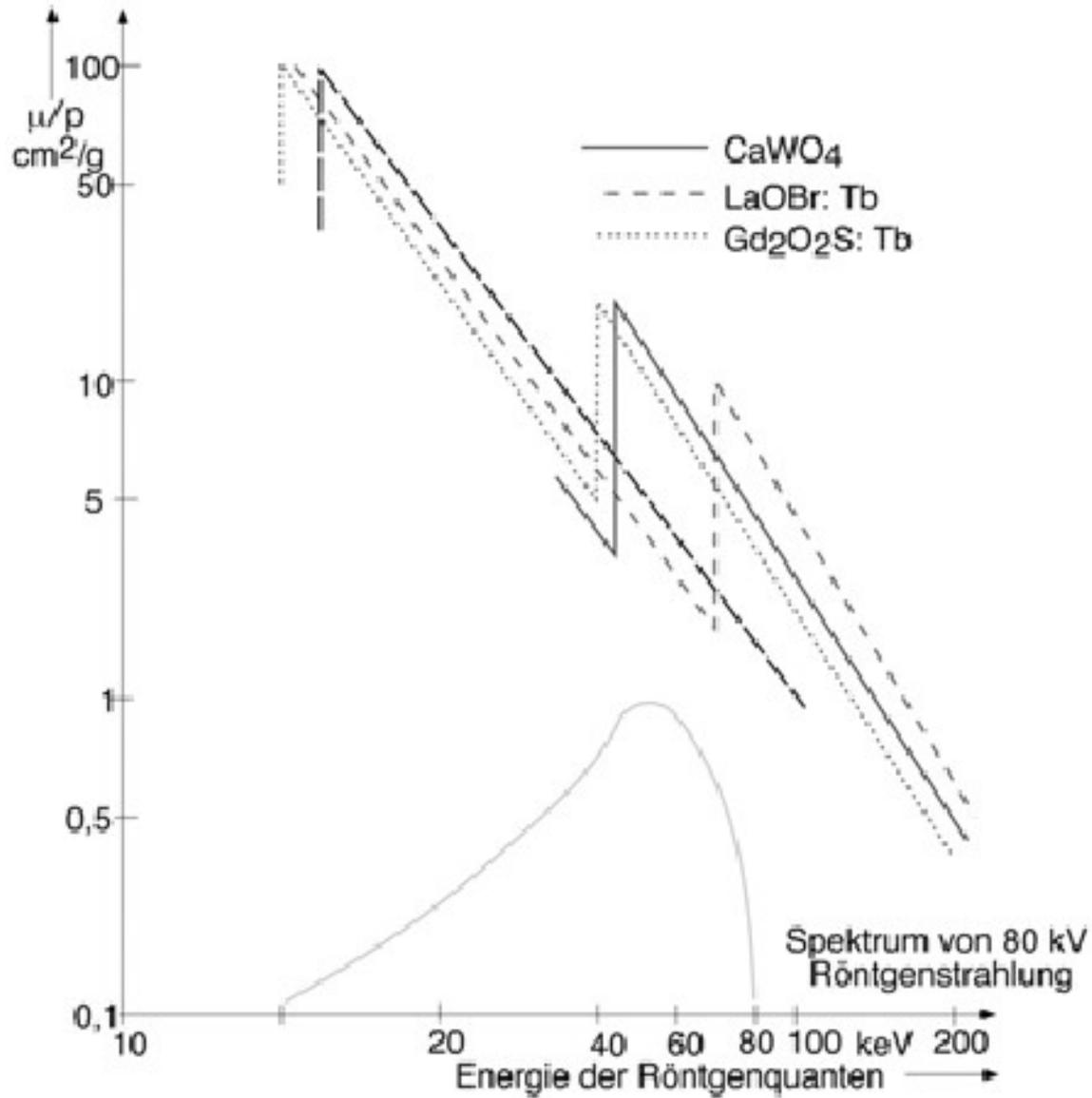
reflection effect



cross-over effect

# Imaging with x-rays

## mass absorption coefficient of different materials



## *Imaging with x-rays*

### **storage phosphor plate (digital luminescence radiography)**

same general principle as with amplifying foils (luminescence)

difference: excited  $e^-$  do not reach ground state (optically forbidden), generation of “traps“

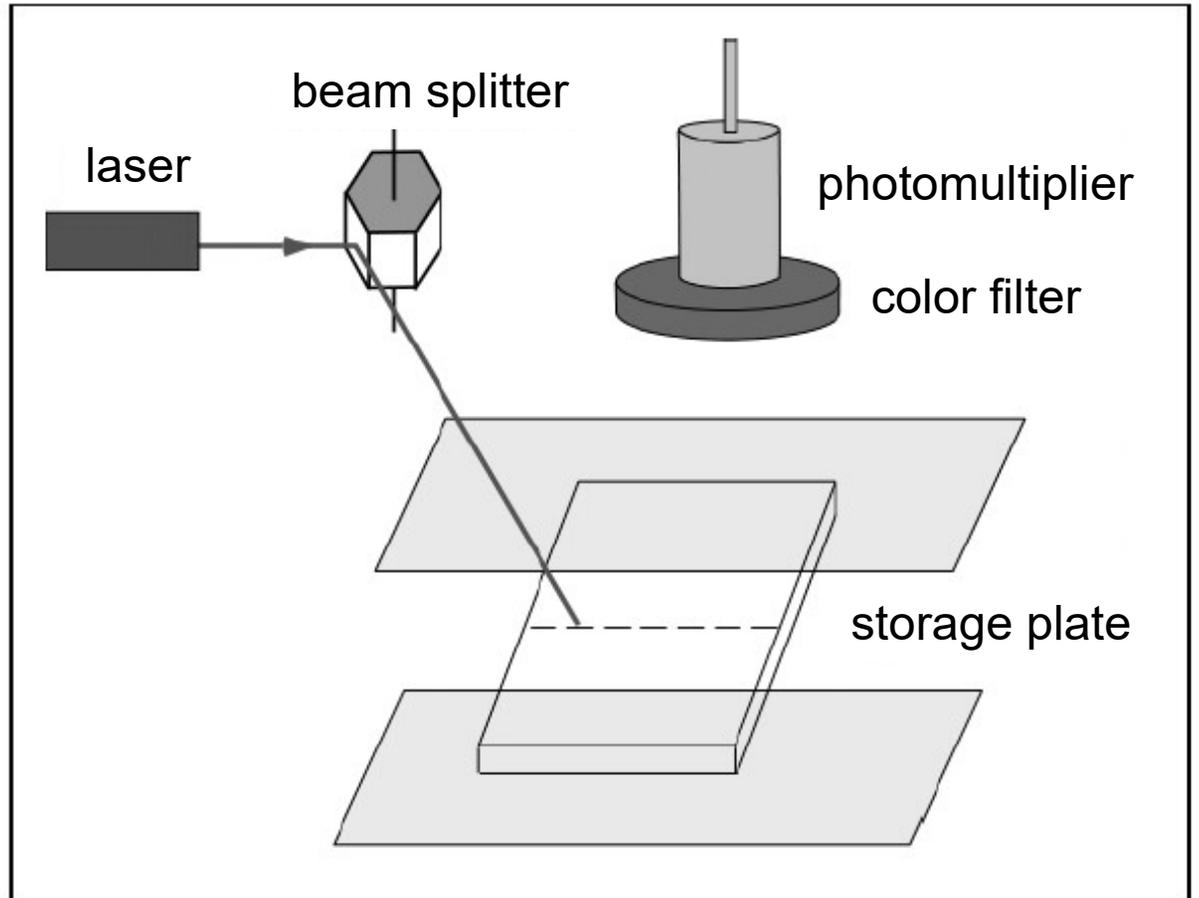
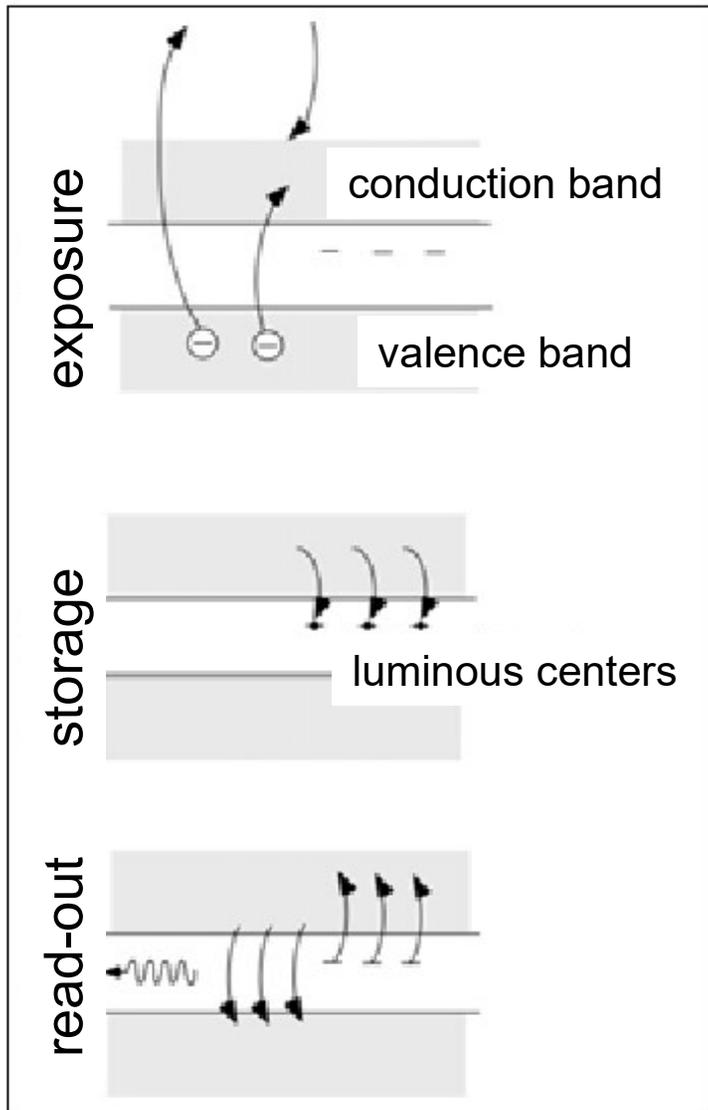
read-out: excitation of traps with laser, relaxation by emission of light (wavelength different to that of laser !!)

scanning of foil with laser scanner, color filter, photomultiplier, digitization

directly digitized image, high resolution, higher dynamic range compared to film

# Imaging with x-rays

## storage phosphor plate (digital luminescence radiography)



## *Imaging with x-rays*

### **selenium film (xeroradiography)**

*basic principle: copy machine*

selenium film mounted on a compact carrier is positively charged up (corona discharge)

$\gamma$ -quanta release  $e^-$  from carrier, neutralization of charges in film

toner clings at positively charged areas only

copy of pattern onto paper

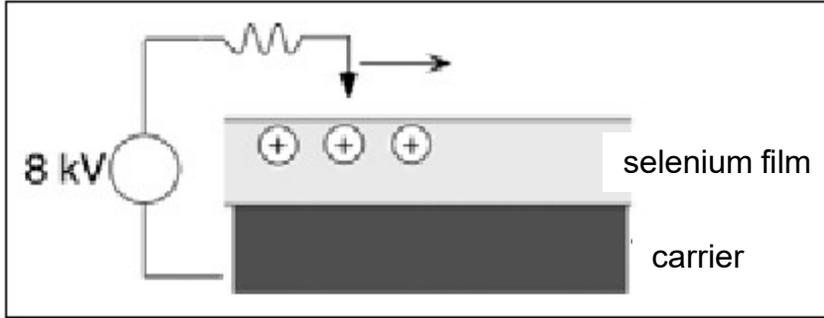
for digital radiography: spatial sampling of “charge image” with comb-like capacitor and digitization

higher dynamic range compared to film

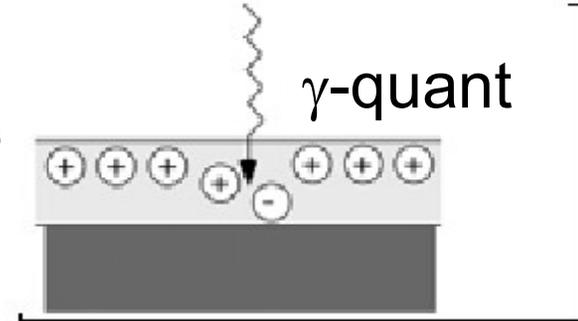
# Imaging with x-rays

## selenium film (xeroradiography)

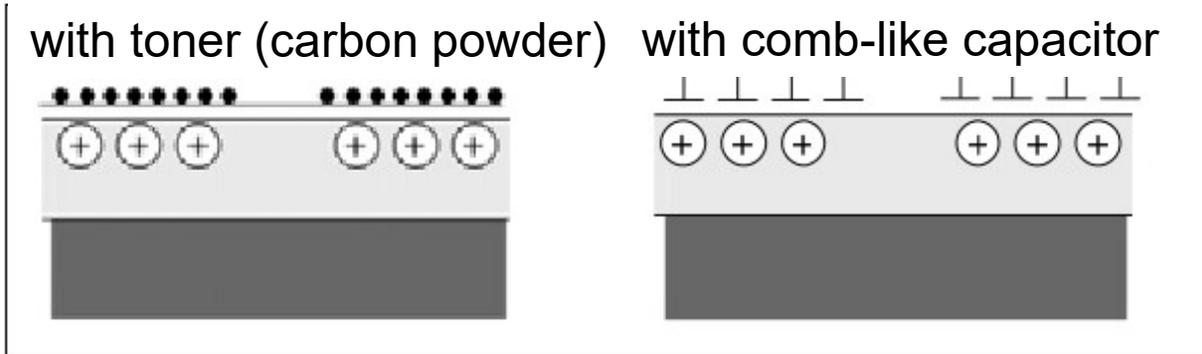
charge up



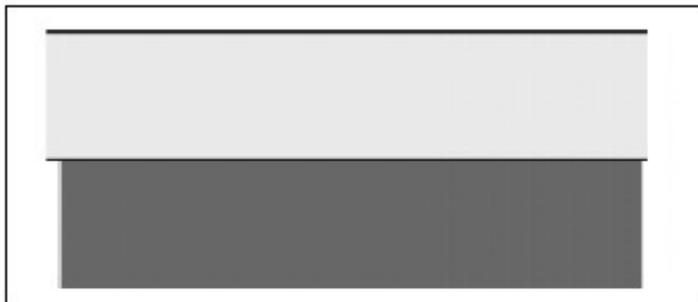
exposure



read-out



delete



## *Imaging with x-rays*

### **CCD camera**

most recent development for digital x-ray imaging

CCD-chips (charge-coupled devices) as camera for lines or planes

technological problems:

- decrease/increase of radiation with suitable optical elements
- requires large sensor areas

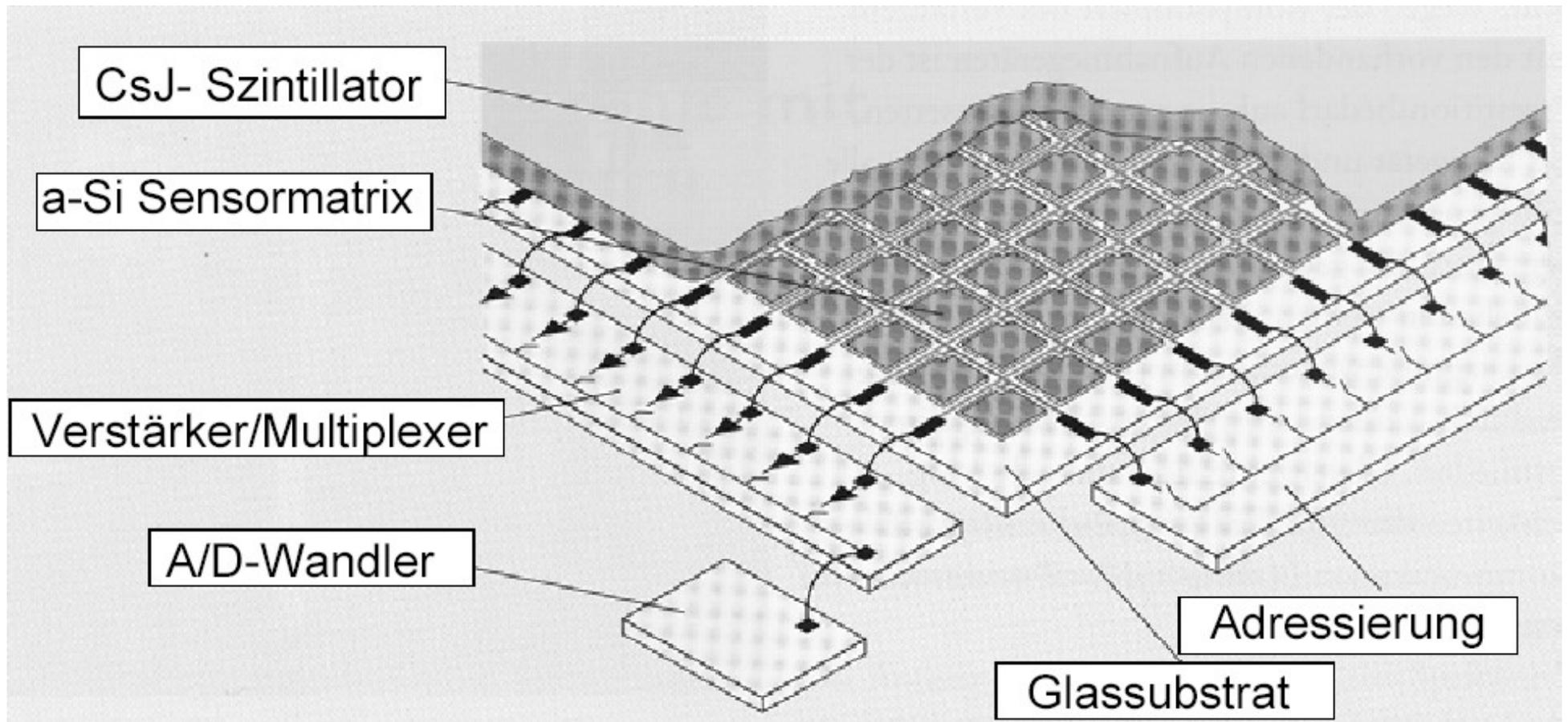
current systems: 1024 x 1024 pixel on an area of 20 cm x 20 cm  
(caesium iodide converter)

pros: flat, low weight, small

cons: expensive, requires higher dose (spatial sampling)

# Imaging with x-rays

## CCD camera



## Imaging with x-rays

### raster (anti-scatter grid)

Compton effect → scattered x-rays

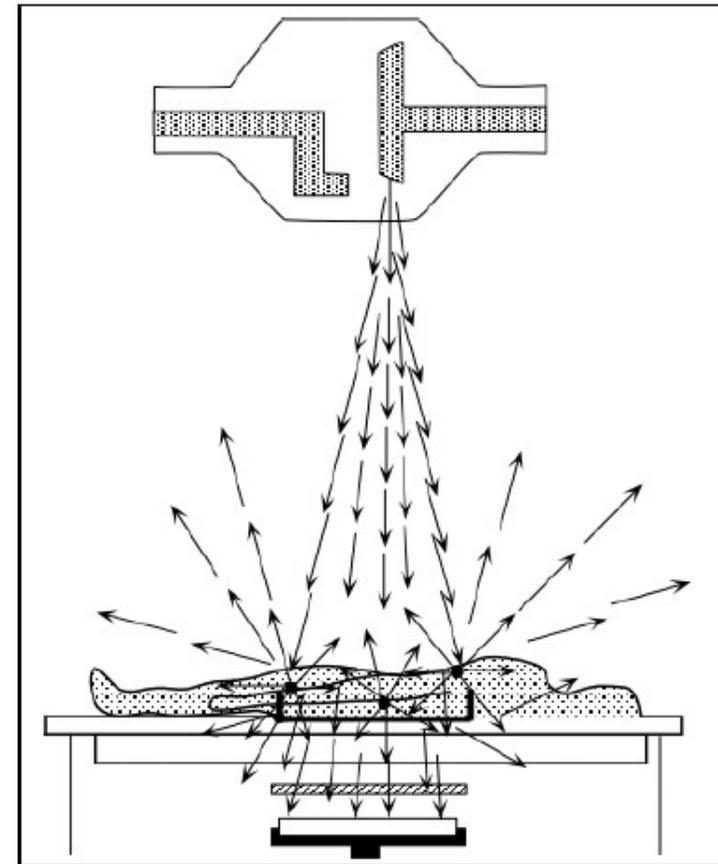
amount of scattered x-rays  $P$ :

$$P = \frac{J_s}{J_p + J_s}$$

where

$J_s$  = intensity of scattered x-rays in detector

$J_p$  = primary intensity: non-scattered x-rays in detector

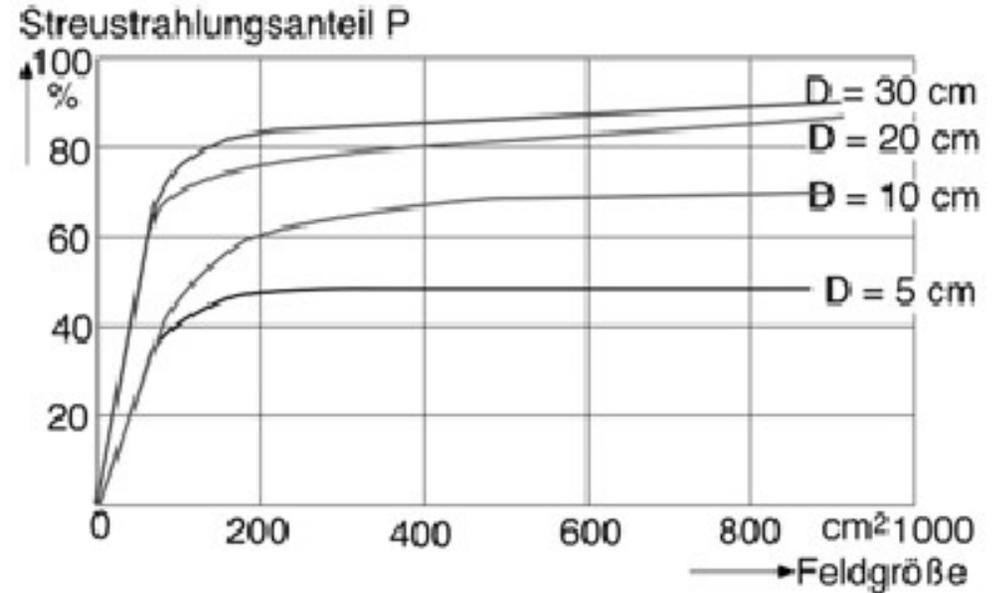
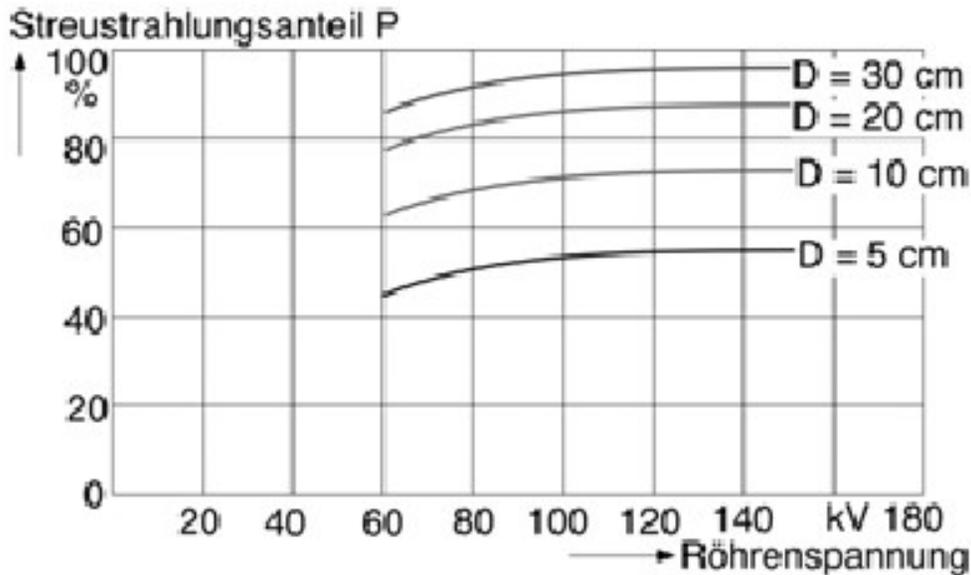


scattered radiation leads to a diminished contrast !!

# Imaging with x-rays

## raster (anti-scatter grid)

amount of scattered x-rays depending on tube potential, object thickness  $D$ , and field of view



# Imaging with x-rays

## raster (anti-scatter grid)

contrast: 
$$K = \frac{J_A - J_B}{J_A + J_B}$$

$J_A$  = x - ray intensity in area A

$J_B$  = x - ray intensity in area B

for areas with small differences  $\Delta J$  in contrast, we have

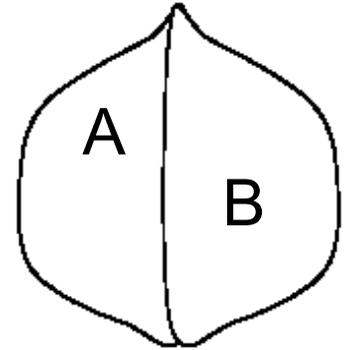
$$K = \frac{\Delta J}{2J} \quad (J = \text{mean x - ray intensity in A and B})$$

contrast without scattered radiation

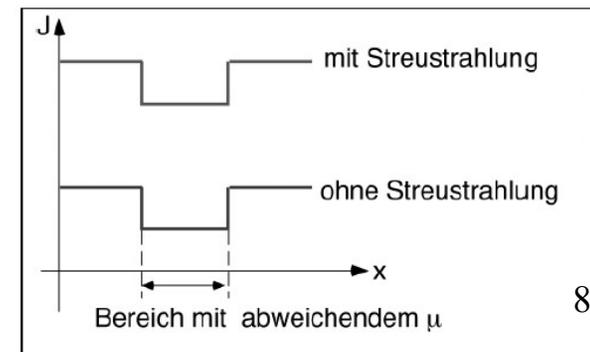
$$K_o = \frac{\Delta J_p}{2J_p}$$

contrast with scattered radiation

$$K_s = \frac{\Delta J_p}{2(J_p + J_s)} = K_o \cdot \frac{J_p}{J_p + J_s} = K_o \cdot \frac{1}{1 + \frac{J_s}{J_p}}$$



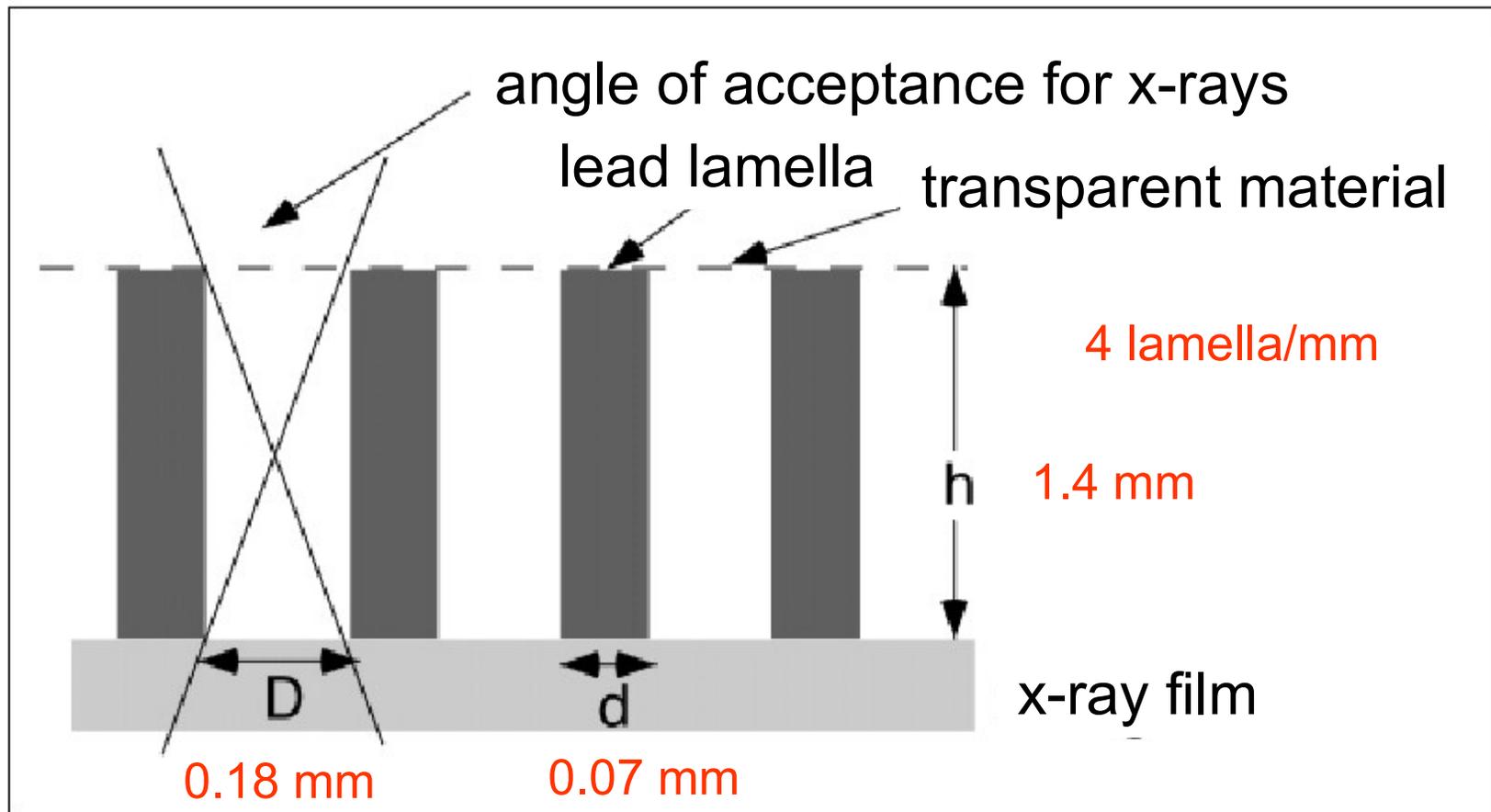
scattered radiation increases total intensity, but does not lead to smearing !!!



# Imaging with x-rays

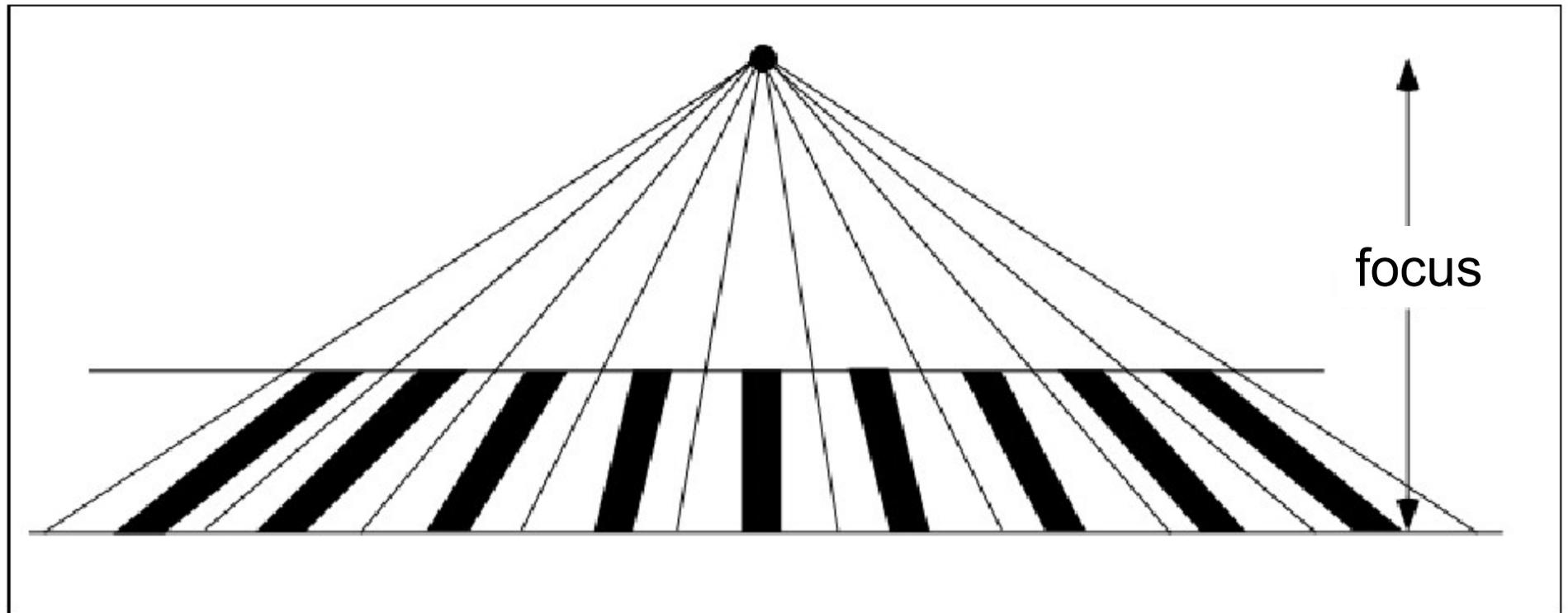
## raster (anti-scatter grid)

- amount of scattered radiation can be reduced with **raster**
- plates : alternating arrangement of lead lamella and material transparent to x-rays
- raster applied directly to x-ray film or amplifying foil



*Imaging with x-rays*

**focusing line raster**



*Imaging with x-rays*

**focusing line raster**



high amount of scattered radiation,  
75 kV, no raster



low amount of scattered radiation,  
75 kV, with raster

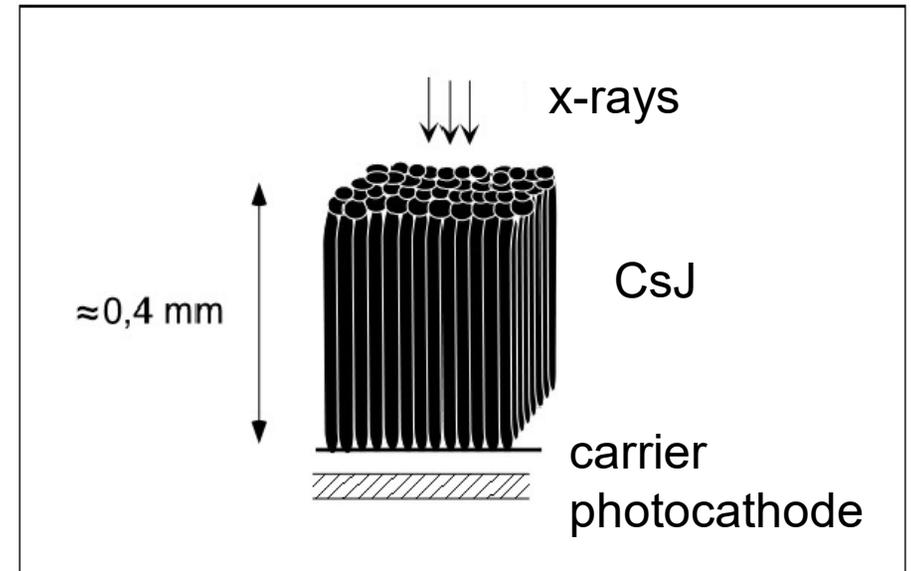
## *Imaging with x-rays*

### **x-ray image amplifier**

aim: visualization of dynamic processes during x-ray imaging  
in the past: watch fluorescent screen, high radiation exposure  
today: video-based image chain (optics, camera, monitor)

principle:  
convert x-rays into visible light  
photo effect; caesium iodide screen

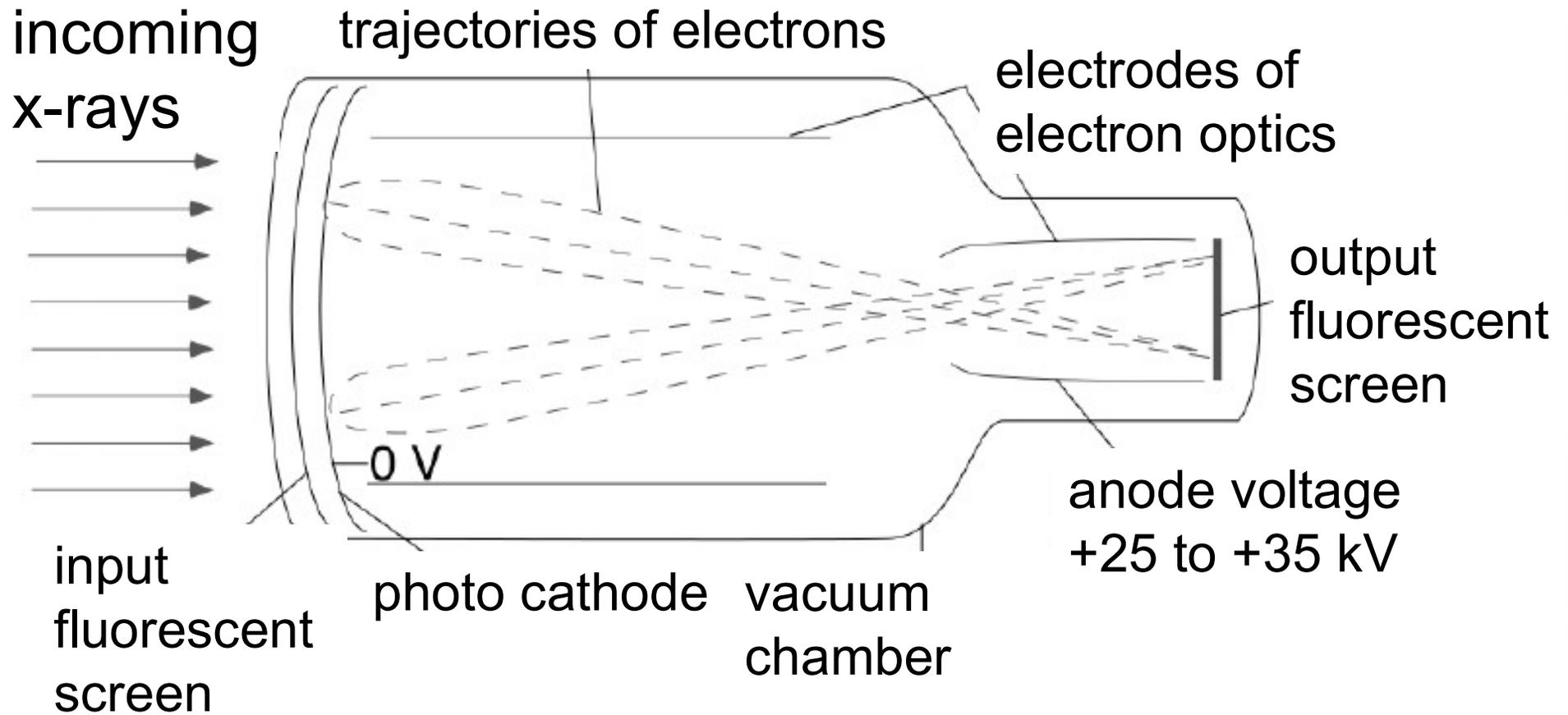
amplification with electron optics



recording of image on ZnCdS:Ag screen with video camera

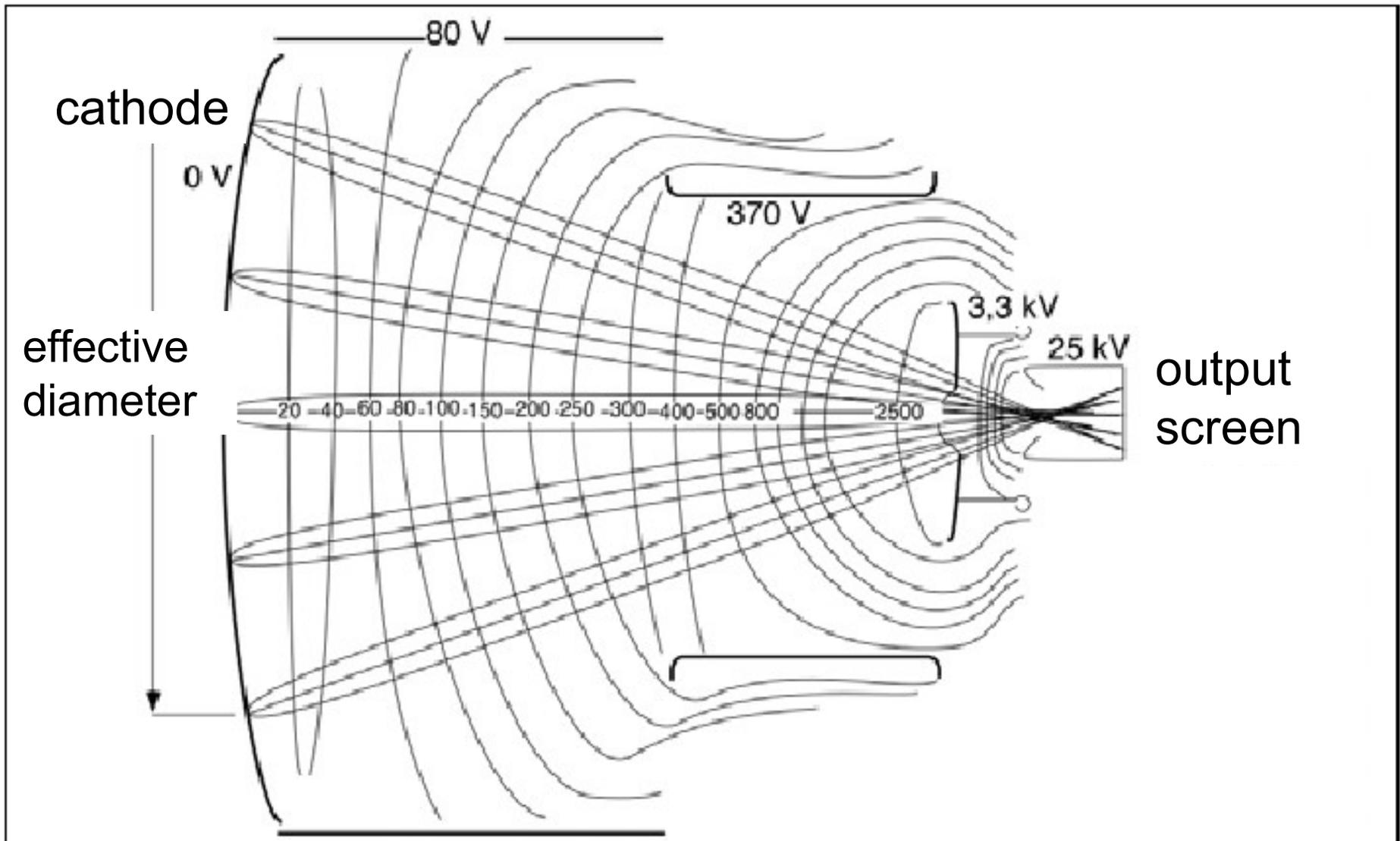
# Imaging with x-rays

## x-ray image amplifier



# Imaging with x-rays

## distribution of potential in x-ray image amplifier



## *Imaging with x-rays*

### **x-ray image amplifier**

example:

dose at fluorescent input screen:  $0.2 \mu\text{Gy/s}$   
(equals to  $5 \cdot 10^5$   $\gamma$ -quanta per cm and s)

absorption in input screen:  $\sim 60 \%$

- per  $\gamma$ -quant approx. 1000 photons on fluorescent input screen
- per electron approx. 1000 photons
- per photon at input screen approx. 100 photons at output screen

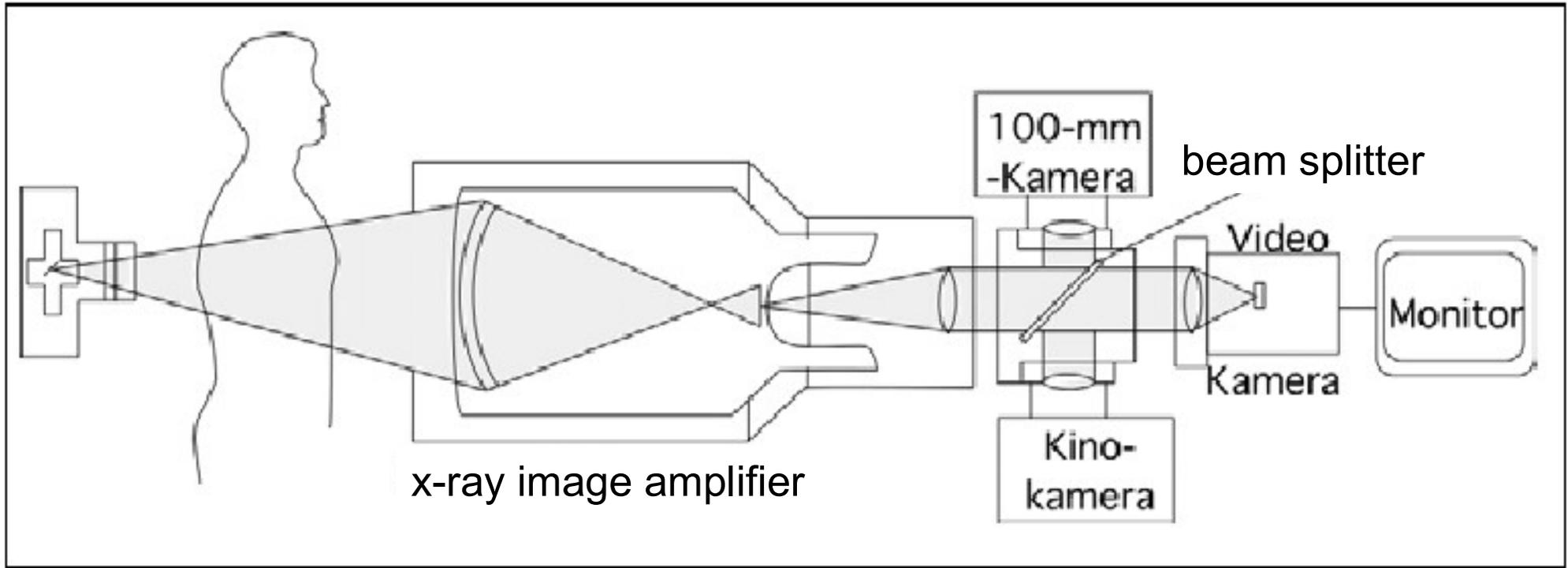
area of output screen about 10x smaller than input screen

⇒ increase of light density by x100

⇒ **total amplification:  $10^4$**

# Imaging with x-rays

## x-ray image amplifier



## *Imaging with x-rays*

### **gas detectors**

principle: ionization chamber

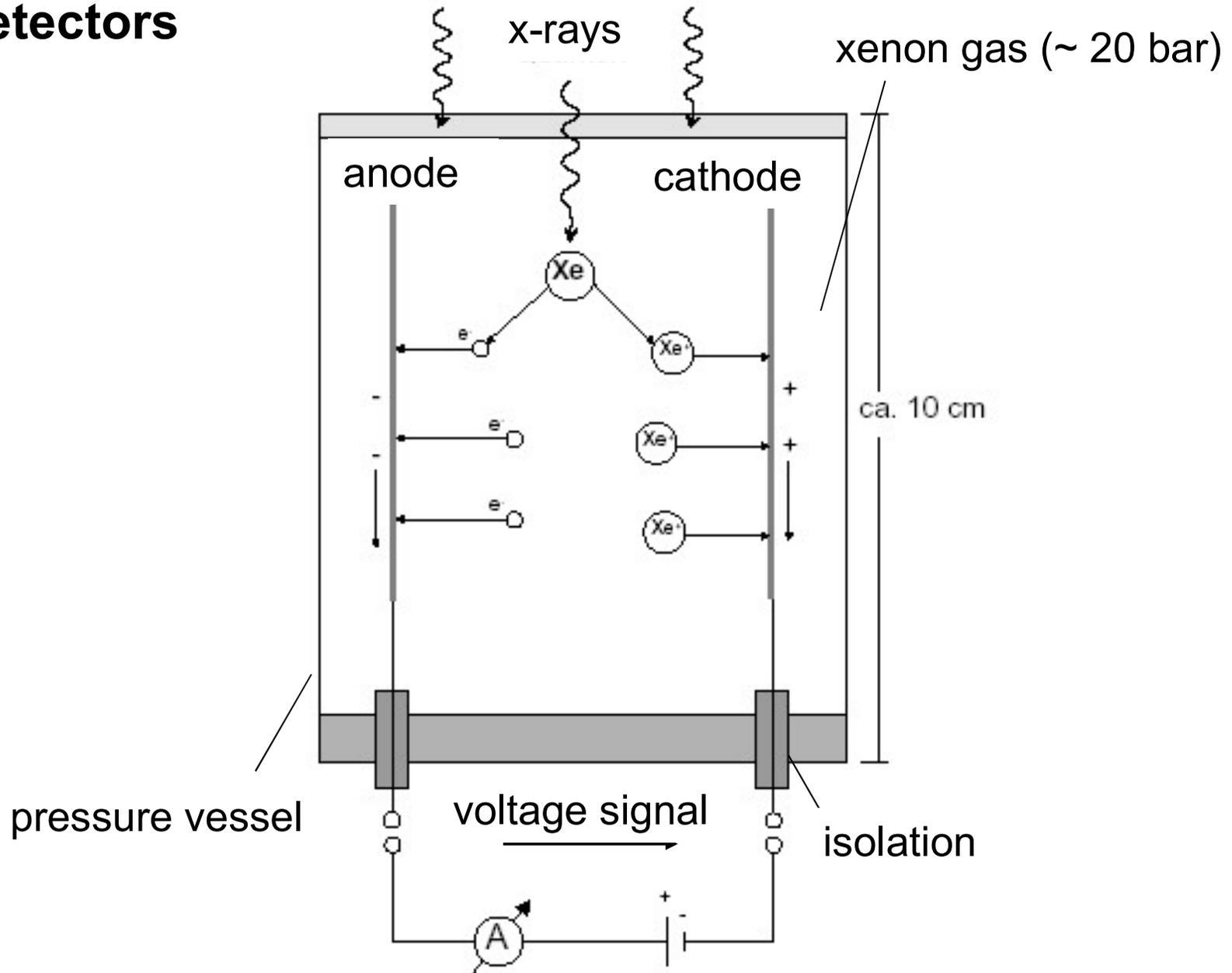
- x-rays ionize of xenon gas (high pressure chamber)
- walls of chamber = capacitor plates (high voltage)
- ionization generates charged particles
- charge separation
- output voltage directly prop. to intensity of x-rays

pros

- fast decay times (short acquisition times)
- insensitive to fluids and temperature

# Imaging with x-rays

## gas detectors

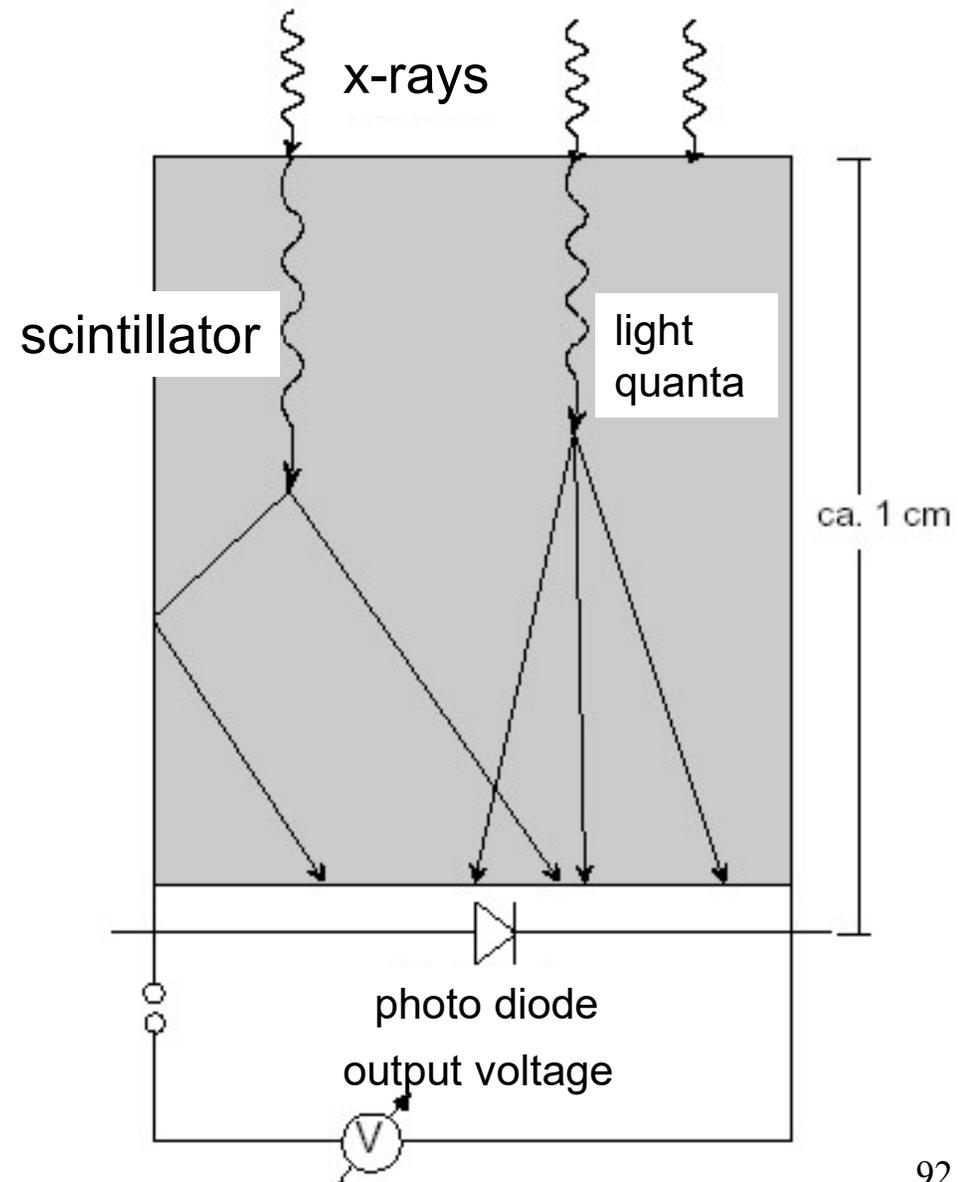


# Imaging with x-rays

## solid-state detectors

principle: scintillation

- x-rays excite  $e^-$  in crystal
- relaxation with emission of photons
- conversion to voltage changes with photo diode
- voltage directly prop. to energy of x-rays



# Imaging with x-rays

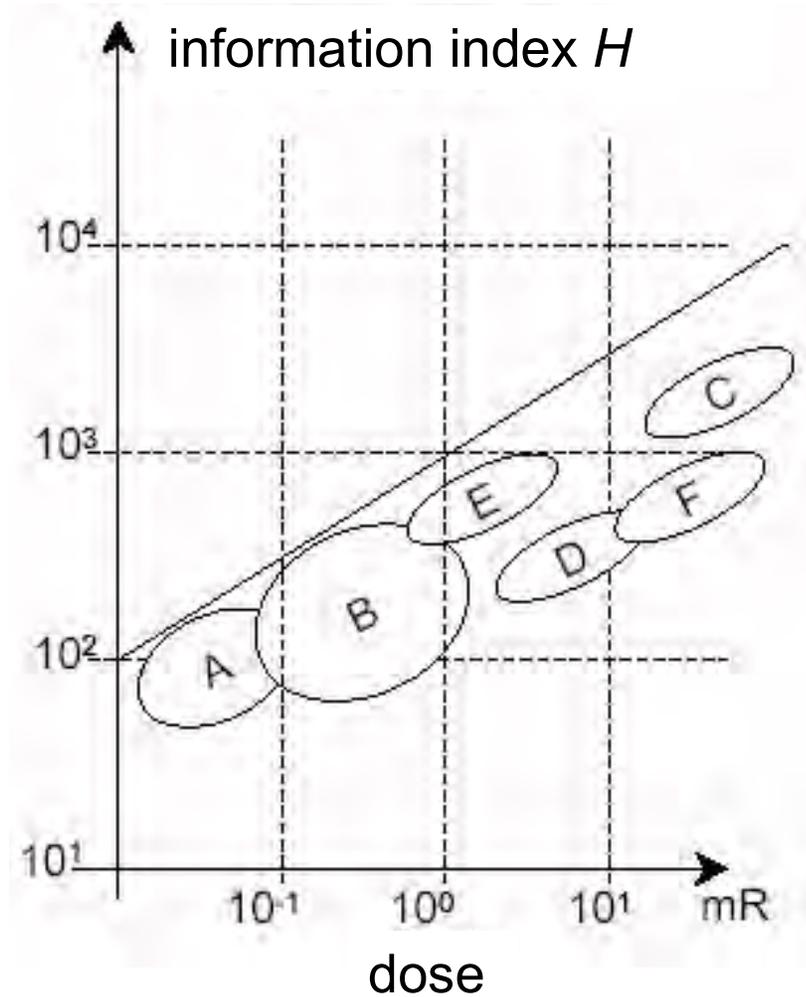
## comparing different detectors

information index  $H$ :

$$H = c \cdot r$$

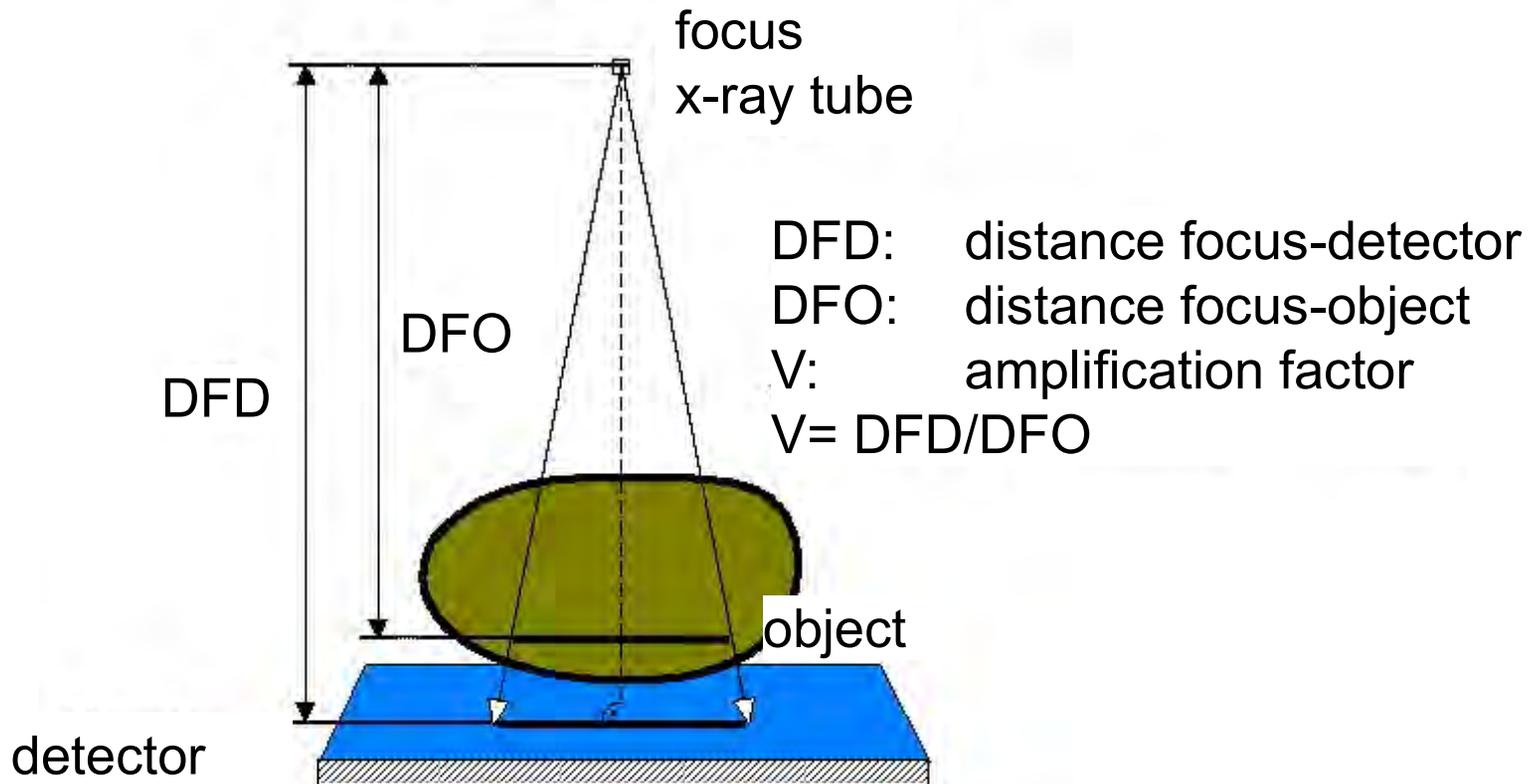
$c$  = contrast sensitivity

$r$  = resolution



- A: x-ray image amplifier
- B: intensifying screen (amplifying foil)
- C: x-ray film
- D: xeroradiography
- E: electroradiography with Xe chamber
- F: CT

**central projection**

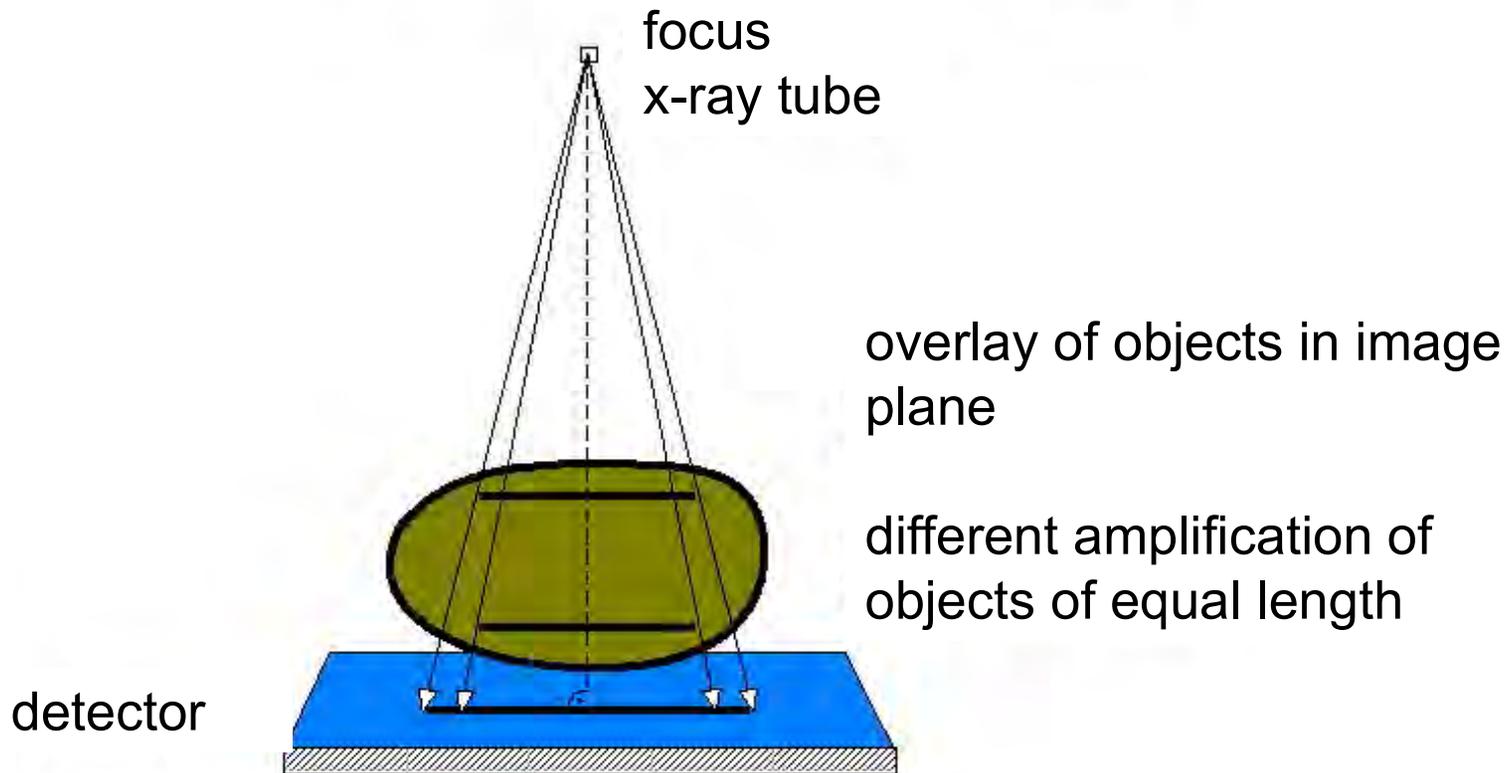


# Imaging with x-rays

impact of geometry

projection laws

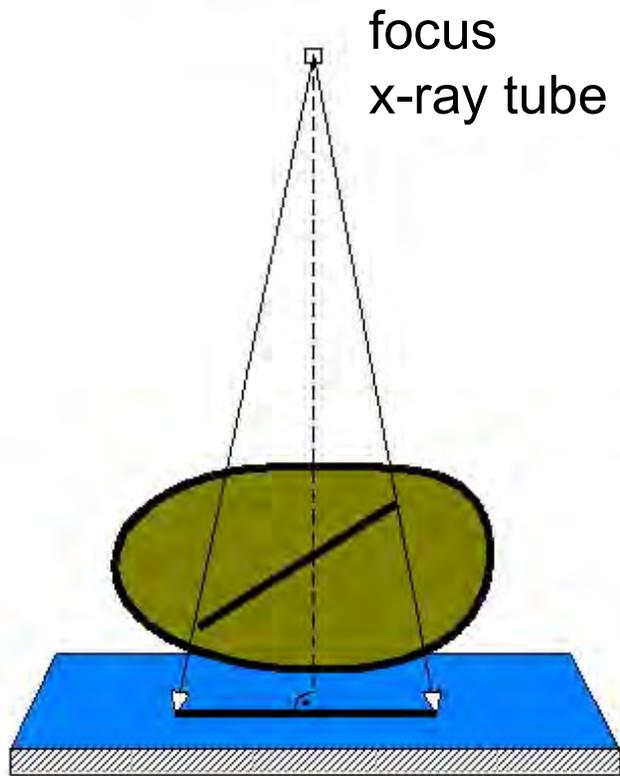
inverted field of view



# Imaging with x-rays

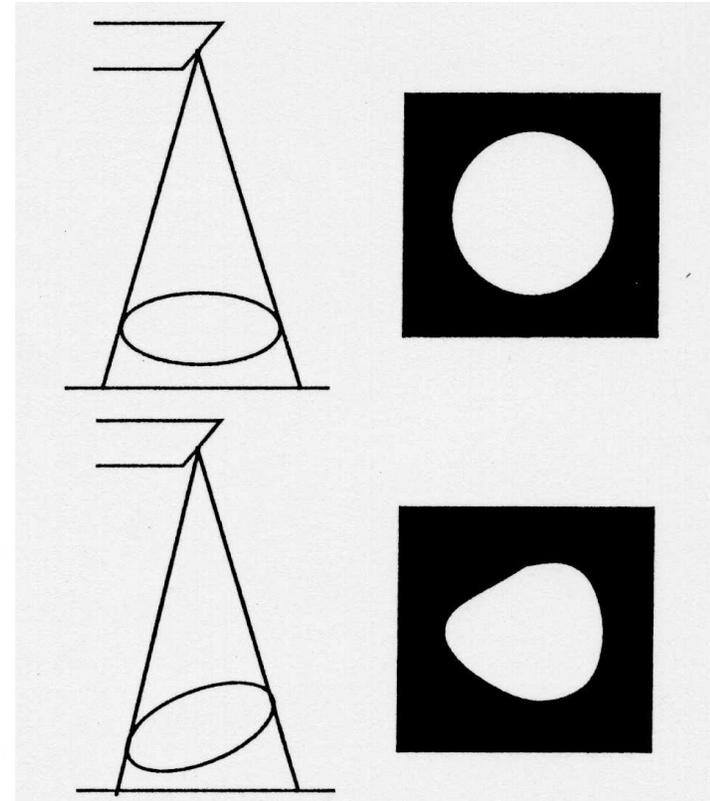
## impact of geometry

### distortions

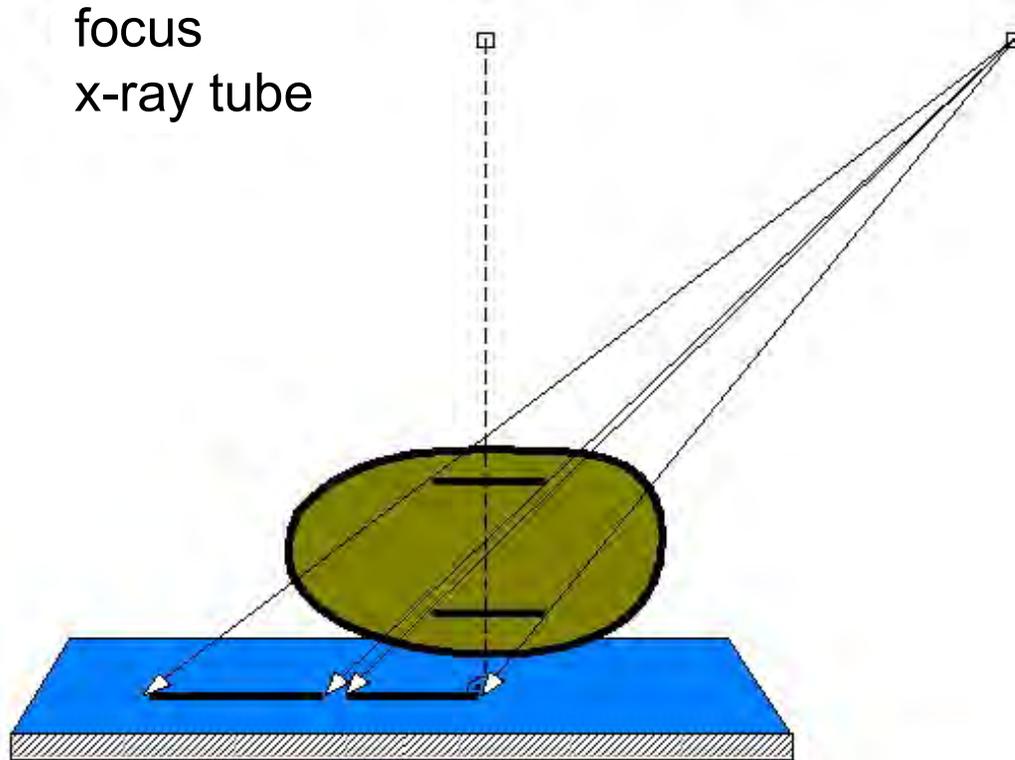


perspective shortening

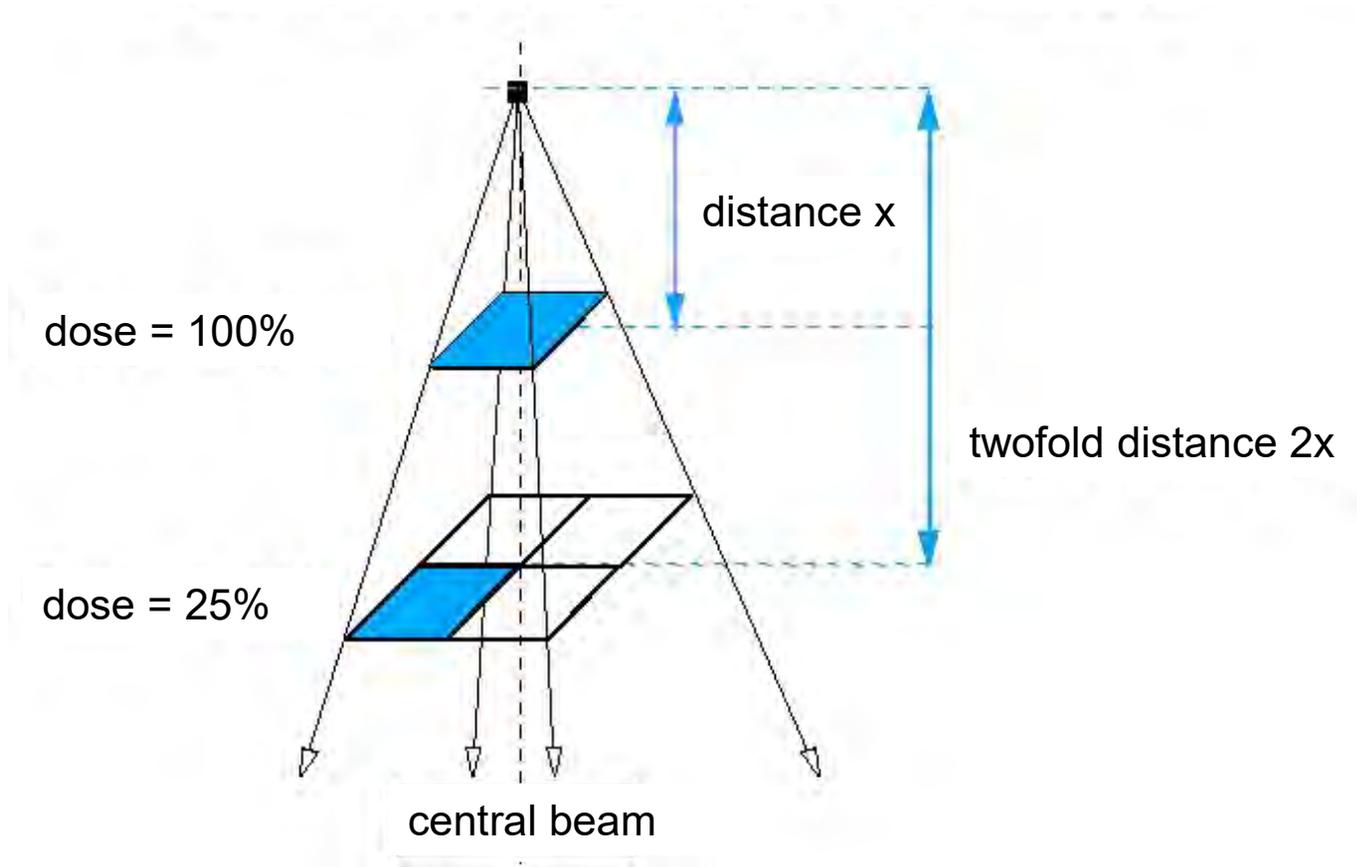
## projection laws



**disentangling of superposed objects**

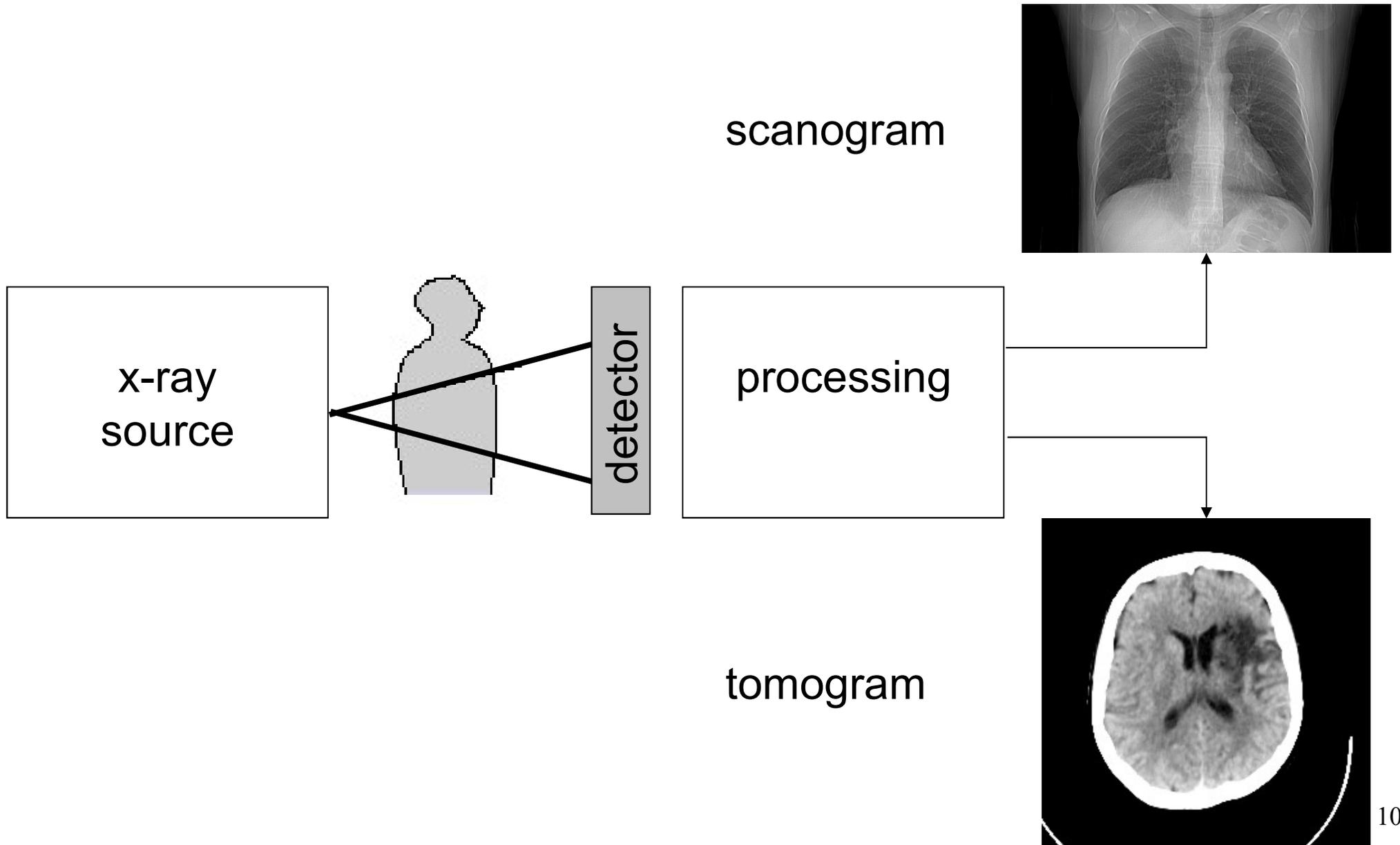


**inverse square law**



# **image quality**

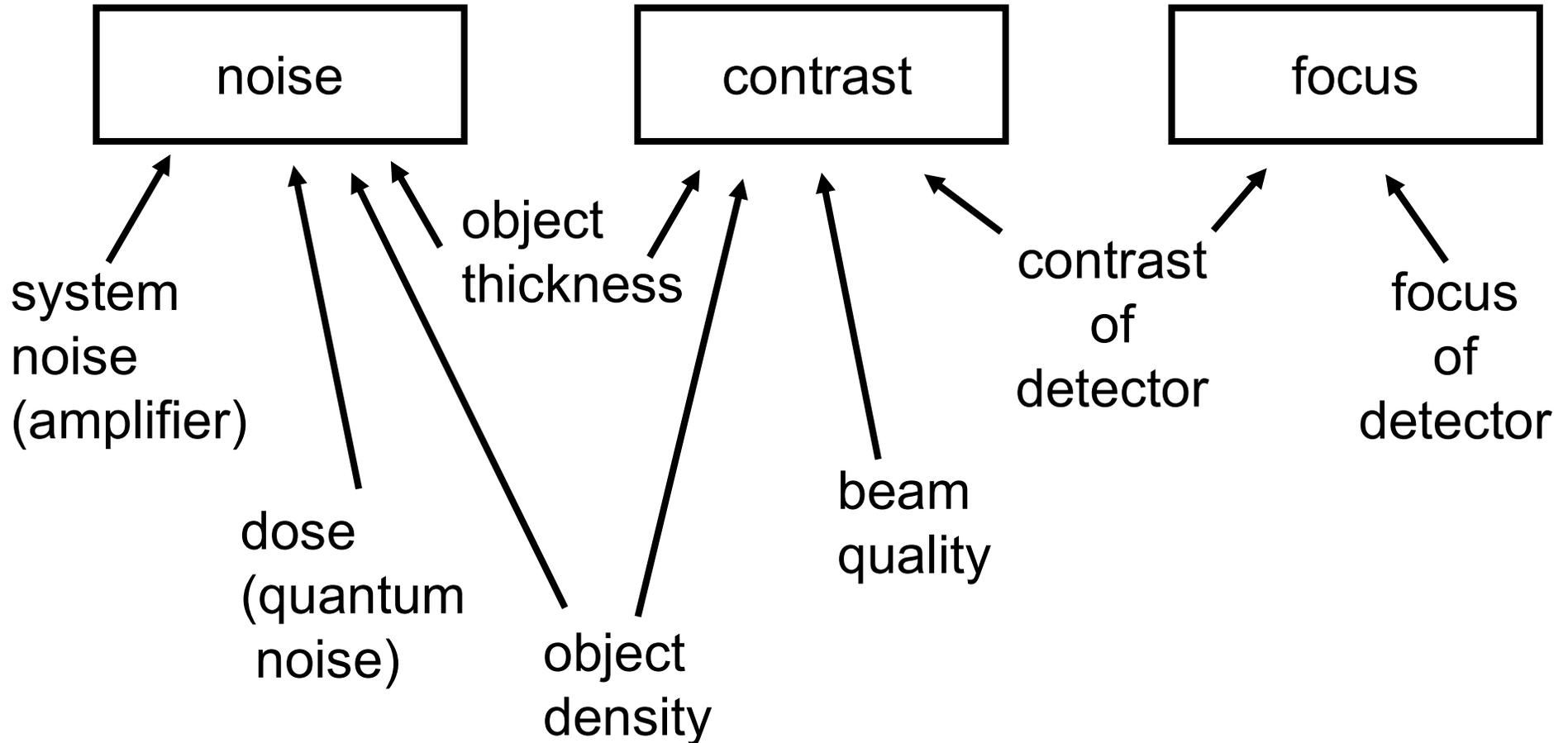
# Imaging with x-rays



# *Imaging with x-rays*

## **image quality of x-ray image amplifier**

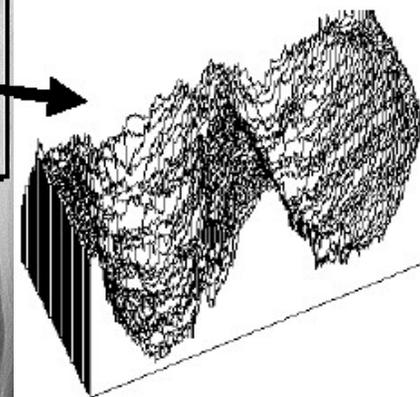
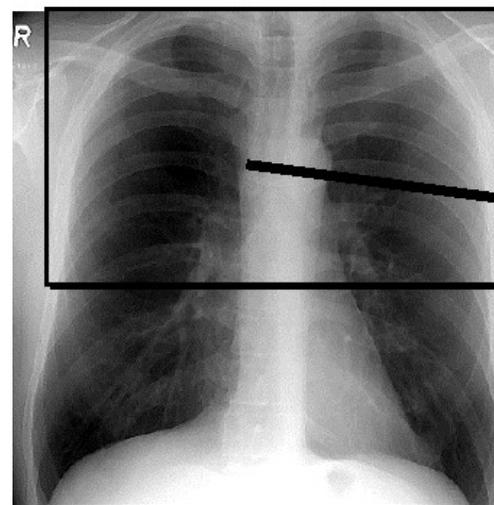
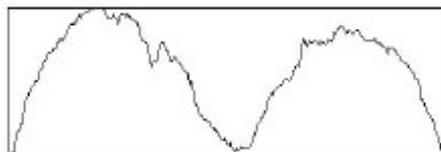
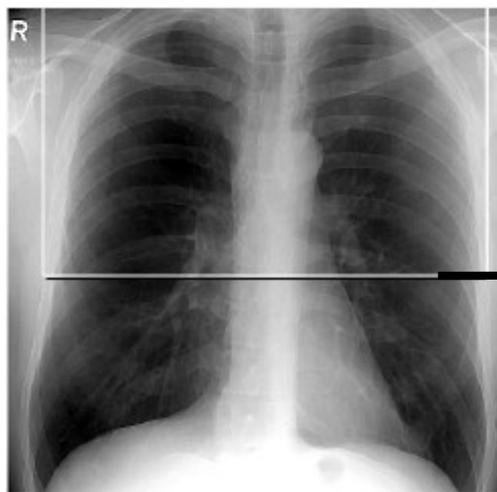
## **influencing factors**



# *Imaging with x-rays*

## **image quality of x-ray image amplifier**

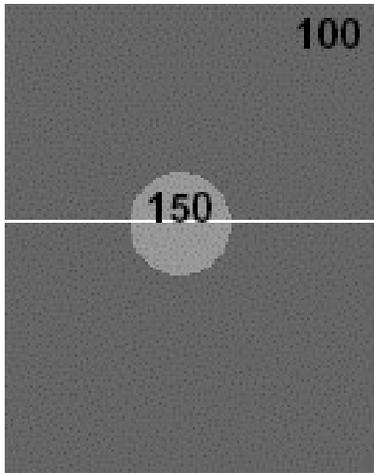
intensity profile



## *Imaging with x-rays*

### **image quality of x-ray image amplifier**

$$\text{contrast} = \frac{\text{object intensity} - \text{background intensity}}{\text{background intensity}}$$



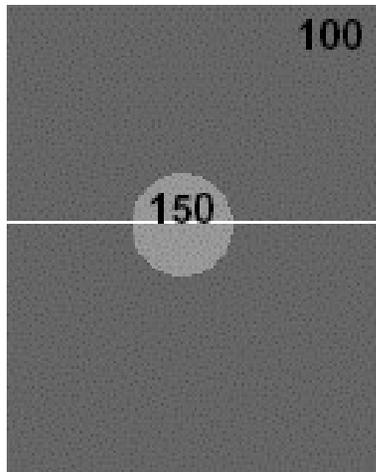
$$\text{contrast} = (150-100)/100 = 0,5$$

## *Imaging with x-rays*

### **image quality of x-ray image amplifier**

$$\text{modulation} = \frac{\text{object intensity} - \text{background intensity}}{\text{background intensity} + \text{object intensity}}$$

$$\text{modulation} \in [0, 1]$$



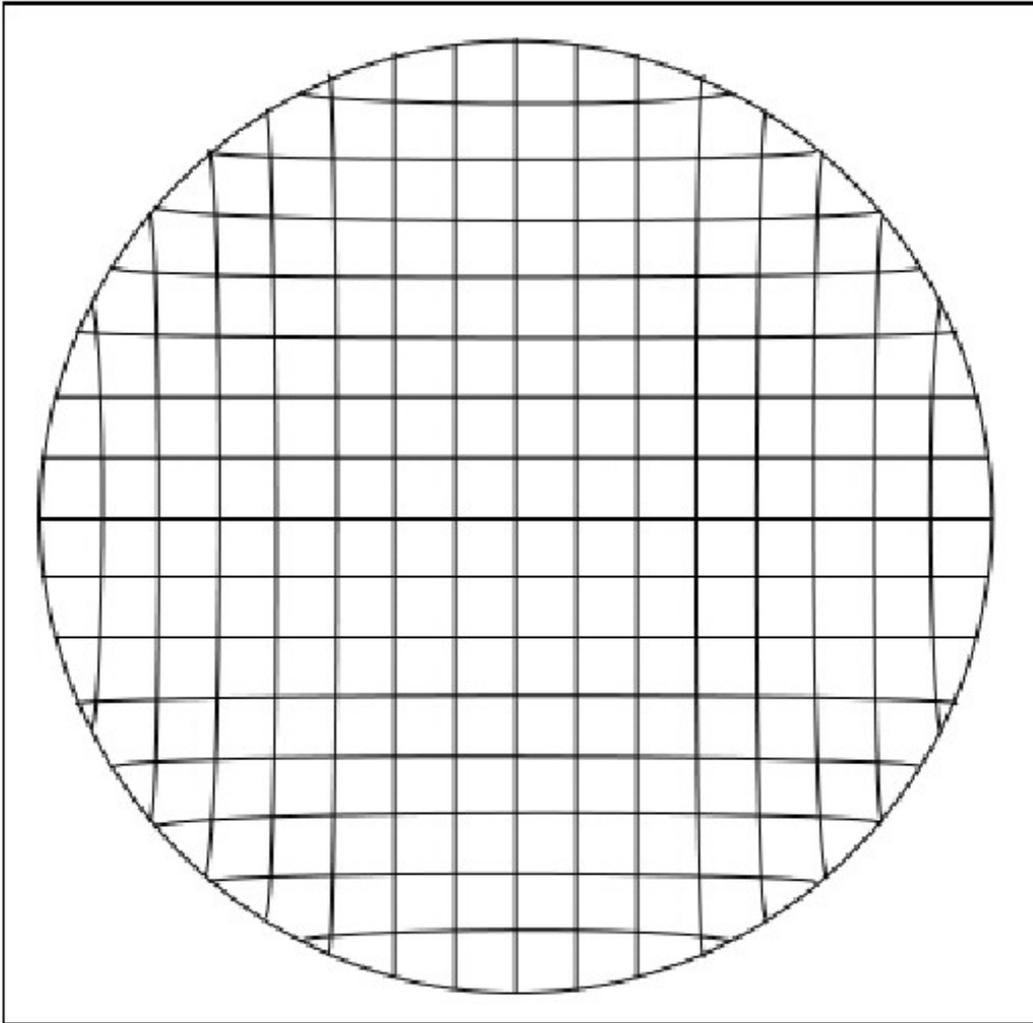
$$\text{modulation} = (150 - 100) / (100 + 150) = 0,2$$

## **image quality of x-ray image amplifier**

### quality criteria

- distortions
- uniform illumination
- conversion factor  
(brightness @output / dose rate @input)
- noise
- spatial resolution

## image quality of x-ray image amplifier

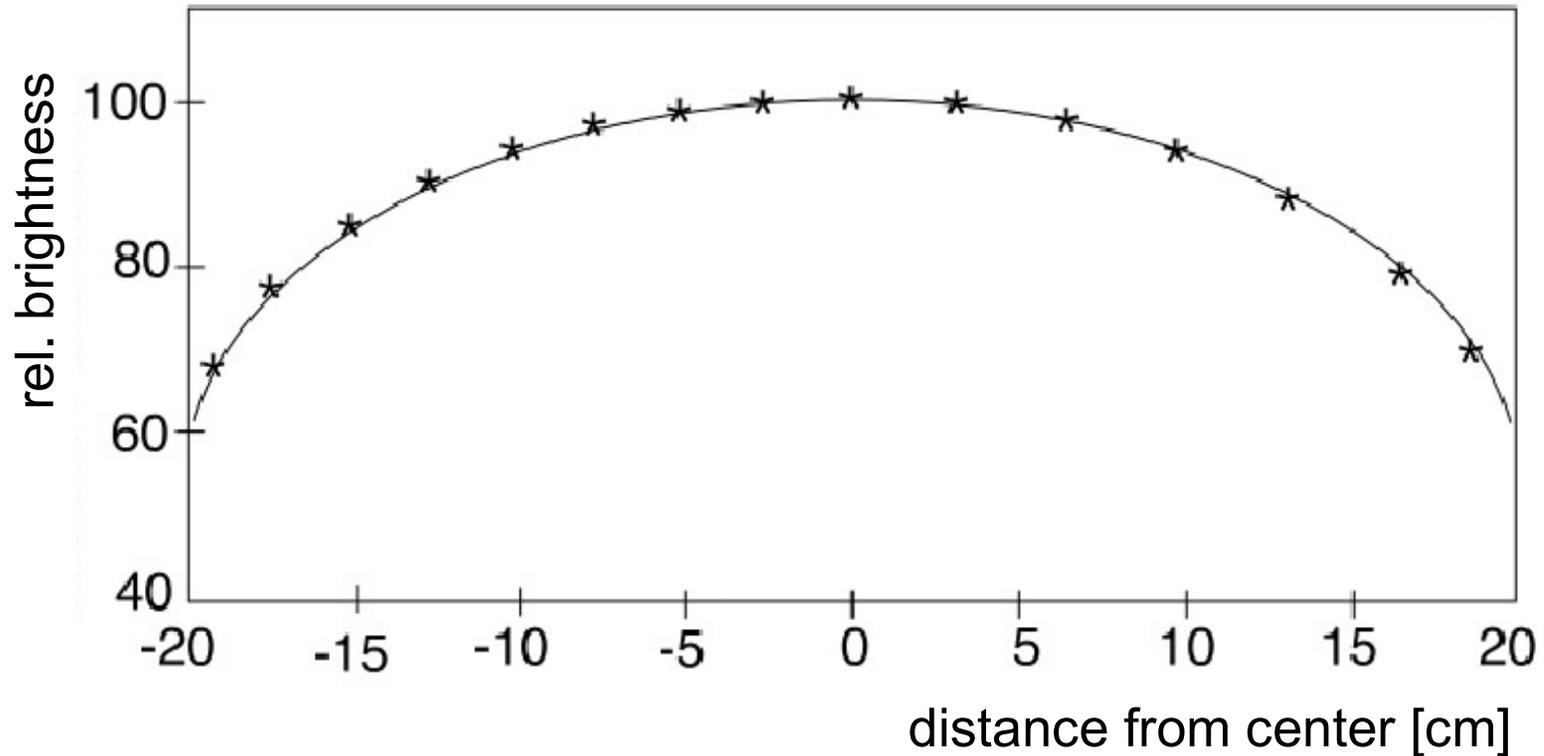


### **saddle-like distortions**

- device-specific
- no strong impact on diagnosis
- identify with quadratic mesh
- digital correction

## *Imaging with x-rays*

### **image quality of x-ray image amplifier**



**„vignetting“**

- device specific effect

## Imaging with x-rays

### image quality of x-ray image amplifier

noise

number of detected  $\gamma$ -quanta ( $x$ ) per unit area and unit time is Poisson distributed:

$$p(x) = \frac{m^x e^{-m}}{x!}$$

$$\sigma_{Poisson}^2 = m$$

Poisson distribution characterized by mean value  $m$  only!

(Gaussian distribution: mean and standard deviation)

the higher the number of  $\gamma$ -quanta, the higher the standard deviation (the smaller the relative spread)

m	$\sigma$	$\sigma$ (%)
10	3,16	31,60
100	10,00	10,00
1000	31,60	3,16

### quantum noise:

number of  $\gamma$ -quanta  $\Rightarrow$  image grey level  $\Rightarrow$  noisy image

# Imaging with x-rays

image quality of x-ray image amplifier

noise

**estimating quantum noise (units)**

*number of quanta per absorbed dose*

$$\text{dose} = \frac{\text{number of quanta}}{\text{unit area}}$$

unit: 1/mm<sup>2</sup>

$$\frac{\text{radiation energy}}{\text{unit area}} = \frac{\text{radiant power} \cdot \text{exposure time}}{\text{unit area}}$$

$$= \frac{\text{number of quanta} \cdot \text{energy of quanta}}{\text{unit area}}$$

unit: J/mm<sup>2</sup>

## *Imaging with x-rays*

**image quality of x-ray image amplifier**

**noise**

***estimating quantum noise (technically)***

*number of quanta per absorbed dose*

$$\text{ion dose} = \frac{\text{charge quantity (of given sign) due to ionization in air}}{\text{air volume in measurement chamber @ 760 Torr}}$$

unit: Röntgen (R)

$$\text{ion dose} = \frac{\text{charge quantity (of given sign) due to ionization in air}}{\text{mass of air in measurement chamber}}$$

unit: C/kg = As/kg

**100 R = 25.8 mC/kg (air @760 Torr)**

## *Imaging with x-rays*

**image quality of x-ray image amplifier**

**noise**

***estimating quantum noise (technically)***

*number of quanta per absorbed dose*

$$\text{absorbed dose} = \frac{\text{energy deposited in body due to radiation}}{\text{mass of object}}$$

unit: J/kg = Gray = Gy

conversion ion dose [Coulomb/kg] → absorbed dose [Gy]  
(air, 100 keV)

$$\mathbf{1 \text{ Gy} = 29.86 \text{ mC/kg}}$$

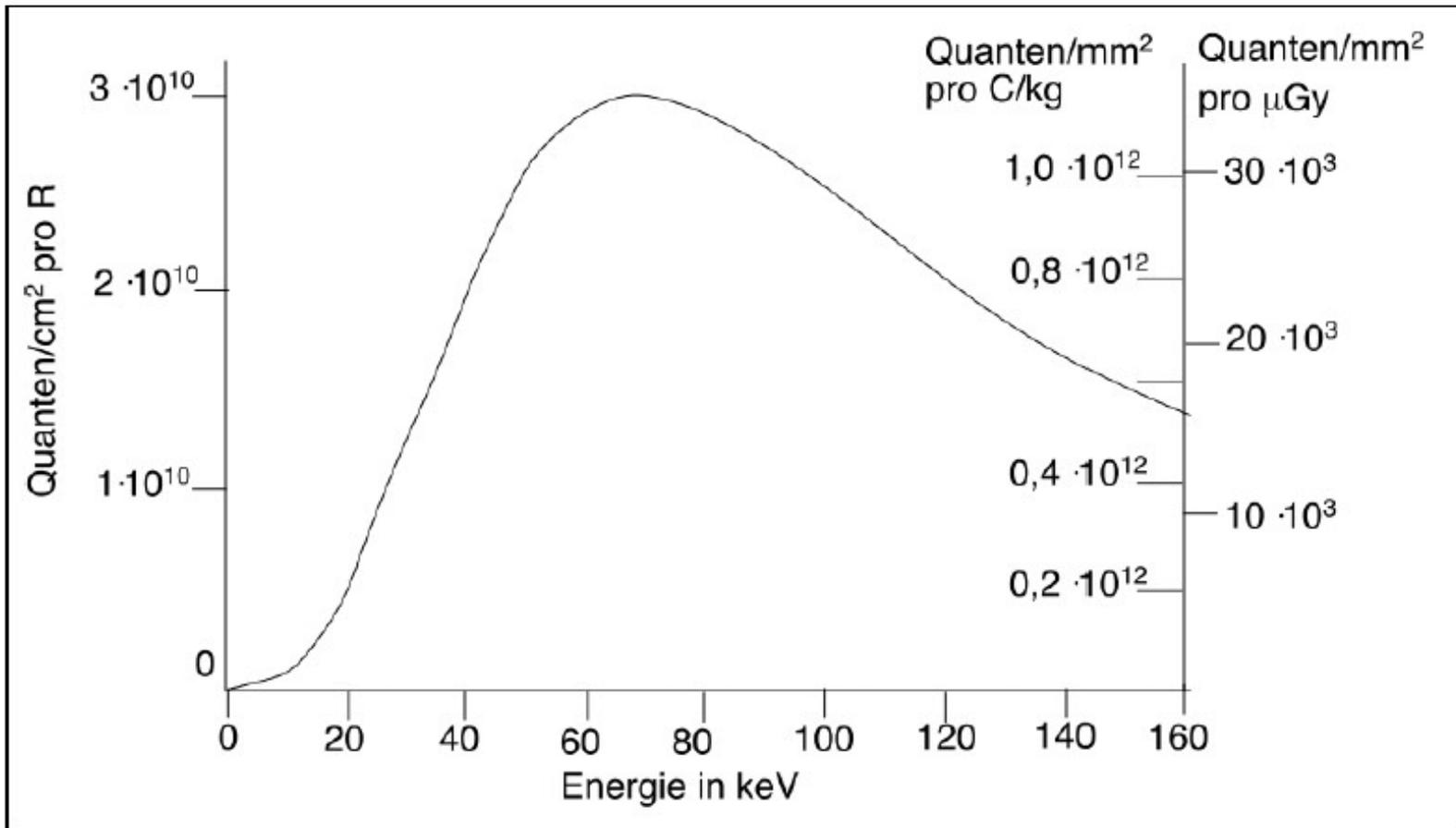
# Imaging with x-rays

image quality of x-ray image amplifier

noise

estimating quantum noise (technically)

calibration curve



## *Imaging with x-rays*

### **image quality of x-ray image amplifier**

**noise**

### ***estimating quantum noise***

***an example***

mean x-ray energy:	80 keV
dose rate:	0.2 $\mu\text{Gy/s}$
pixel size:	0.2 mm x 0.2 mm
exposure time/image:	0.2 s

80 keV  $\Leftrightarrow$   $3,4 \cdot 10^4$  quanta/(mm  $\mu\text{Gy}$ )

quantum noise (incoming  $\gamma$ -quanta):

$\Rightarrow$  54  $\gamma$ -quanta/pixel; std. dev.: 7.3; rel. std. dev.: 13.5 %

(due to the Poisson distribution; can not be optimized further !!)

## *Imaging with x-rays*

**image quality of x-ray image amplifier**

**noise**

***estimating quantum noise***

***an example***

(continued)

impact of measurement chain (I)

absorption in fluorescent screen @input: 10 %

eff. degree of absorption (CsJ (80keV)): 70 %

⇒ detected quanta (screen):  $34 \pm 5,8 = 34 \pm 17,1 \%$

⇒ deterioration of signal/noise-ratio in fluorescent screen @input !!

## *Imaging with x-rays*

**image quality of x-ray image amplifier**

**noise**

***estimating quantum noise***

***an example***

(continued)

impact of measurement chain (II)

conversion  $\gamma$ -quanta to visible light: 2600 photons /  $\gamma$ -quant

$\Rightarrow 2600 \times 34 = 88400$  photons/image/pixel

but: photon generation is also statistical process !

(e.g., assume:  $2600 \pm 100$  photons/image/pixel)

with error propagation:

$\Rightarrow$  number of generated photons  $88400 \pm 15400 = 88400 \pm 17,4 \%$

$\Rightarrow$  only minor change of standard deviation

only minor change of signal/noise ratio !

the signal/noise ratio @output  
is **not** a good measure for the  
quality of an imaging system!

more important is the factor  
by which the system deteriorates  
the signal/noise ratio!

## *Imaging with x-rays*

**image quality of x-ray image amplifier**

**noise**

### ***Detective Quantum Efficiency DQE***

$$DQE = \frac{(\text{signal/noise ratio})^2 \text{ @output}}{(\text{signal/noise ratio})^2 \text{ @input}}$$

$DQE \in [0,1]$

$DQE=1$  ideal system

for quantum noise only, we have:

$$DQE = \frac{\text{mean number of detected } \gamma\text{-quanta}}{\text{mean number of incoming } \gamma\text{-quanta}}$$

$\sigma^2_{\text{output}}/\sigma^2_{\text{input}} = n_{\text{input}}/n_{\text{output}}$  due to Poisson distribution

**image quality of x-ray image amplifier**

**noise**

***Detective Quantum Efficiency DQE***

DQE of an imaging chain:

$$DQE_{\text{chain}} = DQE_1 \cdot DQE_2 \cdot \dots \cdot DQE_N$$

(since output of component 1 = input of component 2 etc.)

DQE for above-mentioned example:

$$DQE_{\text{input screen}} = 34/54 = 0.63$$

after photon conversion:

$$DQE_{\text{conv}} = (88400/15400)^2 / (34/5.8)^2 = 0.96$$

*Imaging with x-rays*

**image quality of x-ray image amplifier**

**noise**

when imaging with x-rays,  
**image quality is always limited**

due to **balance** between

**minimization of dose resp. dose rate**  
(radiation protection)

and

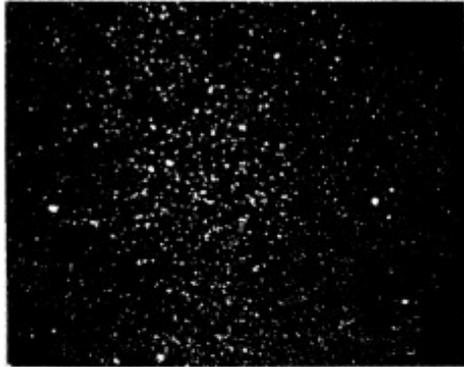
**maximization of signal**  
(detectability of details)

*Imaging with x-rays*

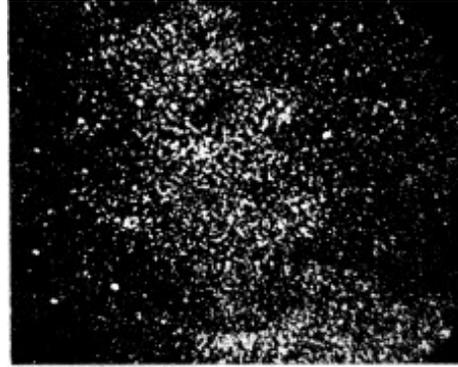
**image quality of x-ray image amplifier**

**noise**

detectability of details



3000 photons



12000 photons



93000 photons



760000 photons



3,6 Mio. photons



28 Mio. photons

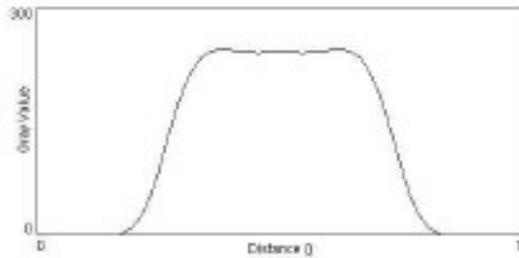
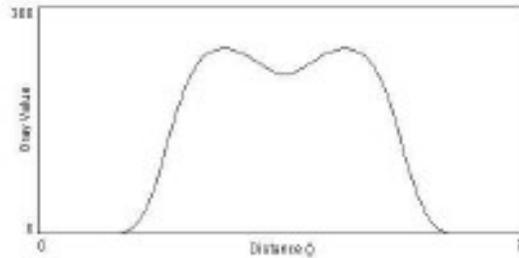
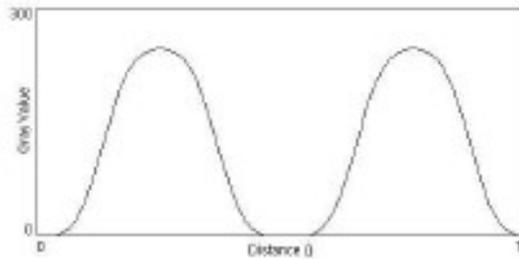
# Imaging with x-rays

## image quality of x-ray image amplifier

### spatial resolution (image sharpness)

general definition:

separability of adjacent objects (Rayleigh criterion)



## *Imaging with x-rays*

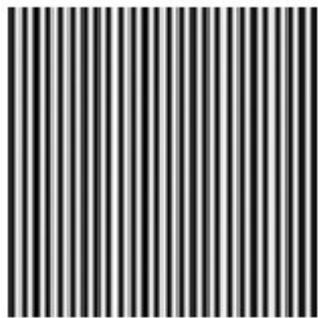
### **image quality of x-ray image amplifier**

### **spatial resolution**

special definition:

### **Modulation Transfer Function MTF**

example: in x-direction sinusoidal modulated image @output



wave length: 20 pixel

$$g(x) = \bar{g} + K_0 \cdot \sin(2\pi \cdot u \cdot x)$$

$g(x)$ : grey level of original at position  $x$

$\bar{g}$ : mean grey level of original

$K_0$ : amplitude of grey-level modulation

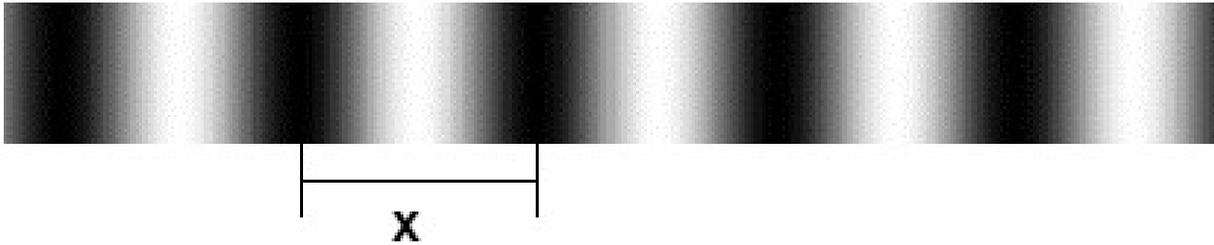
$u=1/\lambda$  spatial frequency of grey-level modulation

$\lambda$  wave length of grey-level modulation

## *Imaging with x-rays*

### **image quality of x-ray image amplifier**

def.: spatial frequency



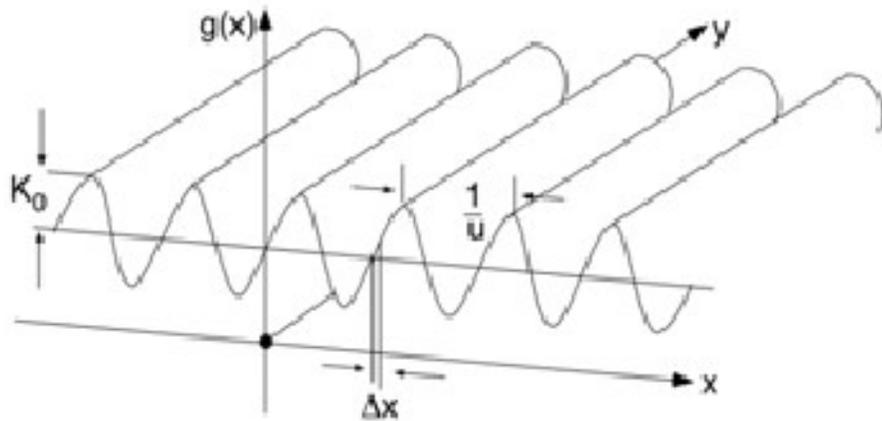
spatial frequency = number of periodically recurring light-dark-modulations (so called line pairs, Lp) per unit length

$$u = 1/x \quad [\text{mm}^{-1}]$$

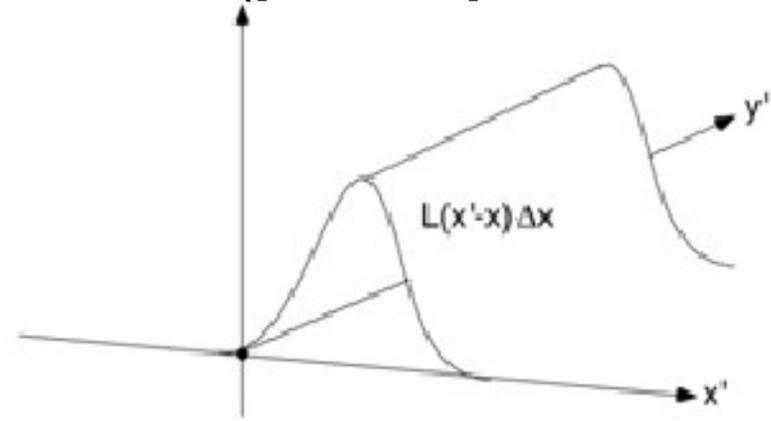
# Imaging with x-rays

## Modulation Transfer Function MTF

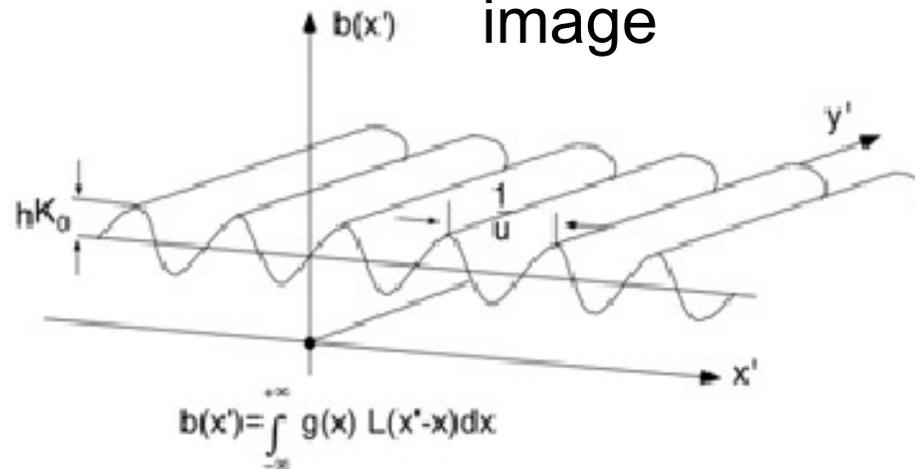
object



imaging system  
(point spread function)



image



## Modulation Transfer Function MTF

$$b(x') = \int_{-\infty}^{+\infty} g(x) \cdot L(x'-x) dx \quad \text{with } L(x'-x) = \text{point spread function (PSF)}$$

$$b(x') = \int_{-\infty}^{+\infty} g(x'-x) \cdot L(x) dx$$

with chosen function  $g(x)$

$$\begin{aligned} b(x') &= \int_{-\infty}^{+\infty} \{ \bar{g} + K_0 \cdot \sin(2\pi u(x'-x)) \} \cdot L(x) dx \\ &= \bar{g} \int_{-\infty}^{+\infty} L(x) \cdot dx + K_0 \int_{-\infty}^{+\infty} \sin(2\pi u(x'-x)) L(x) dx. \end{aligned}$$

## Modulation Transfer Function MTF

$$\int_{-\infty}^{+\infty} L(x) dx = 1 \text{ (normalization of point spread function)}$$
$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\begin{aligned} b(x') &= \bar{g} + K_0 \cdot \int_{-\infty}^{+\infty} L(x) \sin 2\pi u x' \cdot \cos 2\pi u x \cdot dx \\ &\quad - K_0 \cdot \int_{-\infty}^{+\infty} L(x) \cos 2\pi u x' \cdot \sin 2\pi u x \cdot dx \\ &= \bar{g} + K_0 \cdot \sin 2\pi u x' \int_{-\infty}^{+\infty} L(x) \cdot \cos 2\pi u x \cdot dx \\ &\quad - K_0 \cdot \cos 2\pi u x' \cdot \int_{-\infty}^{+\infty} L(x) \sin 2\pi u x \cdot dx \end{aligned}$$

## Imaging with x-rays

### Modulation Transfer Function MTF

with  $\int_{-\infty}^{+\infty} L(x) \sin 2\pi u x \cdot dx = 0$

and  $\eta(u) = \int_{-\infty}^{+\infty} L(x) \cos 2\pi u x \cdot dx$

symmetric PSF:  $L(x)=L(-x)$

$$b(x') = \bar{g} + K_0 \cdot \eta(u) \cdot \sin 2\pi u x'$$

$\eta$  depends on spatial frequency of original

$\eta \in [-1, 1]$  (normalization of PSF and  $-1 < \cos 2\pi u x < 1$ )

$\eta \rightarrow 1$ , if  $\lambda = 1/u \gg L(x)$  (unbiased transmission of large  $\lambda$ )

$\eta \rightarrow 0$ , if  $\lambda = 1/u \ll L(x)$  (complete loss of image information)

## *Imaging with x-rays*

### **MTF and contrast**

“contrast” of original:

$$\frac{\max[g(x)] - \min[g(x)]}{\max[g(x)] + \min[g(x)]} = \frac{\bar{g} + K_0 - (\bar{g} - K_0)}{\bar{g} + K_0 + \bar{g} - K_0} = \frac{K_0}{\bar{g}}$$

“contrast” of image:

$$\frac{\max[b(x)] - \min[b(x)]}{\max[b(x)] + \min[b(x)]} = \frac{\bar{g} + K_0 \cdot \eta(u) - (\bar{g} - K_0 \cdot \eta(u))}{\bar{g} + K_0 \cdot \eta(u) + \bar{g} - K_0 \cdot \eta(u)} = \eta(u) \frac{K_0}{\bar{g}}$$

## Imaging with x-rays

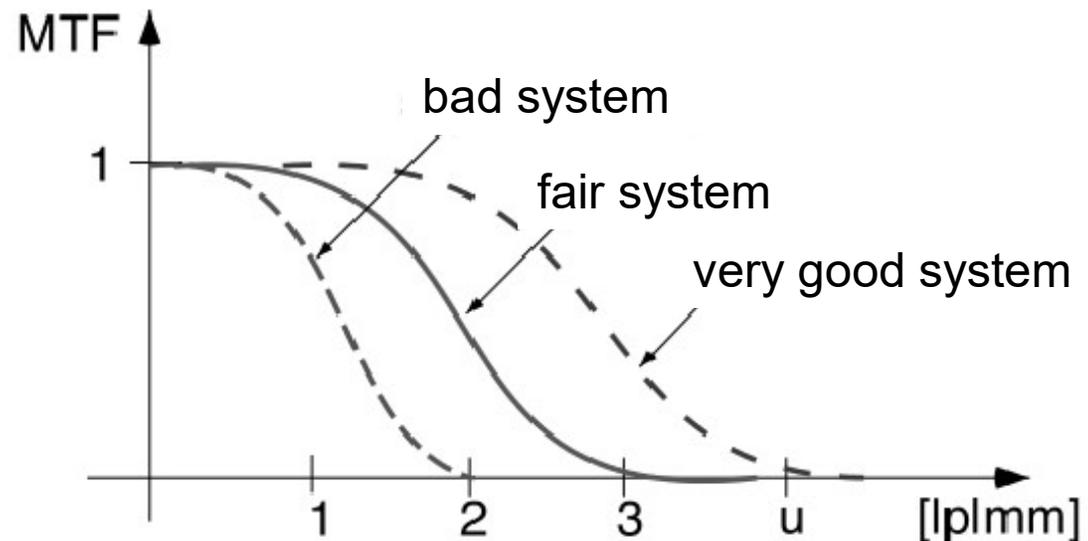
### MTF and contrast

for sinusoidal (!! ) signals, we have:

$$\text{MTF}(u) = \frac{\text{“contrast” of image @output at frequency } u}{\text{“contrast” of original at frequency } u}$$

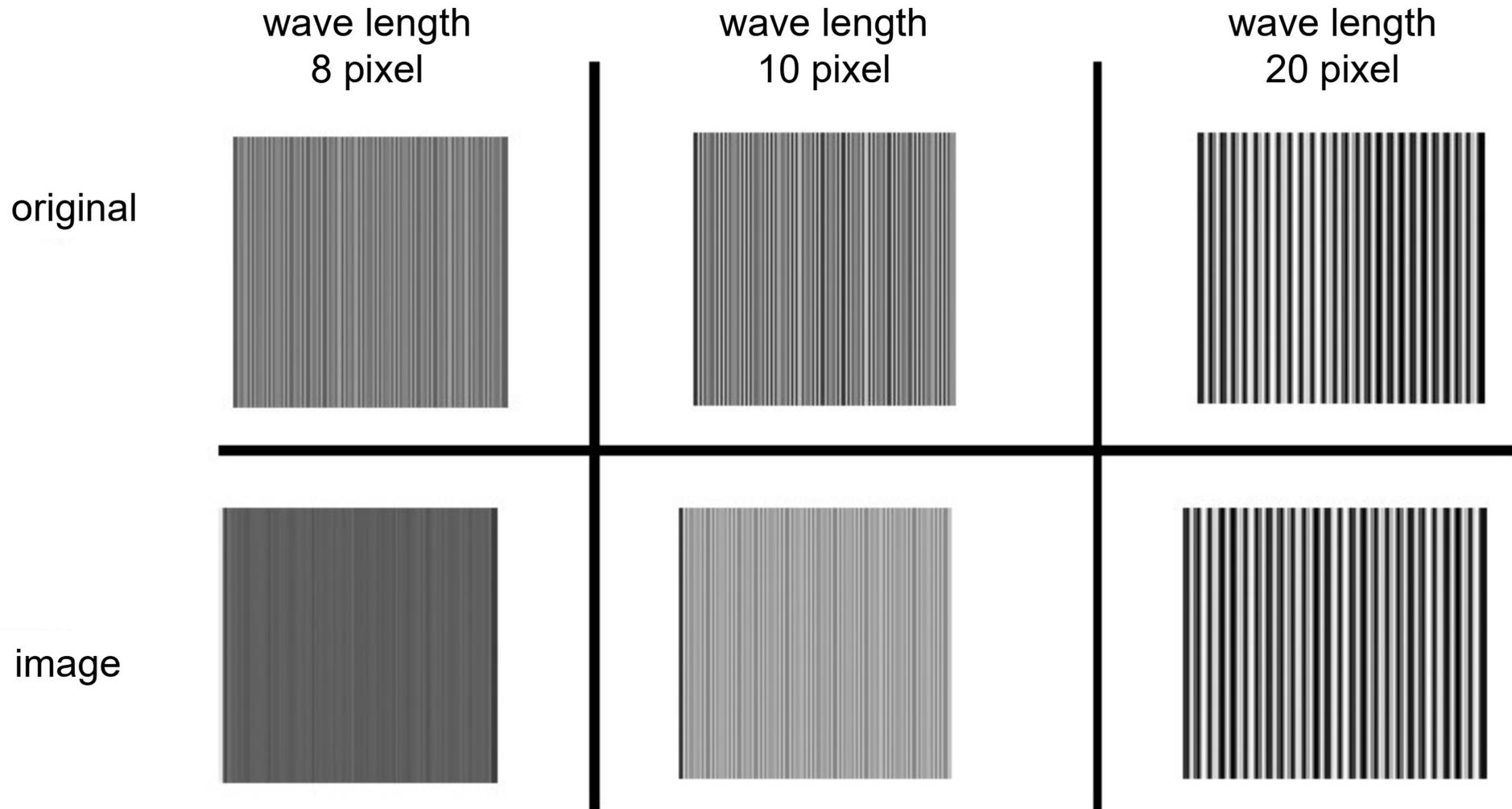
$$\text{MTF}(u) = \eta(u)$$

[u]: line pairs lp/mm



# *Imaging with x-rays*

## **imaging of sine-modulated original**



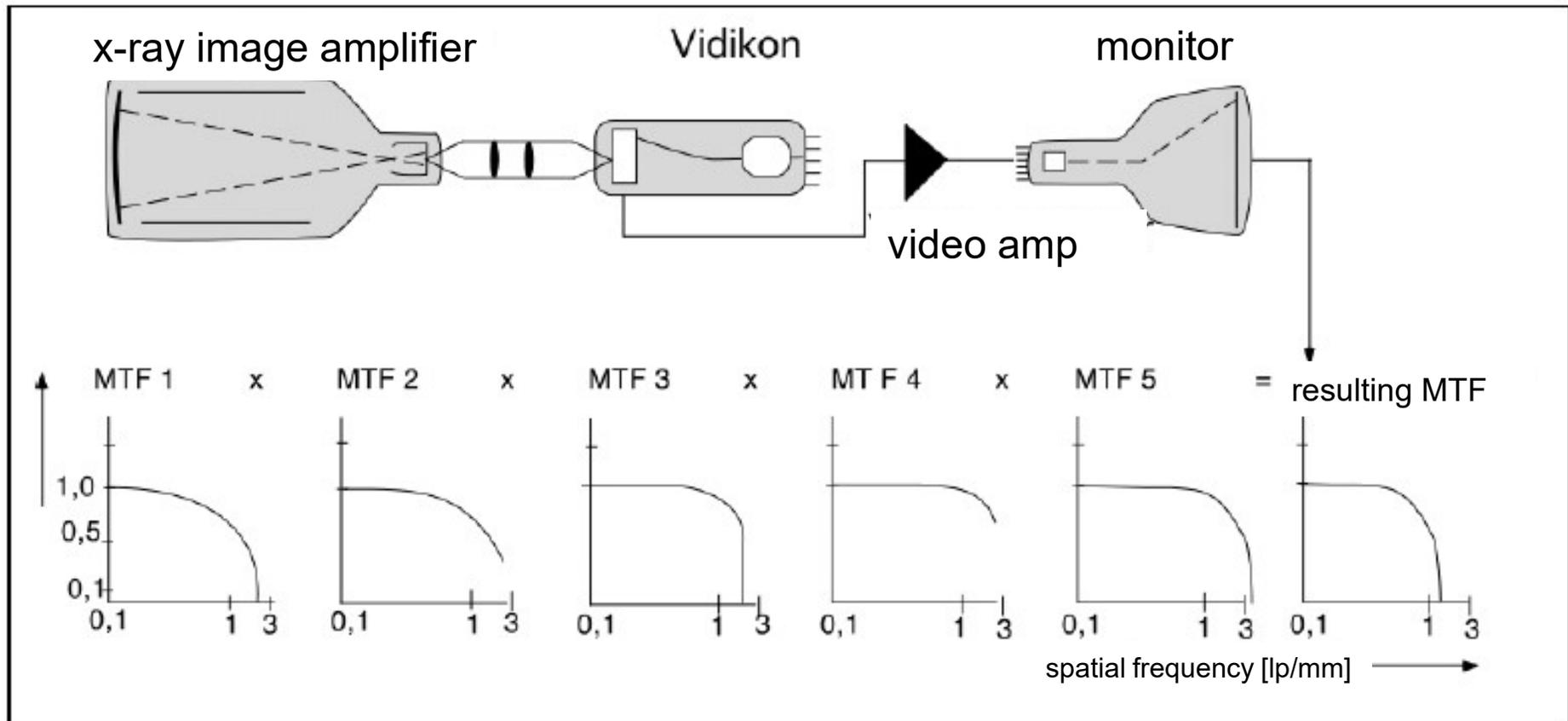
# Imaging with x-rays

## MTF of an imaging chain:

## x-ray image amplifier

$$MTF_{\text{chain}} = MTF(u)_1 \cdot MTF(u)_2 \cdot \dots \cdot MTF(u)_N$$

(since output of component 1 = input in component 2 etc.)

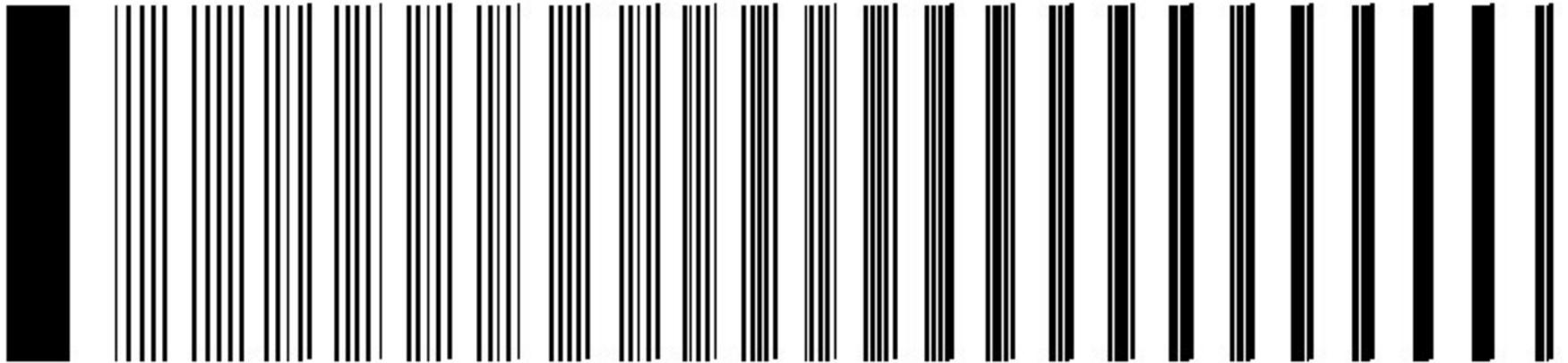


# *Imaging with x-rays*

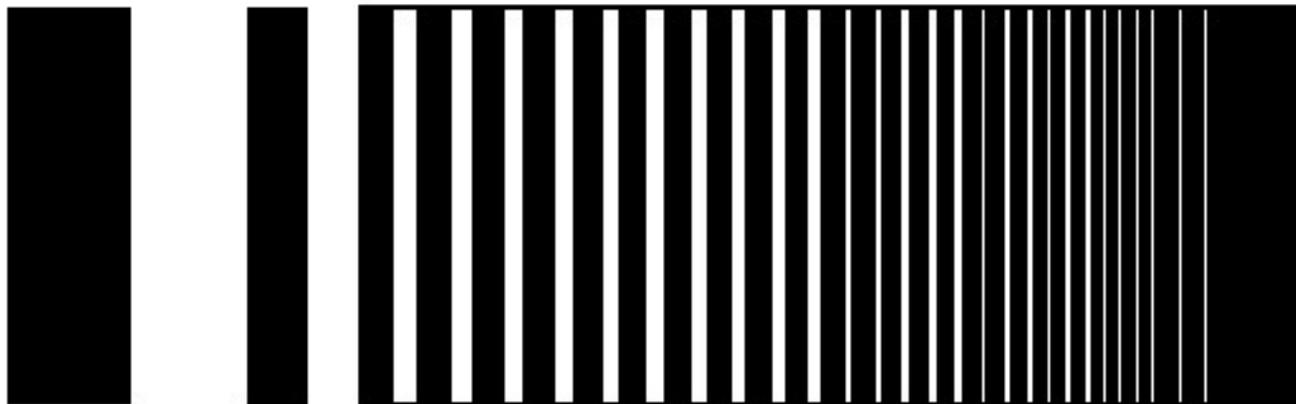
## **experimental assessment of MTF**

lead line test pattern

a)



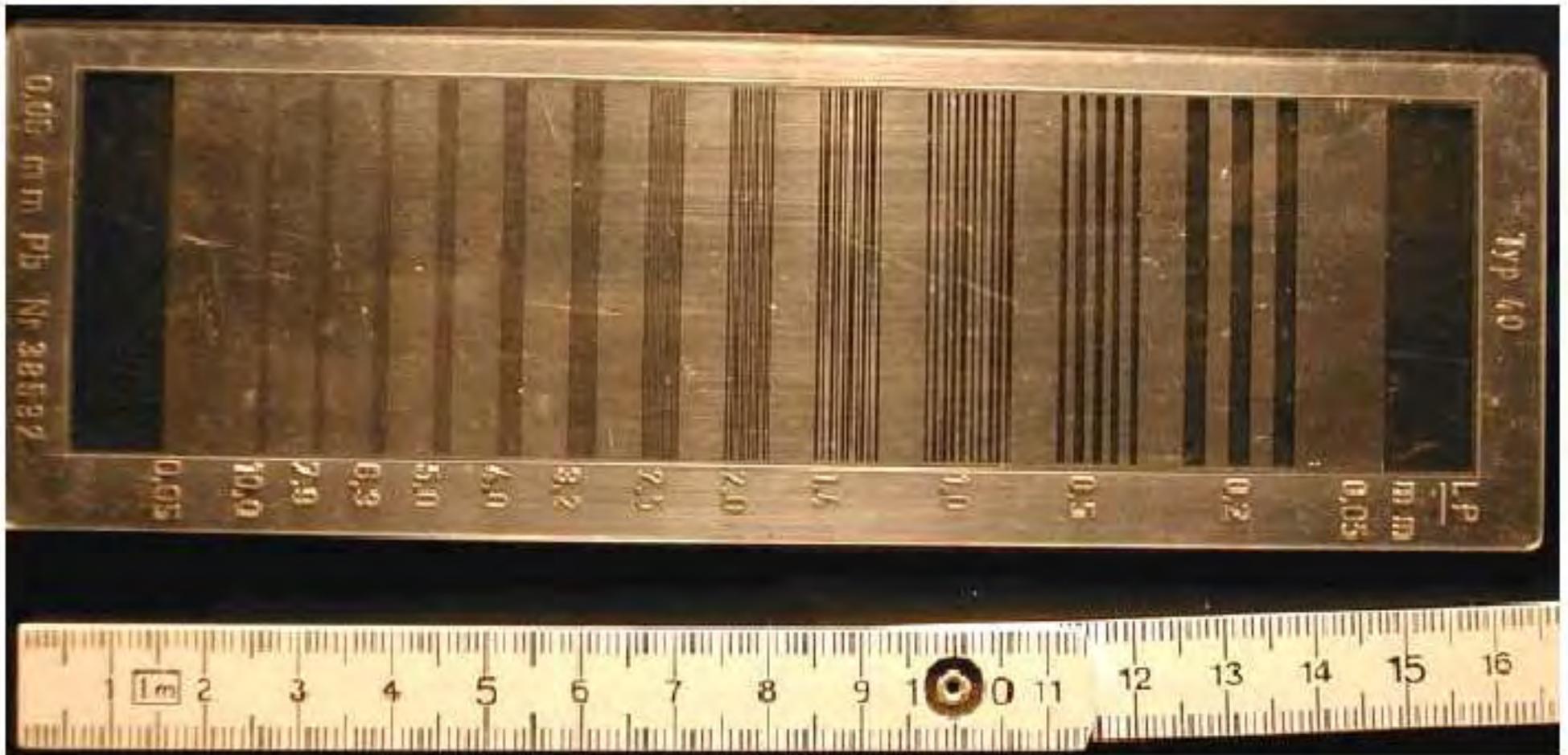
b)



*Imaging with x-rays*

**experimental assessment of MTF**

lead line test pattern

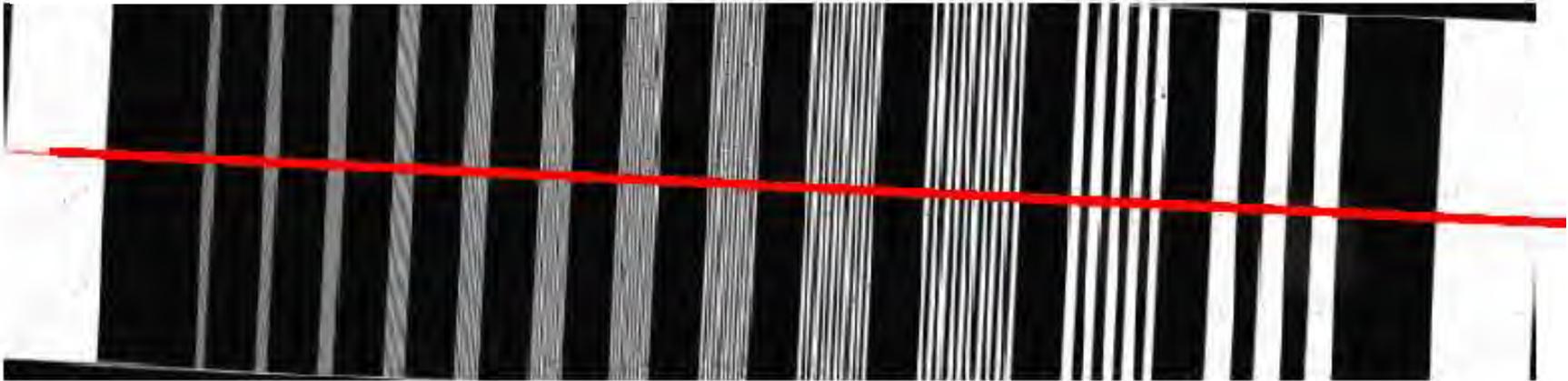


# *Imaging with x-rays*

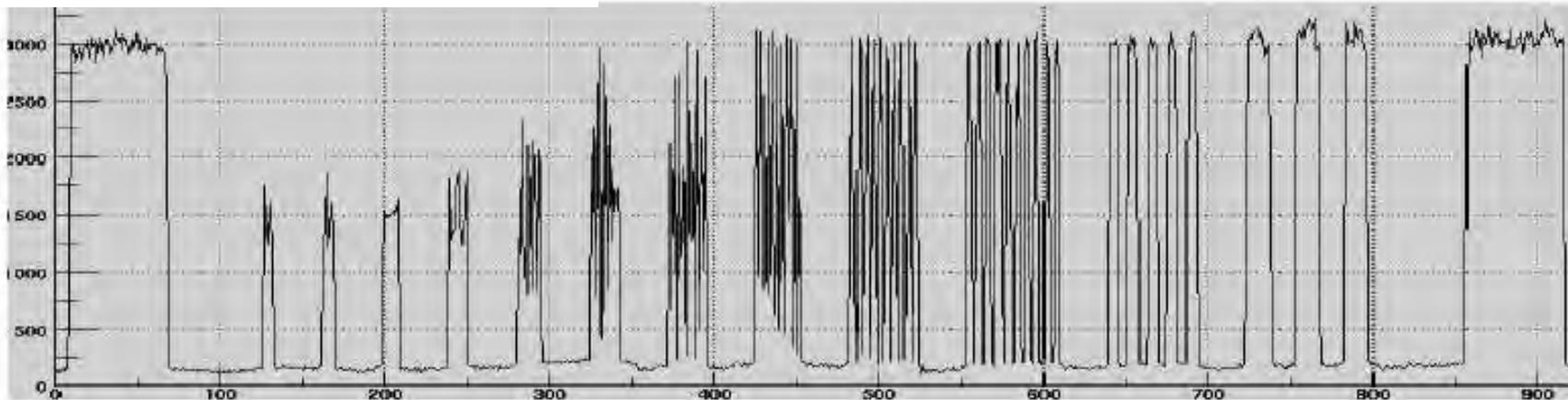
## **experimental assessment of MTF**

lead line test pattern

x-ray image of test pattern



amplitude profile



## *Imaging with x-rays*

### **experimental assessment of MTF**

lead line test pattern

provides information about fundamental frequency AND about odd higher harmonics

this follows from the Fourier series expansion of a rectangular function:

$$\text{MTF}(u) = \frac{\pi}{4} \left| R(u) + \frac{R(3u)}{3} - \frac{R(5u)}{5} + \frac{R(7u)}{7} - \dots \right|$$

where

$$R(u) = \frac{\text{contrast @output}}{\text{contrast @input}} \quad \text{rectangular function}$$

**noise and spatial resolution**

**improving DQE**

**results in**

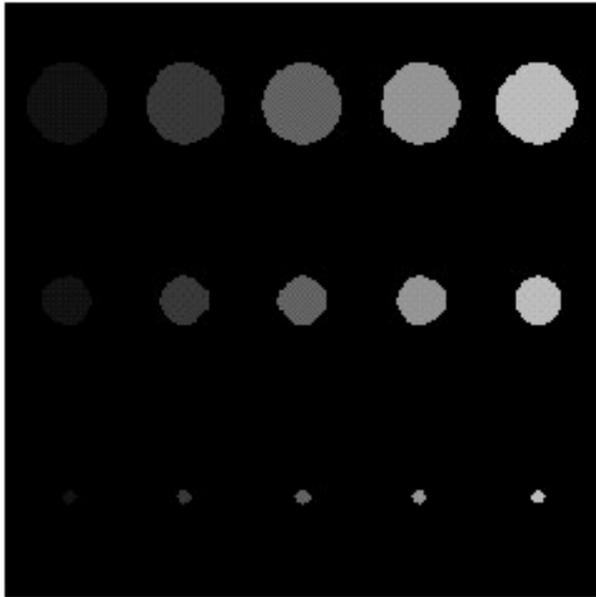
**a deteriorated MTF**

**and vice versa !**

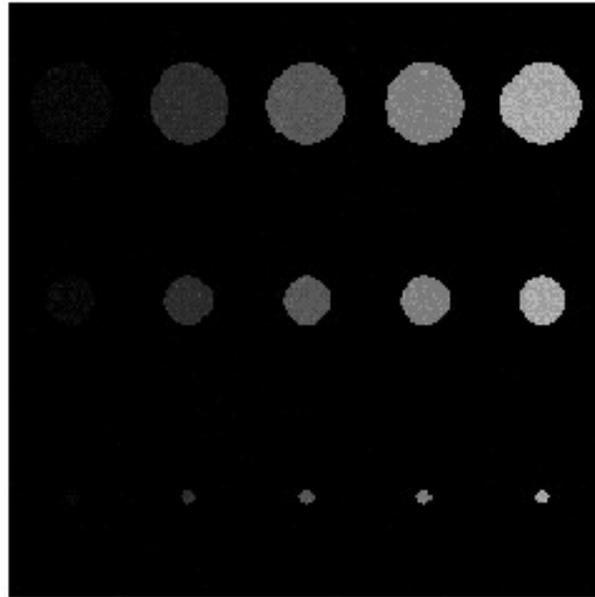
*Imaging with x-rays*

**image quality of x-ray image amplifier**

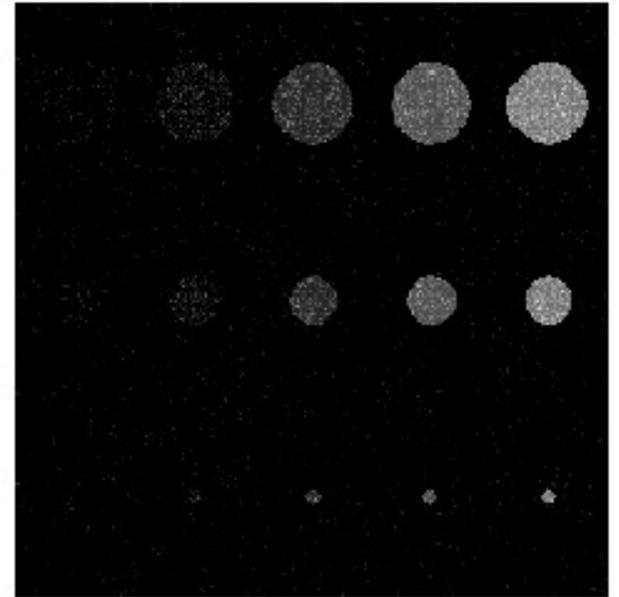
**noise**



no noise



256 quanta/pixel  
noise +/- 16



16 quanta/pixel  
noise +/- 4