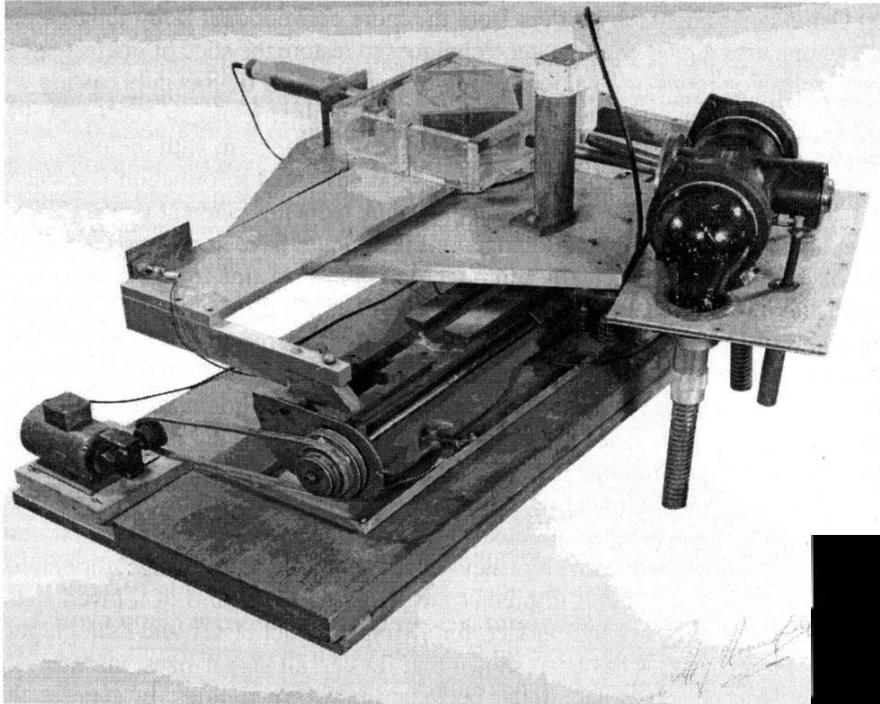


# *x-ray computed tomography (CT)*

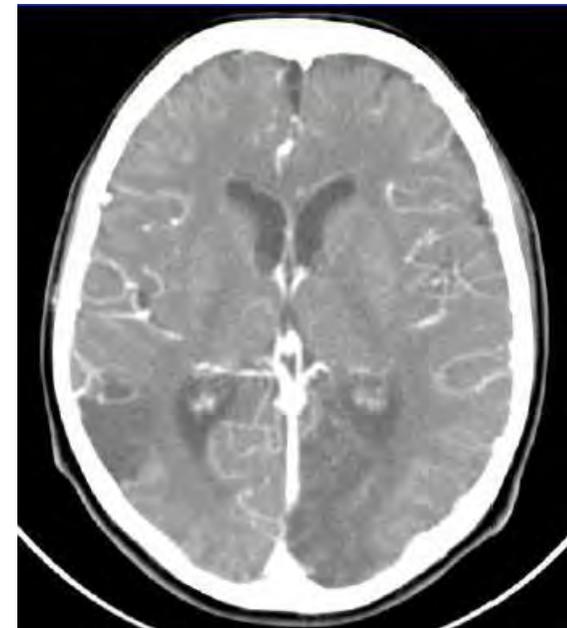


*Godfrey Hounsfield*

Hounsfield, 1969

**additional literature:**

W.A. Kalender: Computertomographie  
Publicis MCD Verlag, 2000



## *x-ray computed tomography (CT)*

### **brief historical overview of CT development**

- 1895 W.C. Röntgen discovers a 'new kind of radiation', later termed "Röntgenstrahlung" (x-rays)
- 1917 J.H. Radon develops mathematical fundamentals to compute tomographic images from transmission measurements
- 1960/1970 improvements of computer technology
- 1972 G.N. Hounsfield und J. Ambrose: first clinical examination with CT
- 1975 first whole-body CT scanner in clinical use
- 1979 Nobel Price awarded to Hounsfield and Cormack
- 1989 first clinical examinations with spiraling CT
- 1998 first clinical examinations with spiraling multi-slice CT
- 2000 approx. 30 000 clinical spiraling CT installations worldwide

*x-ray computed tomography (CT)*

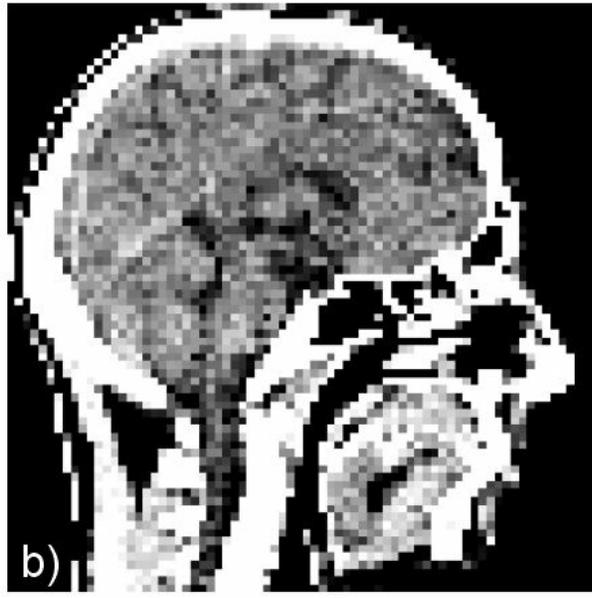
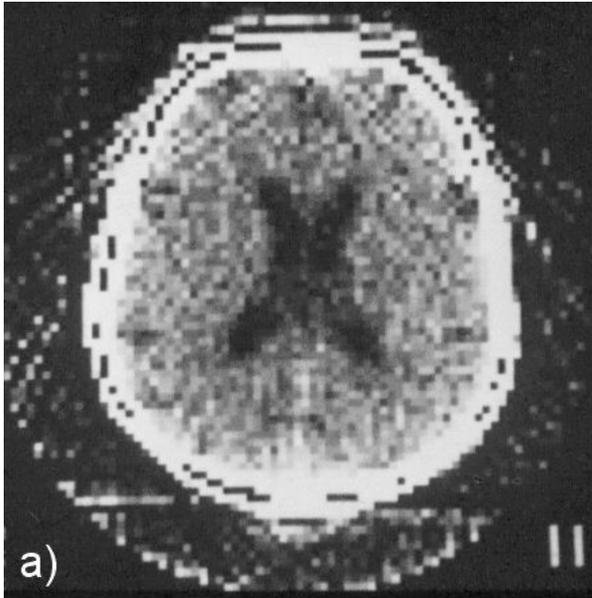
**1974**



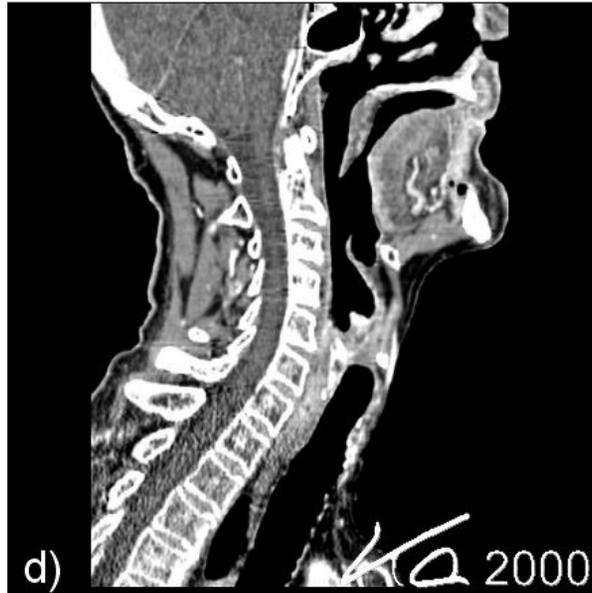
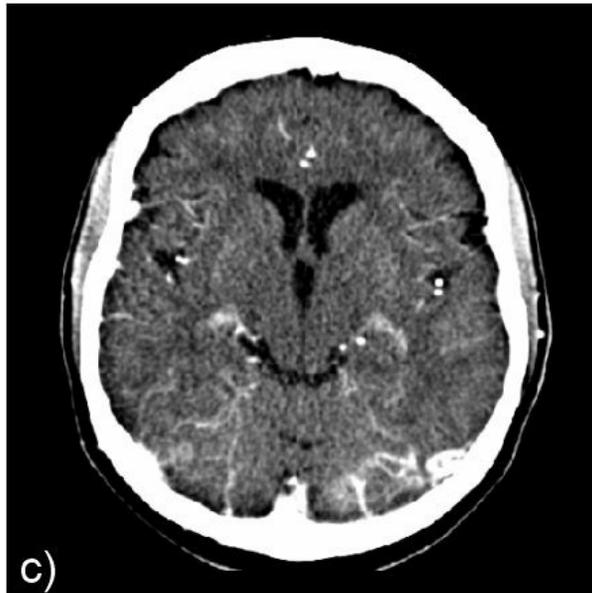
**1994**



# *x-ray computed tomography (CT)*

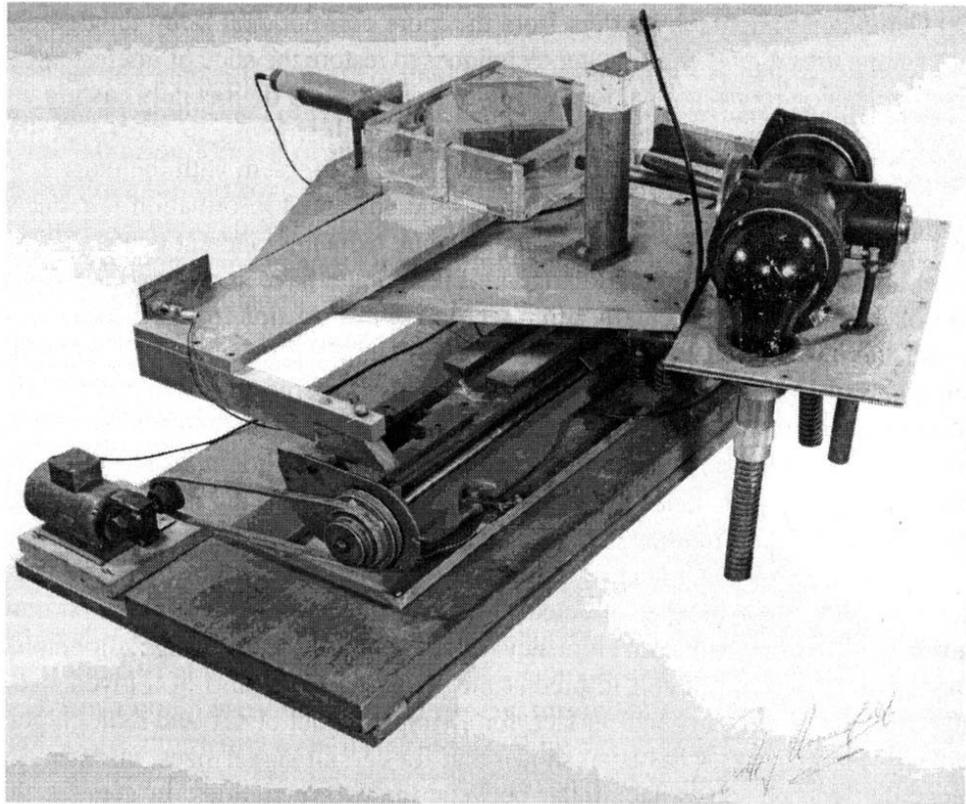


1974  
image matrix: 80 x 80



2000  
image matrix : 512 x 512  
spiraling CT

*x-ray computed tomography (CT)*



Hounsfield, 1969



modern CT scanner

# *x-ray computed tomography (CT)*

## drawbacks of projection radiography:

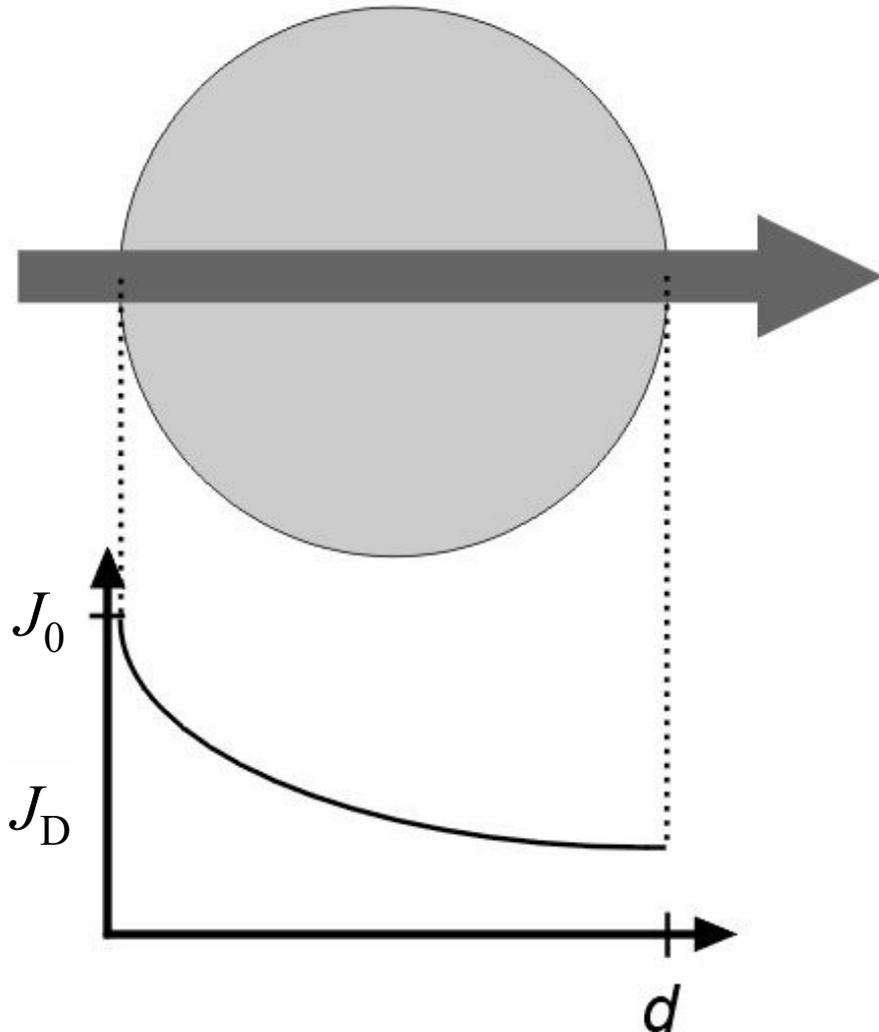
### x-ray image:

- modulated distribution of  $\gamma$ -quanta transmitted through tissue
- 2D projection der attenuation properties of tissue
- all irradiated volume elements contribute to attenuation
- line integral of attenuation:  $J_D = J_0 e^{-\int \mu(x,y,z) dl}$
- contrast mainly from structures with high  $\mu$  (bones) or profound differences in thickness; soft tissue hard to resolve
- projection radiography is not tomography



# *x-ray computed tomography (CT)*

homogeneous object; mono-chromatic radiation



$$J_D = J_0 \cdot e^{-\mu d}$$

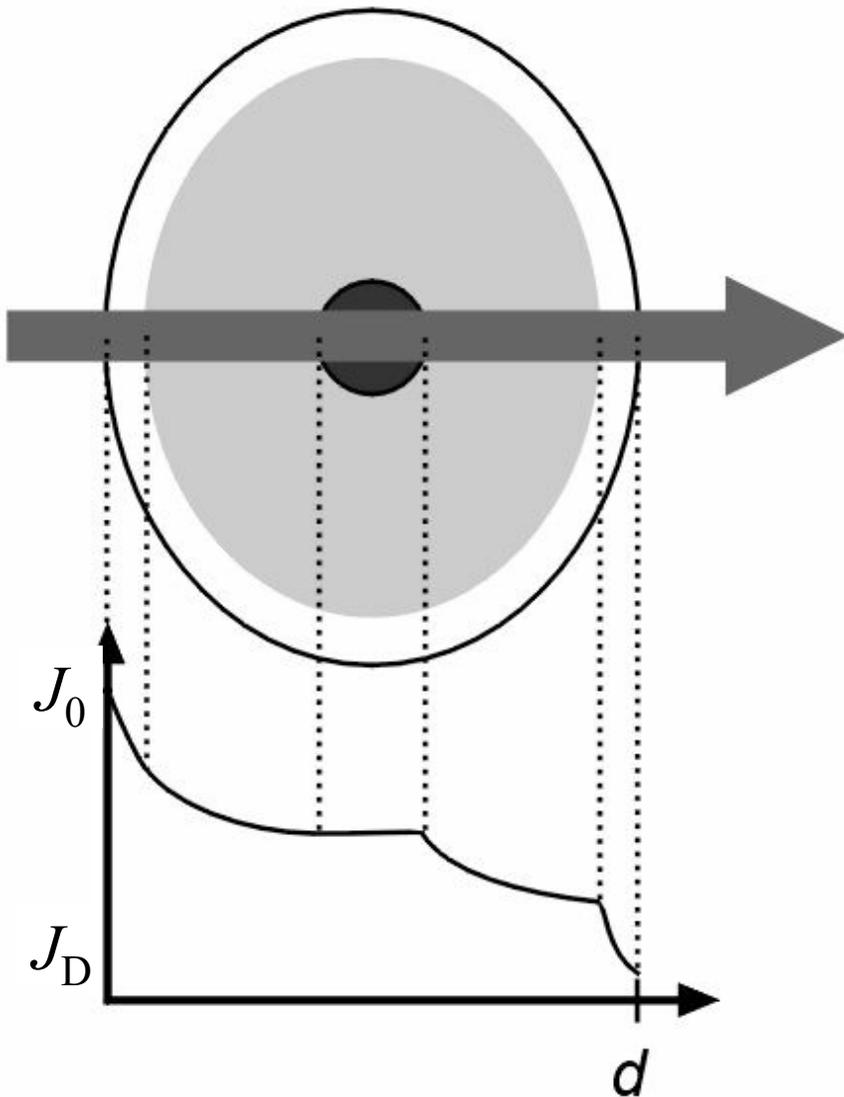
$$P = \ln \frac{J_0}{J_D} = \mu \cdot d$$

$$\mu = \frac{1}{d} \cdot \ln \frac{J_0}{J_D}$$

$P$  = projection value

# x-ray computed tomography (CT)

non-homogeneous object; mono-chromatic radiation



$$J_D = J_0 \cdot e^{-\mu_1 d_1 - \mu_2 d_2 - \mu_3 d_3 - \dots}$$

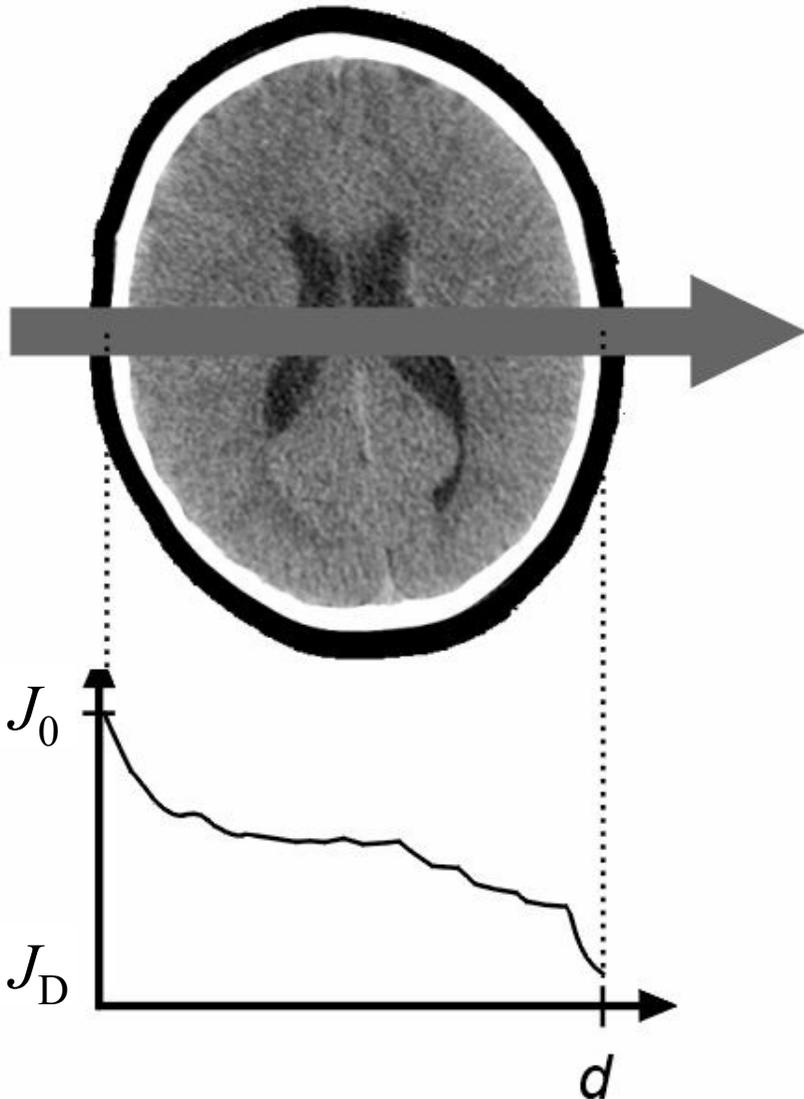
$$= J_0 \cdot e^{-\sum_i \mu_i \cdot d_i} = J_0 \cdot e^{-\int_0^d \mu ds}$$

$$P = \ln \frac{J_0}{J_D} = \sum_i \mu_i \cdot d_i$$

$$\mu_i = ??$$

# x-ray computed tomography (CT)

non-homogeneous object; poly-chromatic radiation



$$J_D = \int_0^{E_{\max}} J_0(E) \cdot e^{-\int_0^d \mu ds} dE$$

$$P = \ln \frac{J_0}{J_D}$$

$$\mu(x, y) = ??$$

*x-ray computed tomography (CT)*

**fundamentals  
of  
x-ray computed tomography**

## **fundamentals of computed tomography\*:**

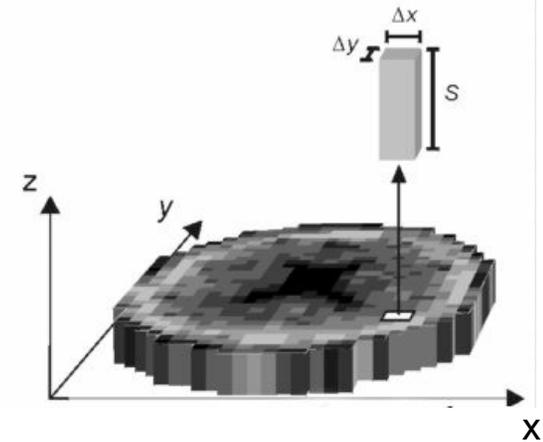
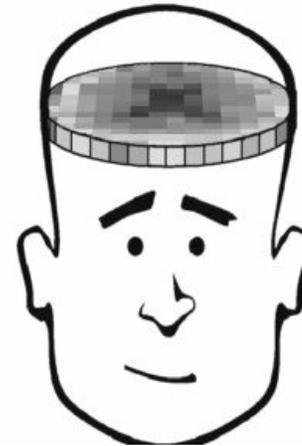
- measure spatial distribution of physical observable of interest (e.g., attenuation coefficient  $\mu(x,y)$ ) of object of interest
- from measured values estimate non-overlapping images (Radon transform and Fourier-slice theorem)

\*holds for any kind of tomographic imaging technique (x-ray CT, PET, MRI, ...)

# *x-ray computed tomography (CT)*

## **idea: record information from single slices**

consider human body as being composed of finite many discrete volume elements



## **in coarse-grained resolution:**

- single transversal slices of thickness  $s$
- slices are composed of discrete cuboid volume elements

*voxel* (volume element)

*pixel* (picture element)

## *x-ray computed tomography (CT)*

### **recording tomographic images and inverse problem**

**inverse problem:** given set of  $N_p$  recordings (tomographic images) outside of some object, estimate the distribution of some physical property (e.g.,  $\mu(x,y)$ ) inside the object

**H. von Helmholtz:** *the inverse problem has no unique solution*

**J. Radon (1917):** the 2D distribution of some object property can be fully characterized with an **infinite amount of line integrals**

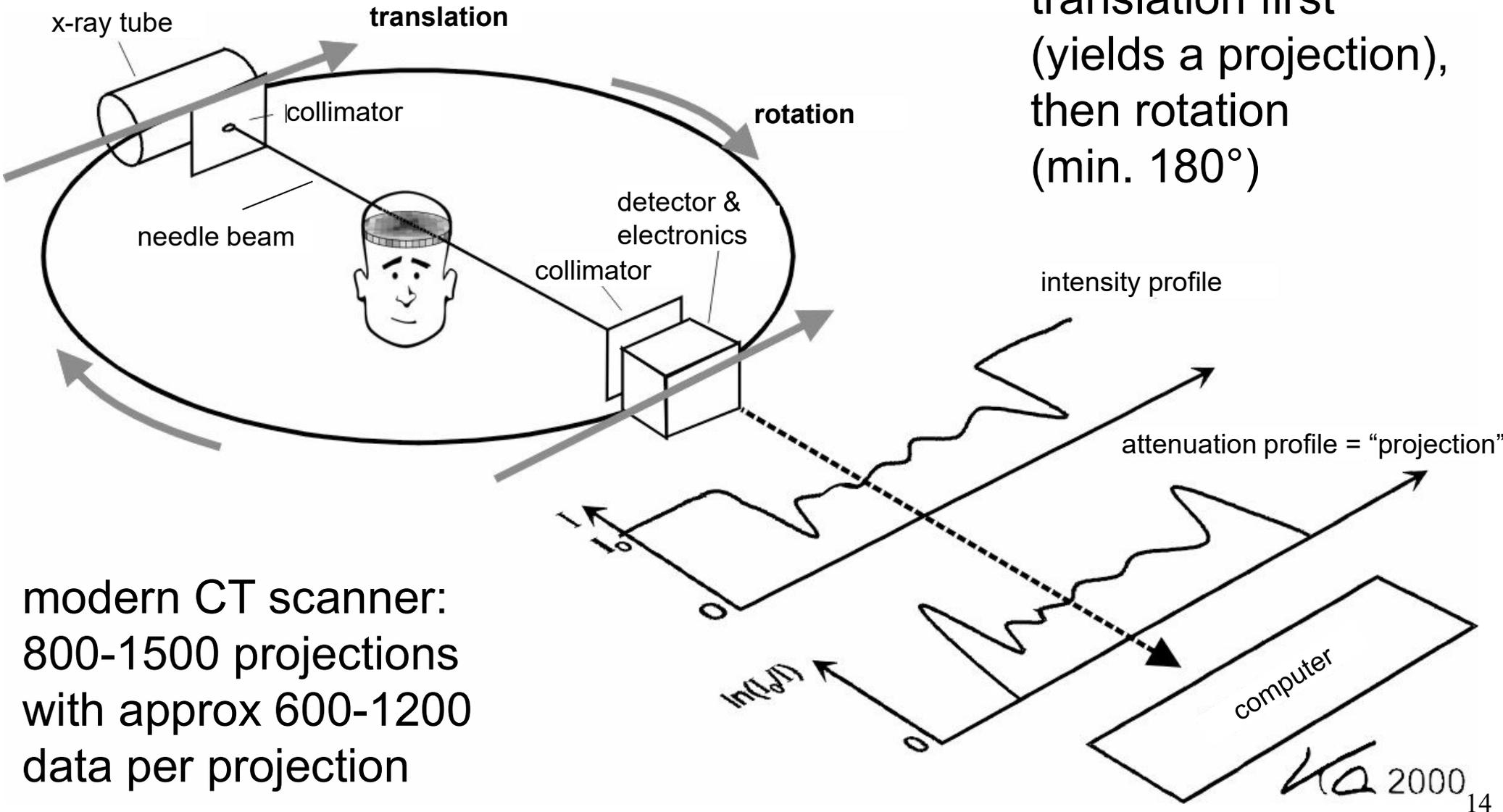
⇒ a finite number  $N_p$  of recordings allows a *sufficiently accurate approximation* of the distribution of some physical property

**(forward problem:** given the distribution of some physical property inside the object, determine result of measurement outside the object)

# *x-ray computed tomography (CT)*

## simplest measurement principle

with Radon's demand:  
translation first  
(yields a projection),  
then rotation  
(min. 180°)

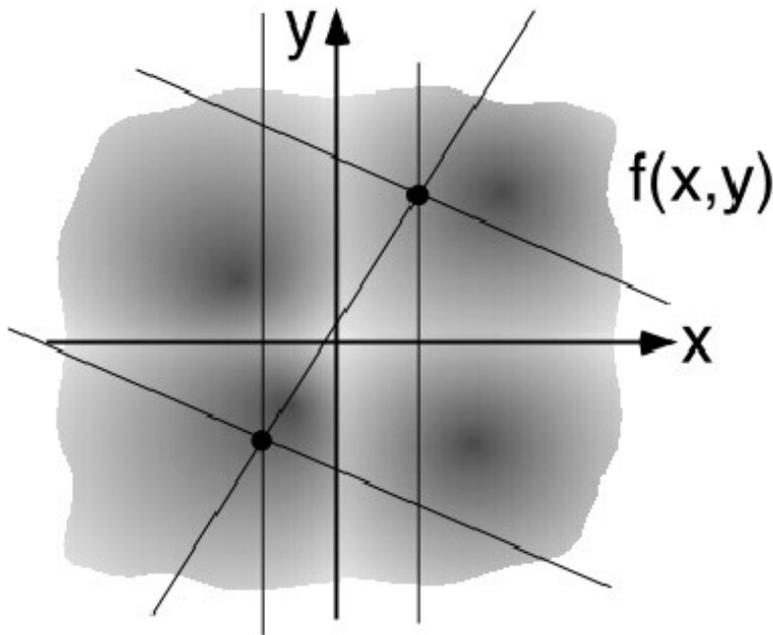


modern CT scanner:  
800-1500 projections  
with approx 600-1200  
data per projection

# *x-ray computed tomography (CT)*

## Radon transform (I)

**J. Radon (1917):** the 2D distribution of some object property can be fully characterized with an **infinite amount of line integrals**



let  $f(x,y)$  denote some arbitrary integrable function

characterize  $f(x,y)$  with all straight line integrals passing through the area on which  $f(x,y)$  is defined:

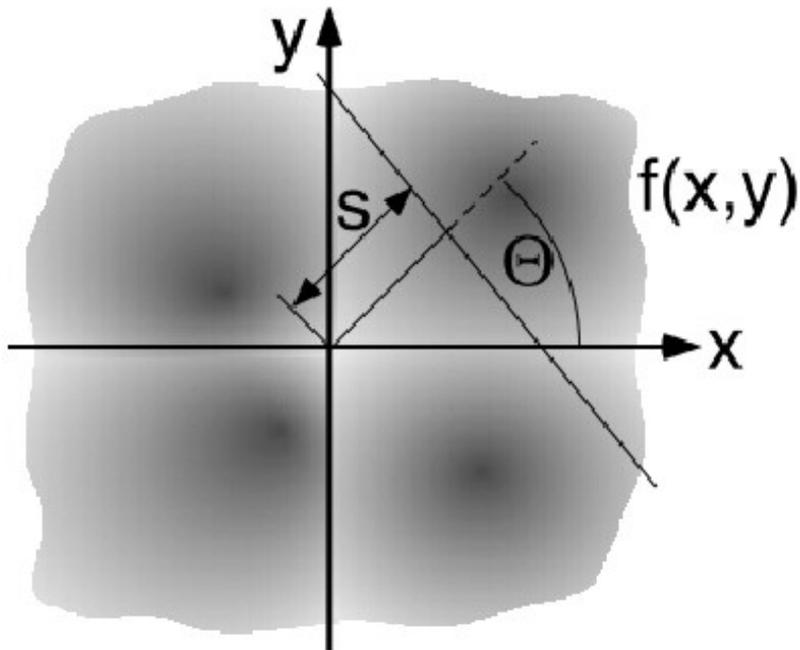
$$\int_{-\infty}^{+\infty} f(x(l), y(l)) dl$$

## Radon transform (II)

naïve ansatz: successive integration through all points and along all directions

⇒ some line integrals are identical

⇒ choose appropriate ordering scheme such that all line integrals are unique (e.g., Hesse normal form)



$$\int_{\vec{e} \cdot \vec{r} = s} f(x, y) dl = p_{\Theta}(s)$$

$\vec{e}$  = unit vector in direction  $\Theta$

$\Theta$  = angle between line of integration  
and normal passing through origin

# *x-ray computed tomography (CT)*

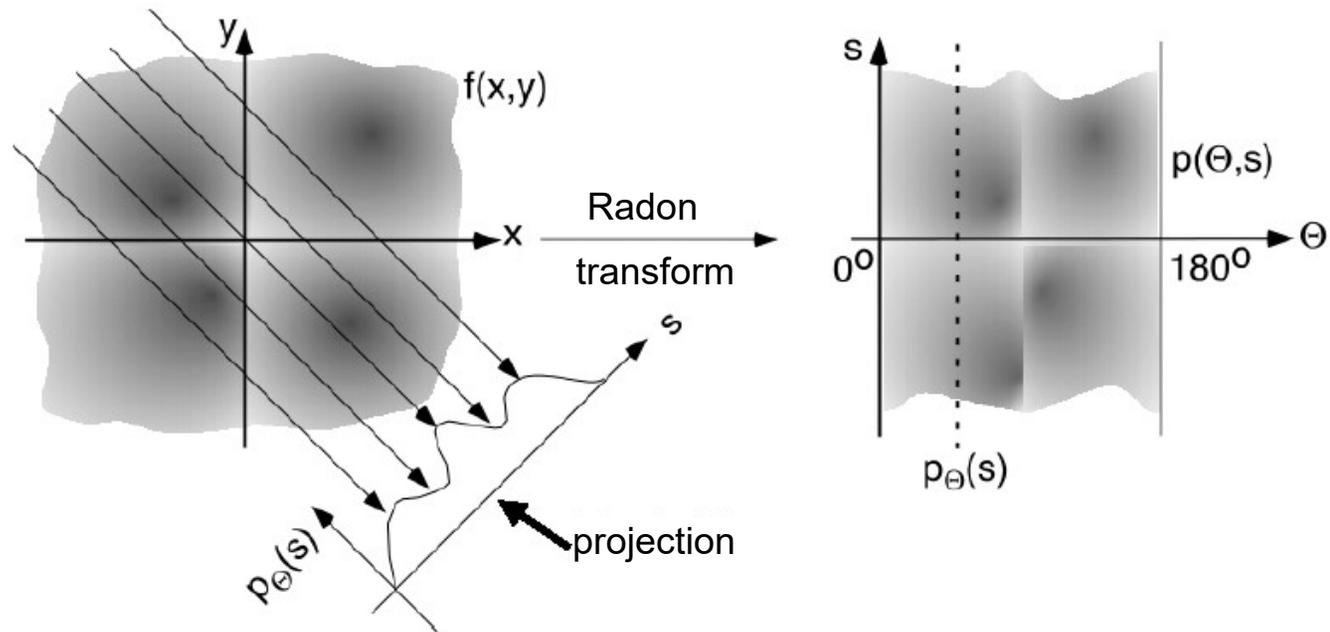
## **Radon transform (II)**

with  $\Theta \in [0^\circ, 180^\circ]$  and with all  $s: (s_{\min} < s < s_{\max})$   
 $\Rightarrow$  all possible line integrals  $p_\Theta(s)$  through function  $f(x,y)$

**assign values of line integrals to  $p(\Theta,s)$ -diagram**

a line in Radon space with  $\Theta = \text{const}$  is called projection  $p_\Theta(s)$

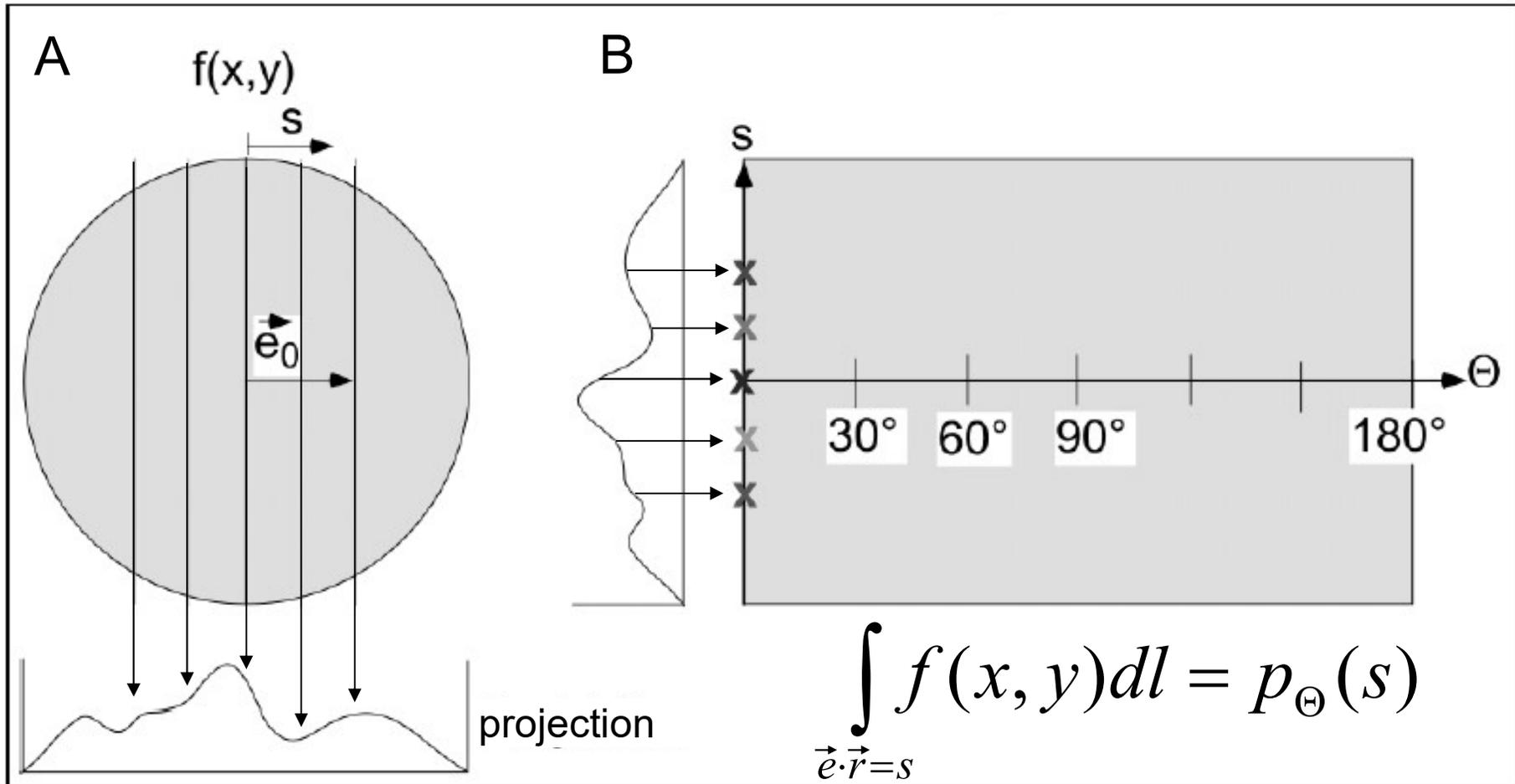
$p_\Theta(s)$  = sequence of values of line integrals passing through  $f(x,y)$  with  $\Theta = \text{const.}$  and distance  $s$  to the origin



# x-ray computed tomography (CT)

## Radon transform (III-1)

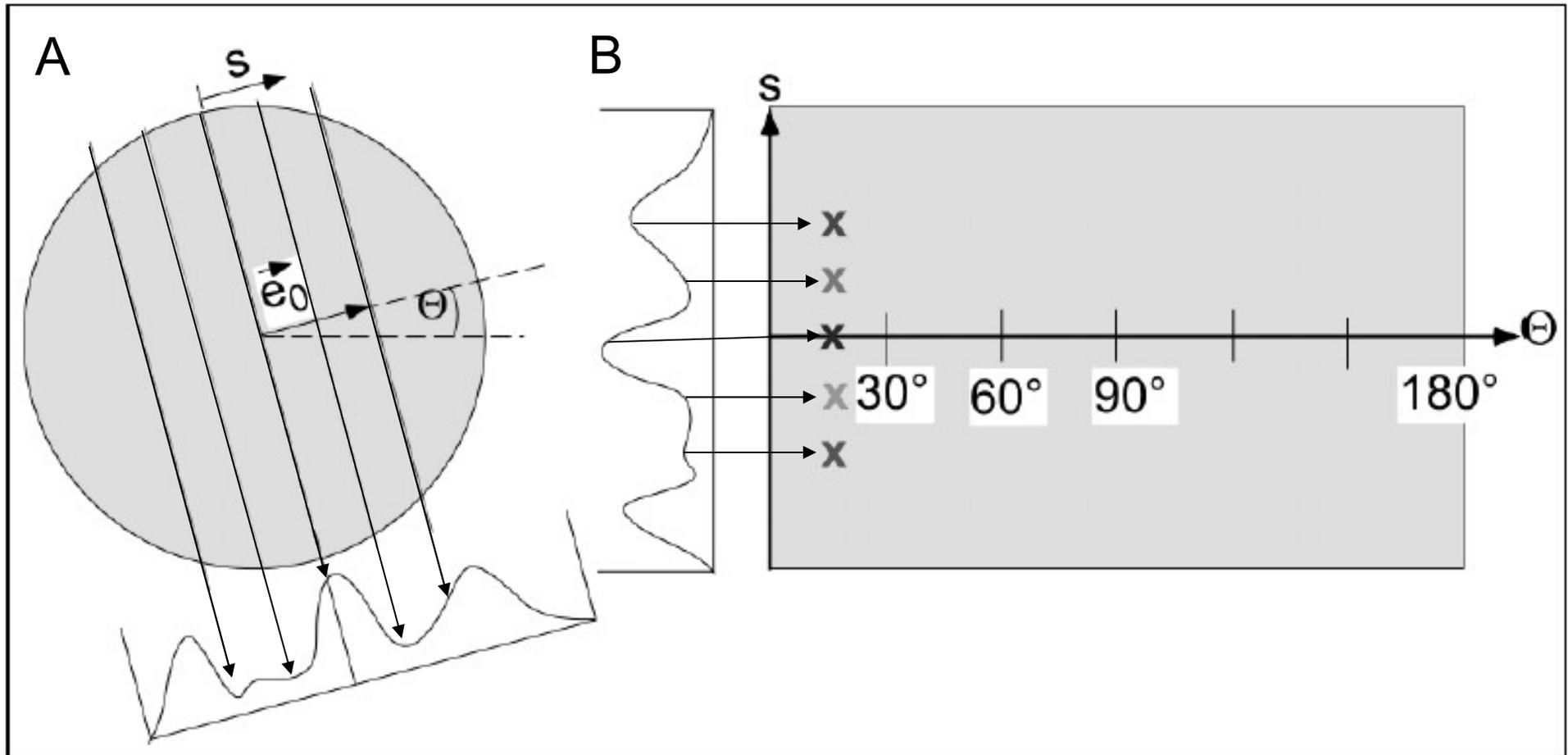
estimate Radon transform for  $\Theta=0$



# *x-ray computed tomography (CT)*

## **Radon transform (III-2)**

estimate Radon transform for  $\Theta \neq 0$



## *x-ray computed tomography (CT)*

**Fourier-slice theorem** (or projection-slice theorem or central slice theorem )

the results of the following two calculations are equal:

- take a 2D function  $f(x,y)$ , project it onto a (1D) line (projection operator  $P_1$ ), and do a Fourier transform ( $F_1$ ) of that projection
- take that same function, but do a 2D Fourier transform ( $F_2$ ) first, and then slice it through its origin (slicing operator  $S_1$ ), which is parallel to the projection line

$$F_1 P_1 = S_1 F_2$$

## *x-ray computed tomography (CT)*

### **Fourier-slice theorem**

- a recording with a needle beam (line integral) for given angle  $\Theta$  and distance  $s$  from the origin (i.e., projection values  $p_{\Theta}(s)$ ) corresponds to the Radon transform of an image.
- the Fourier-slice theorem allows one to determine the function  $f(x,y)$  (i.e.,  $\mu(x,y)$ ) from the Radon transform

### **relation between Radon- und Fourier-transform:**

let  $R f(\vec{e}, s) = \int_{\vec{e} \cdot \vec{r}} f(\vec{r}) d\vec{r}$  denote the Radon transform of function  $f$

with  $G(\alpha) = F(u, v)$  for  $(u, v) = \alpha \cdot (\cos \Theta, \sin \Theta)$

we have

$$G(\alpha) = F_1 \left\{ R_{\Theta} f(s) \right\}$$

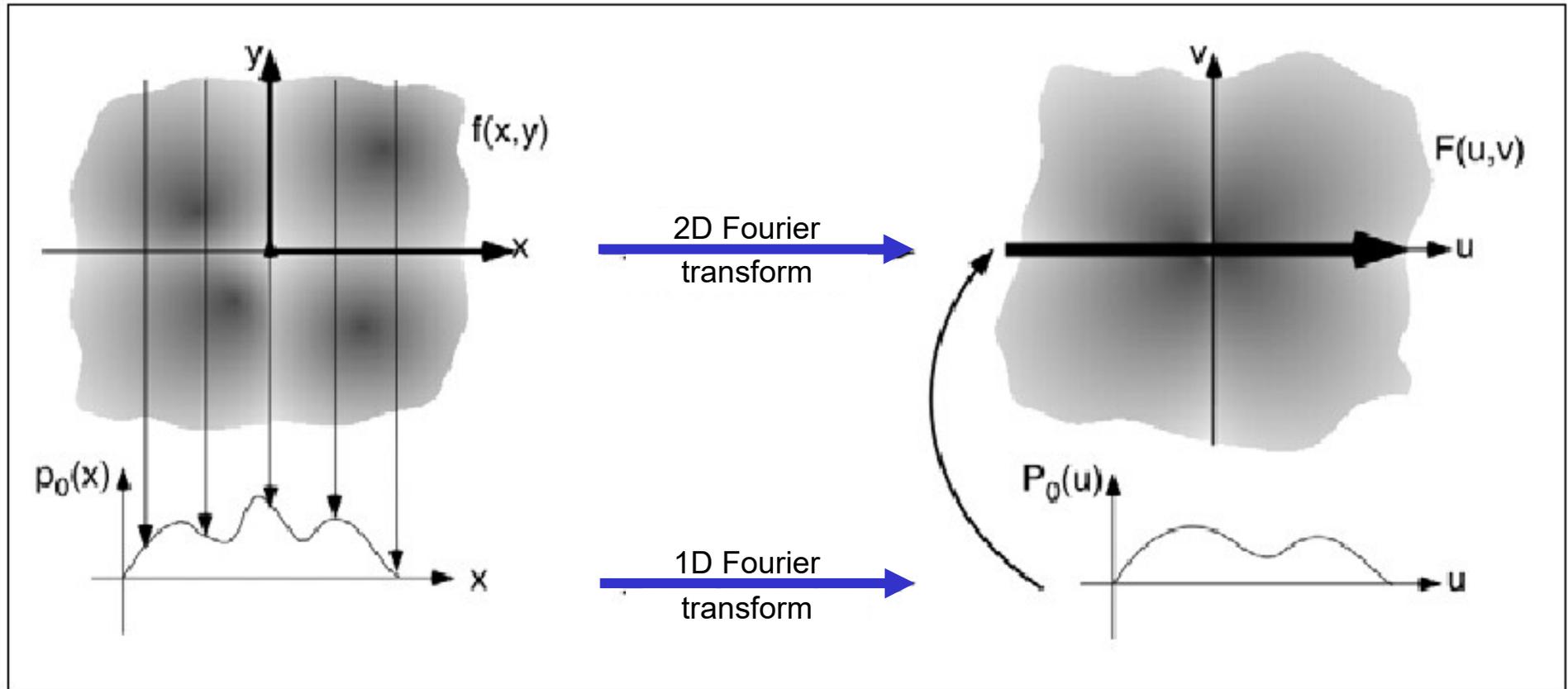
$$F(u, v) = F_2 \left\{ f(x, y) \right\}$$

→ find  $\mu(x,y)$  with inverse 2D-FT

# *x-ray computed tomography (CT)*

## **Fourier-slice theorem (proof for $\Theta=0$ )**

projections for  $\Theta=0$



## *x-ray computed tomography (CT)*

### **Fourier-slice theorem (proof for $\Theta=0$ )**

$$p_0(s) = p_0(x) = \int_{-\infty}^{+\infty} f(x,y)dy \quad \text{for } \Theta = 0^\circ$$

1D Fourier transform of  $p_0(x)$  is defined as:

$$P_0(u) = \int_{-\infty}^{+\infty} p_0(x)e^{-j2\pi ux}dx$$

this can be rearranged as:

$$P_0(u) = \int_{-\infty}^{+\infty} \left[ \int f(x,y)dy \right] e^{-j2\pi ux}dx = \iint f(x,y)e^{-j2\pi(ux+0\cdot y)}dxdy = F(u,0)$$

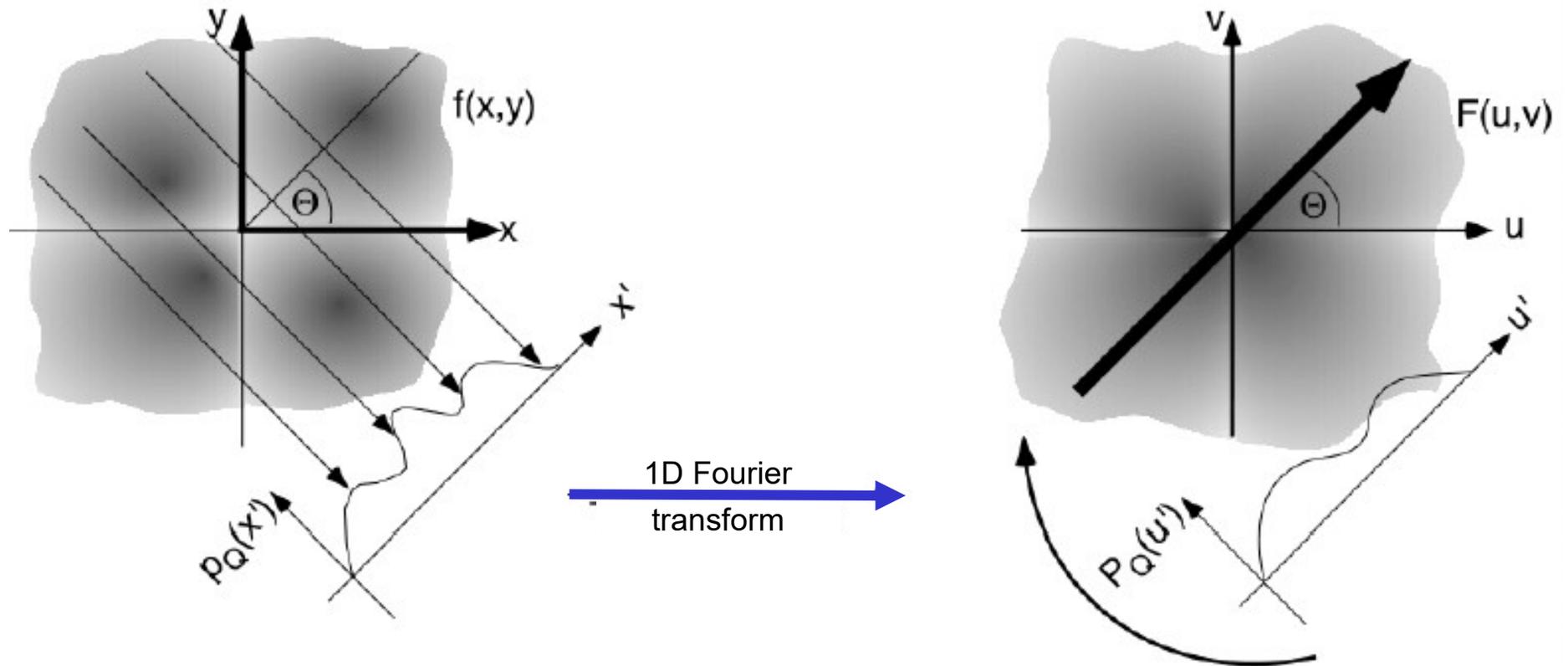
where:  $f(x,y) \xrightarrow{\text{2D-FT}} F(u,v)$

1D-FT( $p_{\Theta=0}(s)$ ) yields results of 2D-FT( $f(x,y)$ ) on the  $u$ -axis

# *x-ray computed tomography (CT)*

## **Fourier-slice theorem (proof for $\Theta \neq 0$ )**

projections for  $\Theta \neq 0$



## **Fourier-slice theorem (proof for $\Theta \neq 0$ )**

projection  $p_{\Theta}(s)$  can be regarded as a projection onto the  $x'$ -axis of a rotated coordinate system.

the same derivation as with  $\Theta=0$  holds:

1D-FT( $p_{\Theta}(s)$ ) yields results of 2D-FT( $f(x,y)$ ) on the  $u'$ -axis

in general, we have:

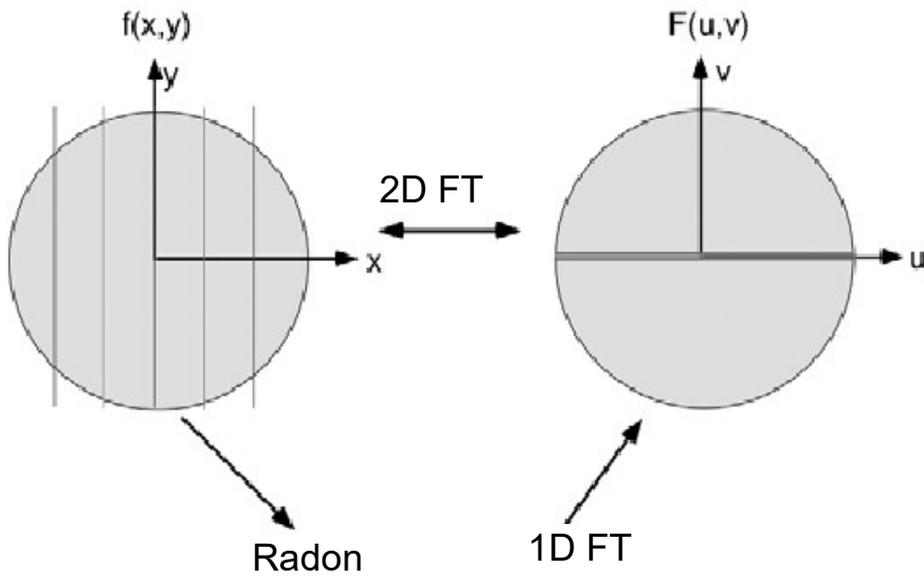
FT of some function  $f(x,y)$  rotated by an angle  $\Theta$  is rotated by the same angle  $\Theta$  with respect to FT of  $F(u,v)$ .

□

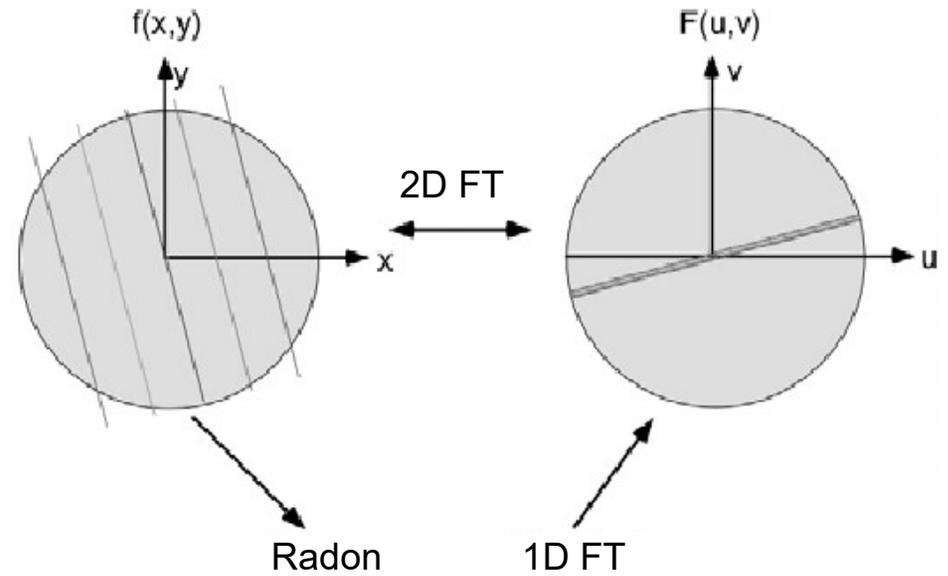
# x-ray computed tomography (CT)

## Fourier-slice theorem

$\Theta = 0$



$\Theta \neq 0$



# *x-ray computed tomography (CT)*

## **Fourier-slice theorem**

given a function  $f(x,y)$  and its 2D FT  $F(u,v)$

$$f(x,y) \quad \circ \text{---} \overset{\text{2D-FT}}{\text{---}} \quad \circ \quad F(u,v)$$

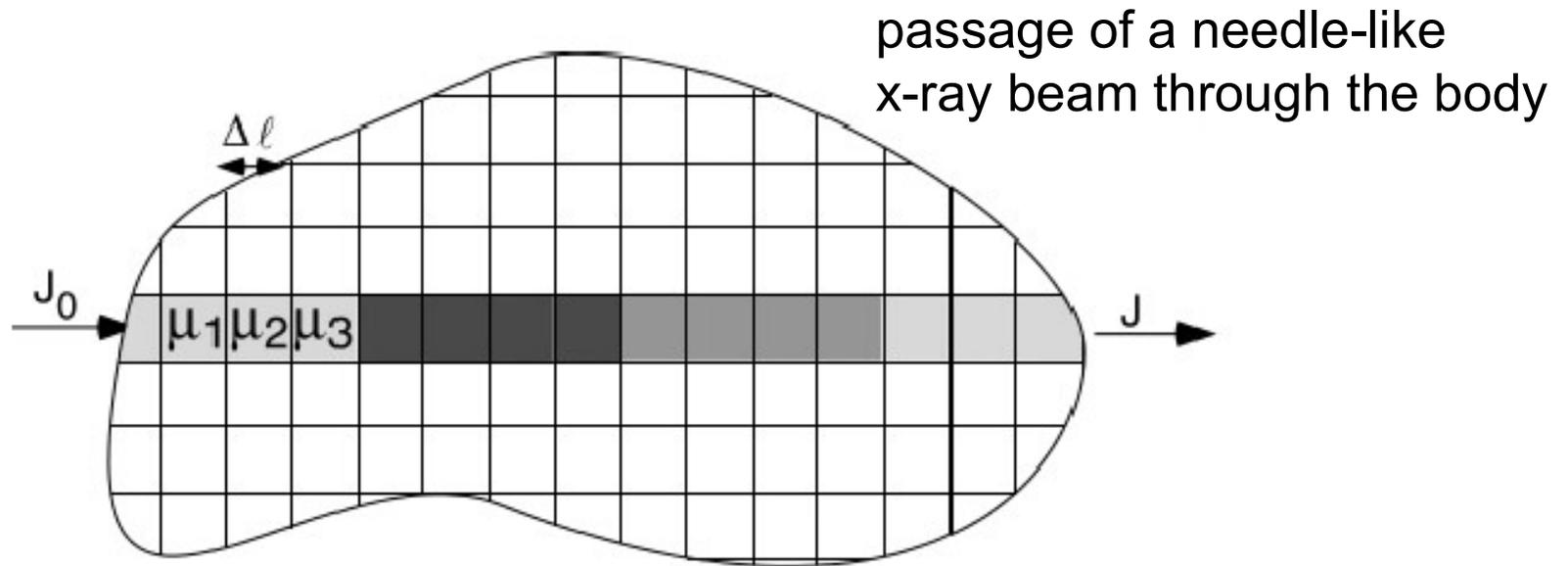
and let  $p_{\Theta}(s)$  denote a projection of  $f(x,y)$  and  $P_{\Theta}(w)$  its 1D FT

$$p_{\Theta}(s) \quad \circ \text{---} \overset{\text{1D-FT}}{\text{---}} \quad \circ \quad P_{\Theta}(w)$$

Then,  $P_{\Theta}(w)$  equals the values of  $F(u,v)$  on a radial beam with angle  $\Theta$ .

# x-ray computed tomography (CT)

## Radon transform and computed tomography



$$J \approx J_0 \cdot e^{-\mu_1 \cdot \Delta l} \cdot e^{-\mu_2 \cdot \Delta l} \cdot \dots \cdot e^{-\mu_N \cdot \Delta l}$$

$$J \approx J_0 \cdot e^{-\sum_{i=1}^N \mu_i \cdot \Delta l}$$

$$J = J_0 \cdot e^{-\int \mu(l) dl}$$

$$\ln\left(\frac{J_0}{J}\right) = \int \mu(l) dl.$$

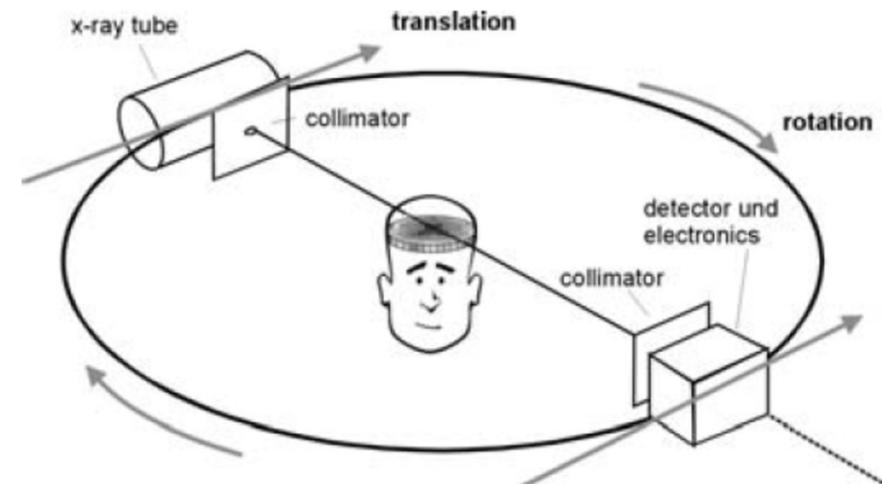
measured values (projections) are the line integrals of the attenuation coefficients

## *x-ray computed tomography (CT)*

**reconstructing  $f(x,y) = \mu(x,y)$  from Radon transform**

given sufficient number of projections  $p_{\Theta}(s)$ :

- estimate all 1D FT of  $p_{\Theta}(s)$  ( $= P_{\Theta}(w)$ )
- for given angle  $\Theta$ , assign these values to matrix  $F(u,v)$
- estimate inverse FT of  $F(u,v)$  ( $= f(x,y)$ )



## *x-ray computed tomography (CT)*

### **Radon transform and computed tomography**

#### **caveat:**

- recorded data stem from radial beams !
- fast Fourier transform (FFT) requires interpolation onto square lattice (polar coordinates → Cartesian coordinates)
- interpolation can lead to serious artifacts !

(modern scanner allow sampling on square lattice)

## *x-ray computed tomography (CT)*

### **iterative image reconstruction (I)**

- unknown distribution  $\mu(x,y)$  only available as projection values (Radon transform) at the end of recording
- use appropriate back transform to find  $\mu(x,y)$

simplest ansatz:

wanted:  $N \times N$  pixels in matrix:  $N^2$  values

given:  $M = N_p \times N_d$   
= number of projections x number of data/projection

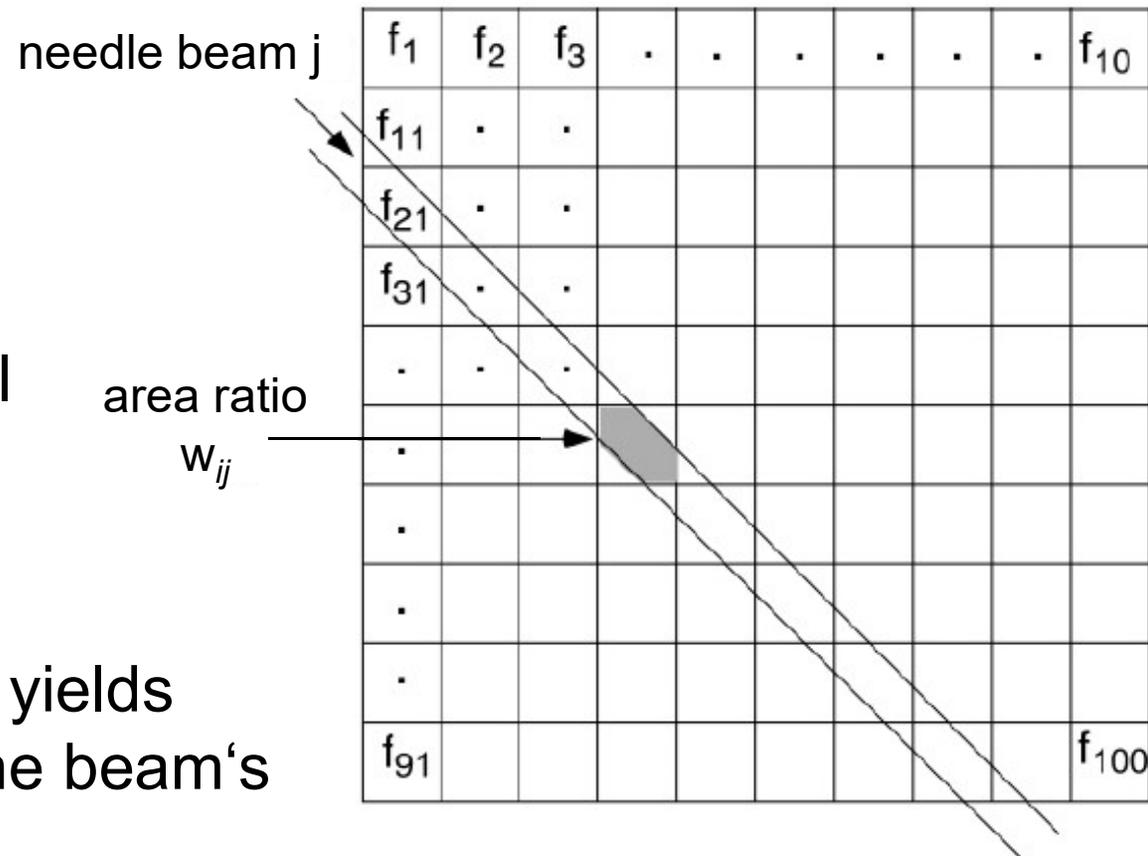
for  $M \geq N$ : over-determined problem  $\Rightarrow$  can be solved !

# *x-ray computed tomography (CT)*

## **iterative image reconstruction (II)**

- let  $f(x,y) = \mu(x,y)$
- for digital processing:  
succession of lines of an  
image matrix yields numerical  
sequence with index  $f_i$
- recording with needle beam  $j$  yields  
integral over values  $f_i$  along the beam's  
trajectory:

values  $f_i$  are multiplied with weight  $w_{ij}$  and summed up  
 $w_{ij}$  denotes area ratio of beam  $j$  to pixel  $i$   
(for a needle beam, most  $w_{ij}=0$ )



## *x-ray computed tomography (CT)*

### **iterative image reconstruction (III)**

recorded data  $p_i$  (line integrals) can be written as:

$$p_1 = w_{11}f_1 + w_{12}f_2 + \dots + w_{1N}f_N$$

$$p_2 = w_{21}f_1 + w_{22}f_2 + \dots + w_{2N}f_N$$

⋮

$$p_M = w_{M1}f_1 + w_{M2}f_2 + \dots + w_{MN}f_N$$

⇒ linear mapping !!

example: matrix size:  $512 \times 512 \Rightarrow N^2 = 262.144$

given: 1.000 projection values/detector with 800 detectors

⇒  $M = 1.000 \times 800 = 800.000$  (over-determined problem!)

***800.000 equations with 262.144 unknown !!***

# *x-ray computed tomography (CT)*

## **iterative image reconstruction (IV)**

solving the system of linear equations:

(1) direct methods

e.g., Gauss elimination      impracticable for large matrices

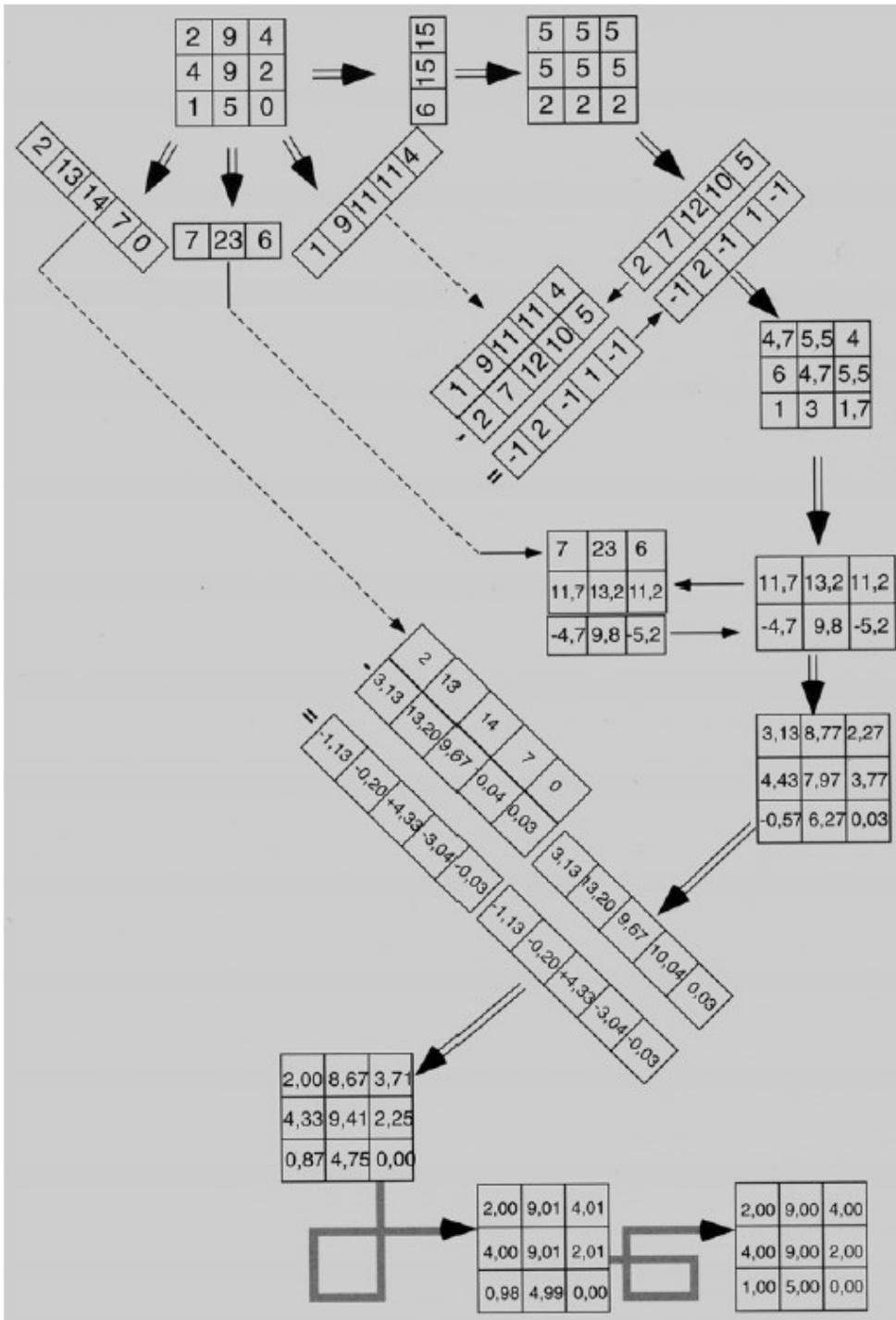
(2) iterative methods:

$$\vec{f}^{(k)} = \vec{f}^{(k-1)} - \frac{\left(\vec{f}^{(k-1)T} \cdot \vec{w}_k\right) - p_k}{\left(\vec{w}_k \cdot \vec{w}_k\right)} \cdot \vec{w}_k$$

$\vec{f}^{(k)} = \left(f_1^{(k)}, \dots, f_N^{(k)}\right)^T$  vector containing solutions after k-th iteration

$\vec{w}_j = \left(w_{j1}, \dots, w_{jN}\right)^T$  weight factors for needle beam j

$p_j =$  data of beam j



## iterative reconstruction scheme

- processing of data from 1. projection: distribute data equally to all pixel that contribute to that projection
- processing of data from 2. projection: use difference between “forward” calculated data and true data as correction value; distribute correction value equally to all pixels that contributed to that projection
- process data from remaining projections in the same manner

*x-ray computed tomography (CT)*

## **iterative image reconstruction (V)**

- method always converges
- method no longer in use in x-ray CT
- method repeatedly applied for PET/SPECT imaging

## *x-ray computed tomography (CT)*

### **image reconstruction with filtered back projection**

#### ***recap:***

a recording with needle beam under given angle  $\Theta$  und distance  $s$  from origin (i.e., projections  $p_{\Theta}(s)$ ) corresponds to the ***Radon transform*** of the spatial distribution of  $\mu(x,y)$ .

with ***Fourier-slice theorem***, we have:

- the 1D-FT of projection  $p_{\Theta}(s)$  is the 2D-FT of  $\mu(x,y)$  along a line in direction of  $\Theta$
- reconstruct  $\mu(x,y)$  by inverse 2D-FT

***but:*** data in polar coordinates;  
FFT requires Cartesian coordinates

## *x-ray computed tomography (CT)*

### **image reconstruction with filtered back projection (I)**

#### ***back projection:***

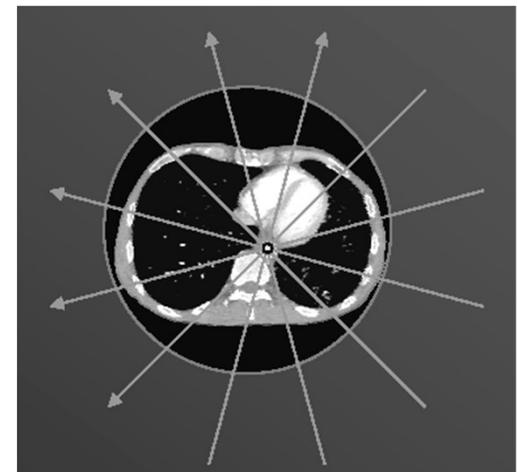
data (projection values  $p_{\Theta}(s)$ ) are line integrals of  $\mu(x,y)$ .

however:

integral value = sum over all contributions **without** location information

ansatz:

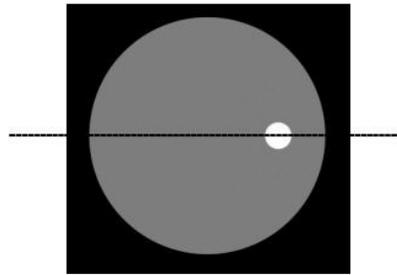
- distribute integral value equally along the initial line of integration
- superposition of all back projections at  $(x,y)$  approximates  $\mu(x,y)$



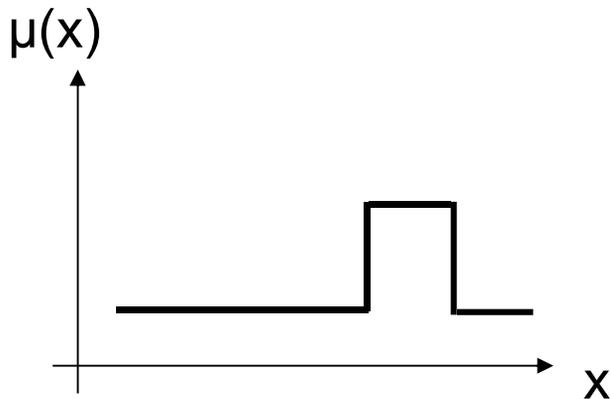
# *x-ray computed tomography (CT)*

## image reconstruction with filtered back projection (II)

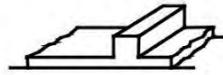
***back projection:***



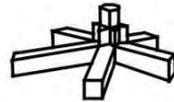
attenuation profile:



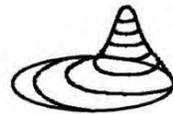
0. back projection



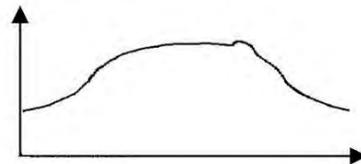
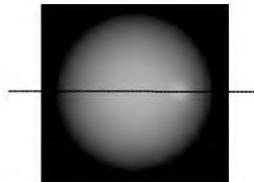
1. back projection



3. back projection



N. back projection



attenuation profile  
after  $N (\neq \infty)$  back projections

## *x-ray computed tomography (CT)*

### **image reconstruction with filtered back projection (III)**

#### ***back projection:***

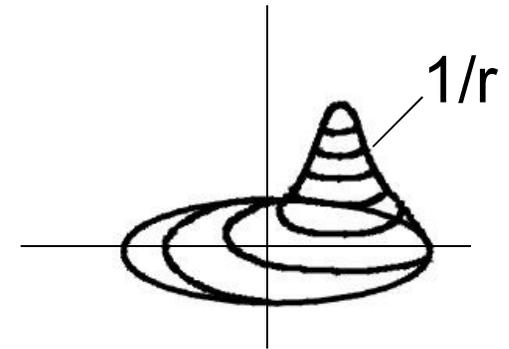
b.p. with finite number of projections  
has **point spread function (PSF)  $\sim 1/r$**

such a PSF can be assigned to each pixel,  
weighted with the local  $\mu(x,y)$

**back projection =  $1/r * \mu(x,y)$**  (convolution)

back projection leads to **blurred image:**

differences in contrast and/or fine structures not recognizable



*x-ray computed tomography (CT)*

## **image reconstruction with filtered back projection (IV)**

### ***filtered back projection:***

#### **idea:**

modify point spread function such that blurring is minimized (ideally: prevented).

#### **ansatz:**

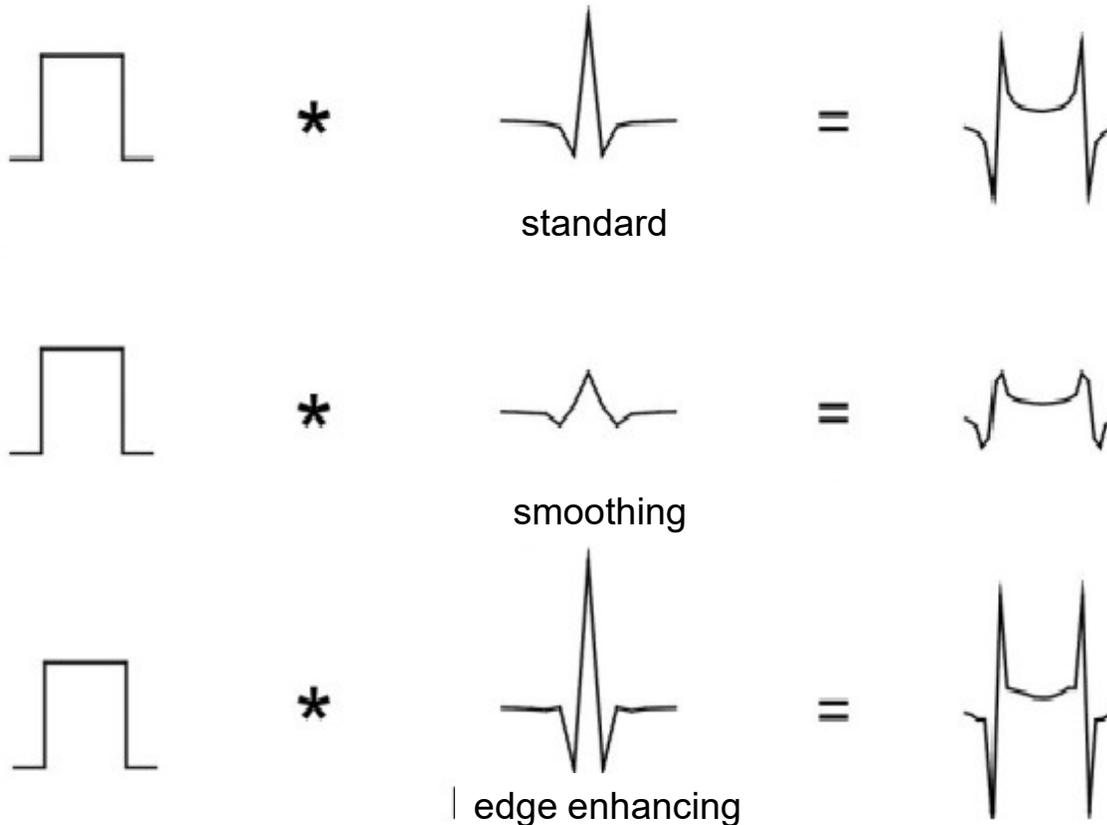
convolve attenuation profile with suitable filter (convolution kernel)

# *x-ray computed tomography (CT)*

## **image reconstruction with filtered back projection (IV-a)**

### ***filtered back projection:***

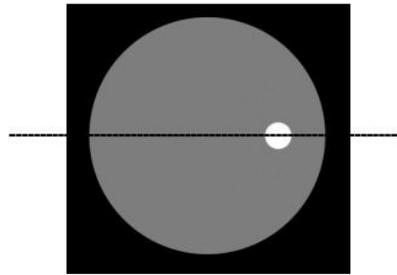
attenuation profile \* convolution kernel = filtered profile



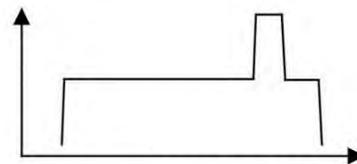
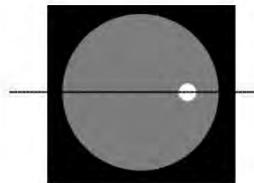
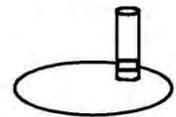
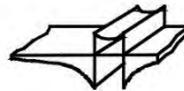
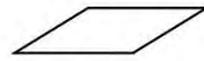
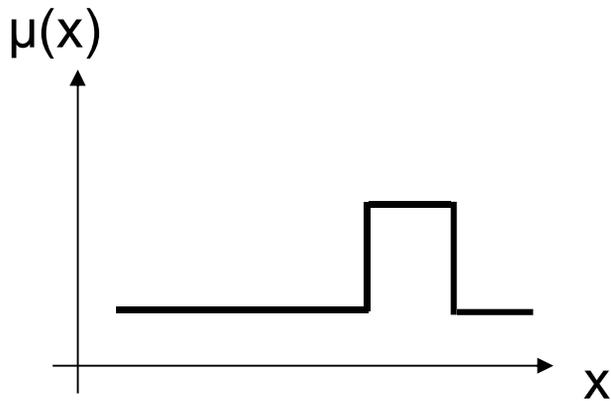
# *x-ray computed tomography (CT)*

## **image reconstruction with filtered back projection (V)**

### ***filtered back projection.***



attenuation profile:



0. back projection

1. back projection

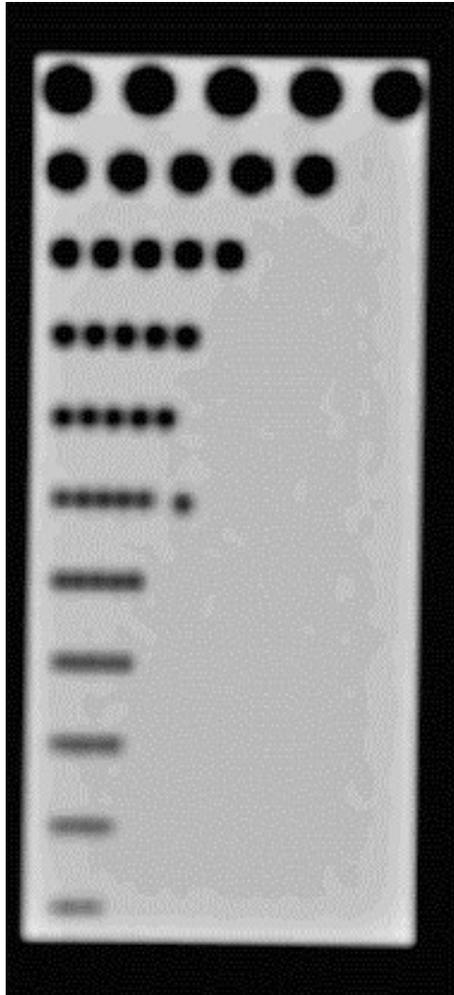
3. back projection

N. back projection

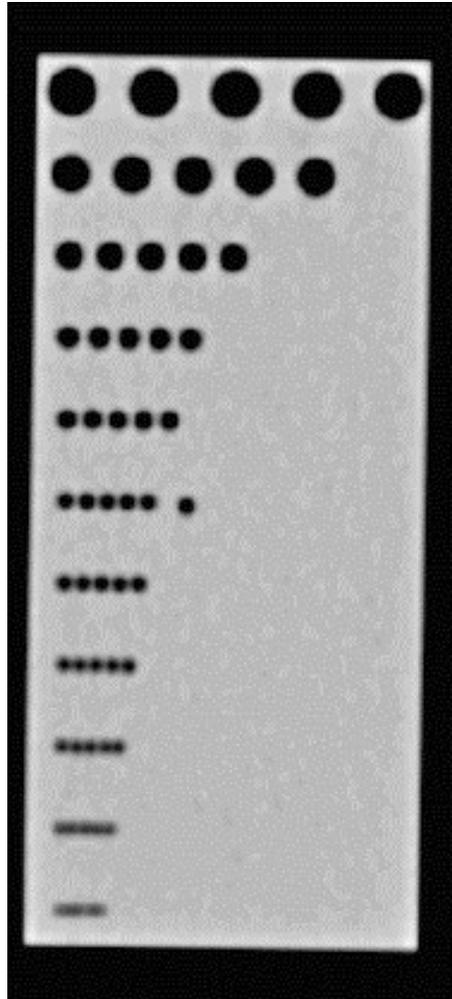
attenuation profile  
after  $N (\neq \infty)$  filtered  
back projections

*x-ray computed tomography (CT)*

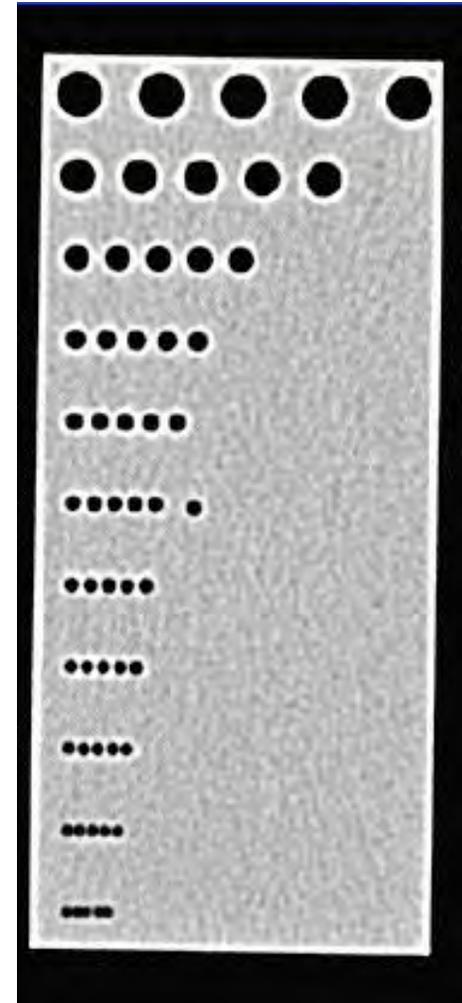
**image reconstruction with filtered back projection (V-a)**



smoothing



standard



edge enhancing

*x-ray computed tomography (CT)*

**image reconstruction with filtered back projection (V-b)**

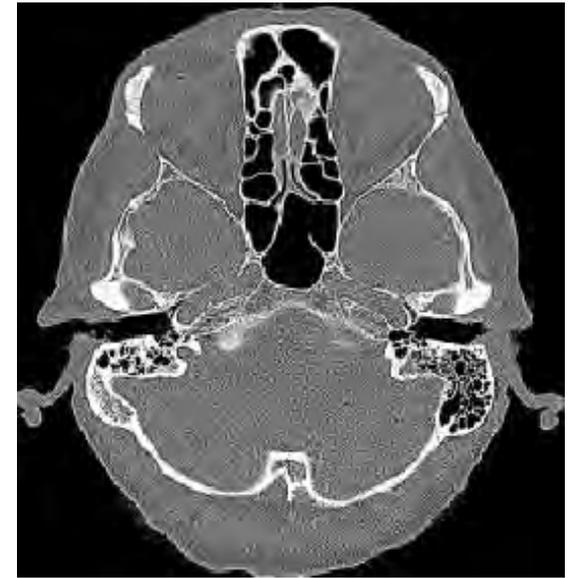
**impact of convolution kernel**



smoothing  
"soft"



standard



edge enhancing  
"bone"

## *x-ray computed tomography (CT)*

### **image reconstruction with filtered back projection (VI)**

#### **can we derive an analytic form for the filter?**

wanted:  $f(x,y) = \mu(x,y)$  from inverse 2D-FT of  $F(u,v)$ , i.e.:

$$f(x,y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(u,v) \cdot e^{j \cdot 2\pi \cdot (ux+vy)} du dv$$

with cylindrical coordinates in Fourier space

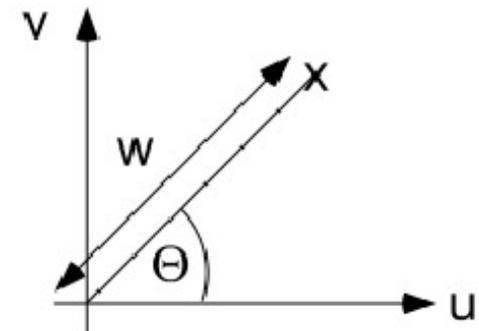
$$u = w \cdot \cos \Theta$$

$$v = w \cdot \sin \Theta$$

$$du dv = w \cdot dw \cdot d\Theta,$$

we have:

$$f(x,y) = \int_0^{2\pi} \int_0^{\infty} F(w,\Theta) \cdot e^{j \cdot 2\pi \cdot w(x \cos \Theta + y \sin \Theta)} w dw d\Theta$$



## *x-ray computed tomography (CT)*

### **image reconstruction with filtered back projection (VI-a)**

integration can be reduced to  $[0, \pi]$  (Hesse normal form):

$$f(x, y) = \int_0^{\pi} \int_{-\infty}^{+\infty} F(w, \Theta) \cdot e^{j \cdot 2\pi \cdot w(x \cos \Theta + y \sin \Theta)} |w| dw d\Theta$$

(use absolute value of  $w$ , to avoid negative radii)

$$F(w, \Theta) = P_{\Theta}(w) \quad (\text{with Fourier-slice theorem})$$

$$p_{\Theta}(s) \quad \circ \xrightarrow{\text{1D-FT}} \circ \quad P_{\Theta}(w) \quad (\text{1D FT of a projection})$$

$$f(x, y) = \int_0^{\pi} \left[ \int_{-\infty}^{+\infty} P_{\Theta}(w) \cdot |w| \cdot e^{j \cdot 2\pi \cdot w s} dw \right] d\Theta$$

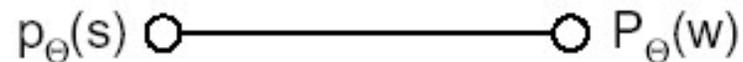
$$s = x \cos \Theta + y \sin \Theta$$

# *x-ray computed tomography (CT)*

## **image reconstruction with filtered back projection (VI-b)**

substitution: 
$$\tilde{p}_\theta(s) = \int_{-\infty}^{+\infty} P_\theta(w) \cdot |w| \cdot e^{j \cdot 2\pi \cdot ws} dw$$

if  $|w|$  were not part of the integral, an inverse FT would immediately recover the original projection  $p_\theta(s)$ , i.e.:



due to the multiplication with  $|w|$  in Fourier space, we have a filtering of the projection  $p_\theta(s)$ :

$$\tilde{p}_\theta(s) = p_\theta(s) * h(s) \quad \text{---} \quad P_\theta(w) \cdot |w|$$

A diagram illustrating the filtering process. It shows two parts: on the left, a circle labeled  $p_\theta(s)$  followed by an asterisk and a circle labeled  $h(s)$ , with a horizontal line connecting them; on the right, a circle labeled  $P_\theta(w)$  followed by a dot and  $|w|$ , with a horizontal line connecting them. A long horizontal line connects the two parts, indicating the overall relationship.

$\tilde{p}_\theta(s)$  is the **filtered projection** (using the convolution theorem).  
 $h(s)$  is the impulse response function of the filter (= convolution kernel)



# x-ray computed tomography (CT)

## image reconstruction with filtered back projection (VI-c)

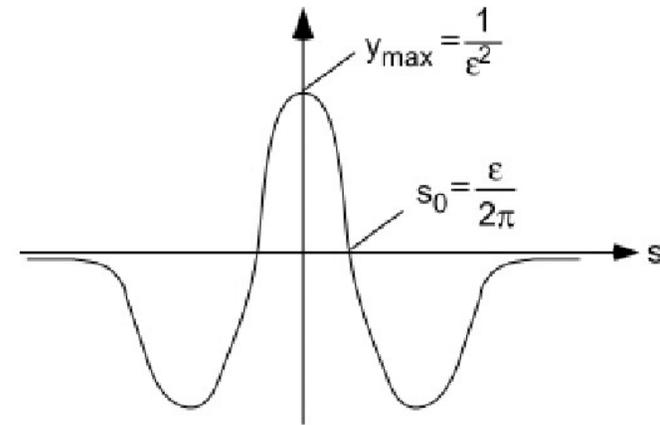
is there a function  $h(s)$  that represents FT of  $|w|$  ?

problem:  $h(s)$  only defined as limiting process

with ansatz:  $|w| \approx |w| \cdot \exp(-\varepsilon |w|)$

we find:  $\frac{\varepsilon^2 - (2\pi s)^2}{(\varepsilon^2 + (2\pi s)^2)^2} \circ \text{---} \circ |w| \cdot e^{-\varepsilon |w|}$

inverse FT of  $|w| \cdot \exp(-\varepsilon |w|)$



consider limes  $\varepsilon \rightarrow 0$ :

- l.h.s. turns into  $-1/2\pi s^2$  and r.h.s. turns into  $|w|$
- peak at  $s=0$  the more narrow and larger the smaller  $\varepsilon$

# x-ray computed tomography (CT)

## image reconstruction with filtered back projection (VI-d)

what is the meaning of the integral over  $\Theta$  ?

$$\begin{aligned} f(x,y) &= \int_0^\pi \tilde{p}_\Theta(s) d\Theta \\ &= \int_0^\pi \tilde{p}_\Theta(x \cos \Theta + y \sin \Theta) d\Theta \end{aligned}$$

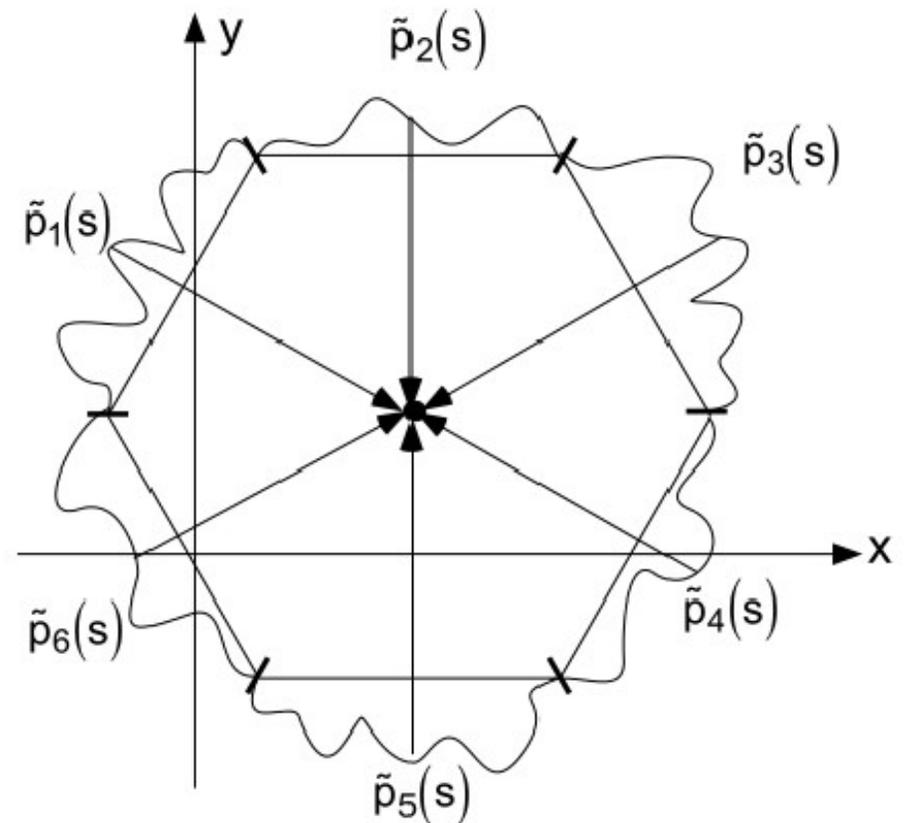
**back projection:**

in order to derive  $f(x,y)=\mu(x,y)$

at position  $(x,y)$ , take all filtered

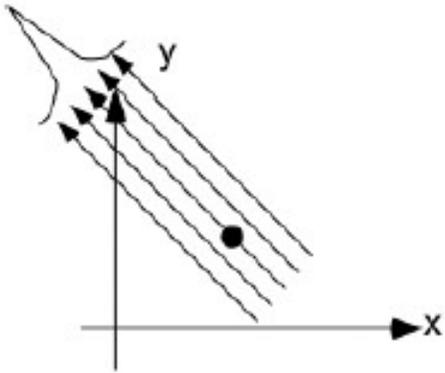
projections  $\tilde{p}_\Theta(s)$  at position

$x \cos \Theta + y \sin \Theta$  und sum them up.

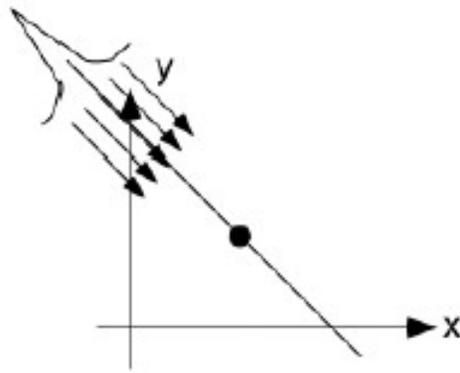


# *x-ray computed tomography (CT)*

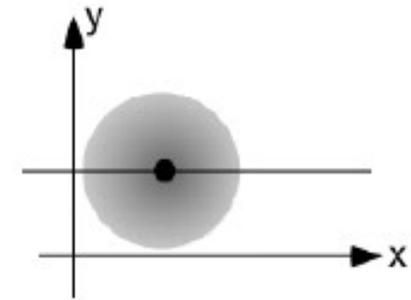
## image reconstruction with filtered back projection (VII)



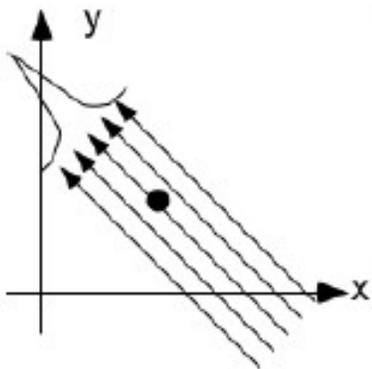
projection  
(measurement)



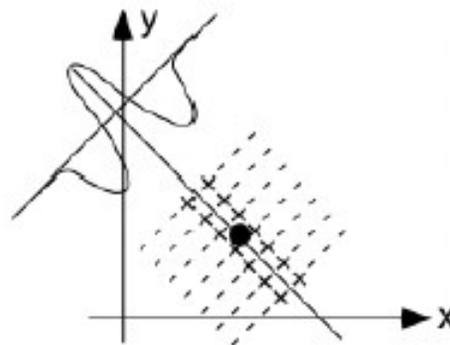
unfiltered back projection  
(single measurement)



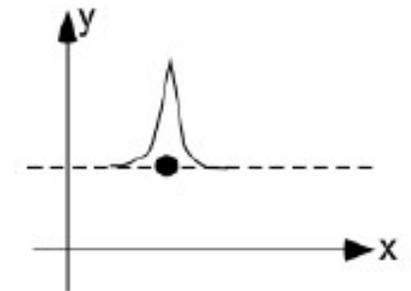
unfiltered back projection  
(all measurements)



projection  
(measurement)



filtered back projection  
(single measurement)



filtered back projection  
(all measurements)

# *x-ray computed tomography (CT)*

## **image reconstruction with filtered back projection (VIII)**

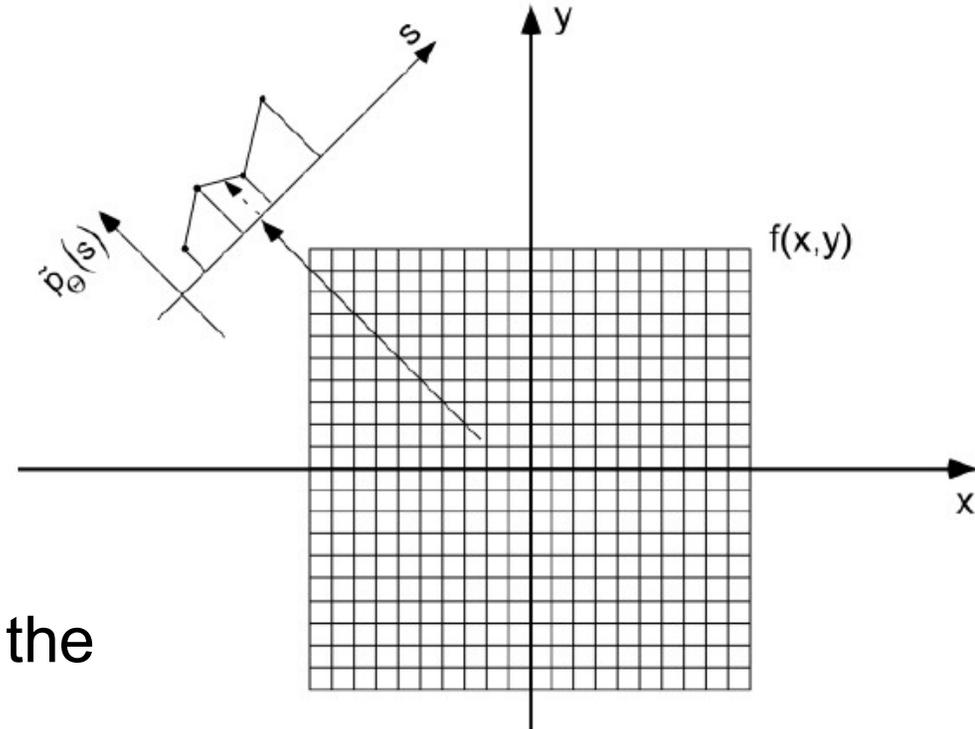
### **linear interpolation:**

*problem:*

a back projection may not always hit the center of quadratically arranged pixel

*solution (go backward)*

- 1) from the center of a pixel target at the filtered projection under angle  $\Theta$
- 2) perform *linear interpolation* between adjacent values to approximate measurement position of the detector.



## image reconstruction with filtered back projection (IX)

### Nyquist theorem and noise:

- digital sampling leads to maximum frequency  $w_{\max} = 1/2\Delta S$  in the projections, where  $\Delta S$  denotes the distance between detectors
- since spatial frequencies larger than  $w_{\max}$  are not known a priori, the filtered back projection has a degraded performance in applications (high spatial frequencies over-emphasized)
- at spatial frequencies  $w < w_{\max}$  the spectrum  $P_{\Theta}(w)$  is dominated by noise
- multiplication with  $|w|$  enhances noise additionally

**ansatz:** replace  $|w|$  with a “more suitable“ filter function

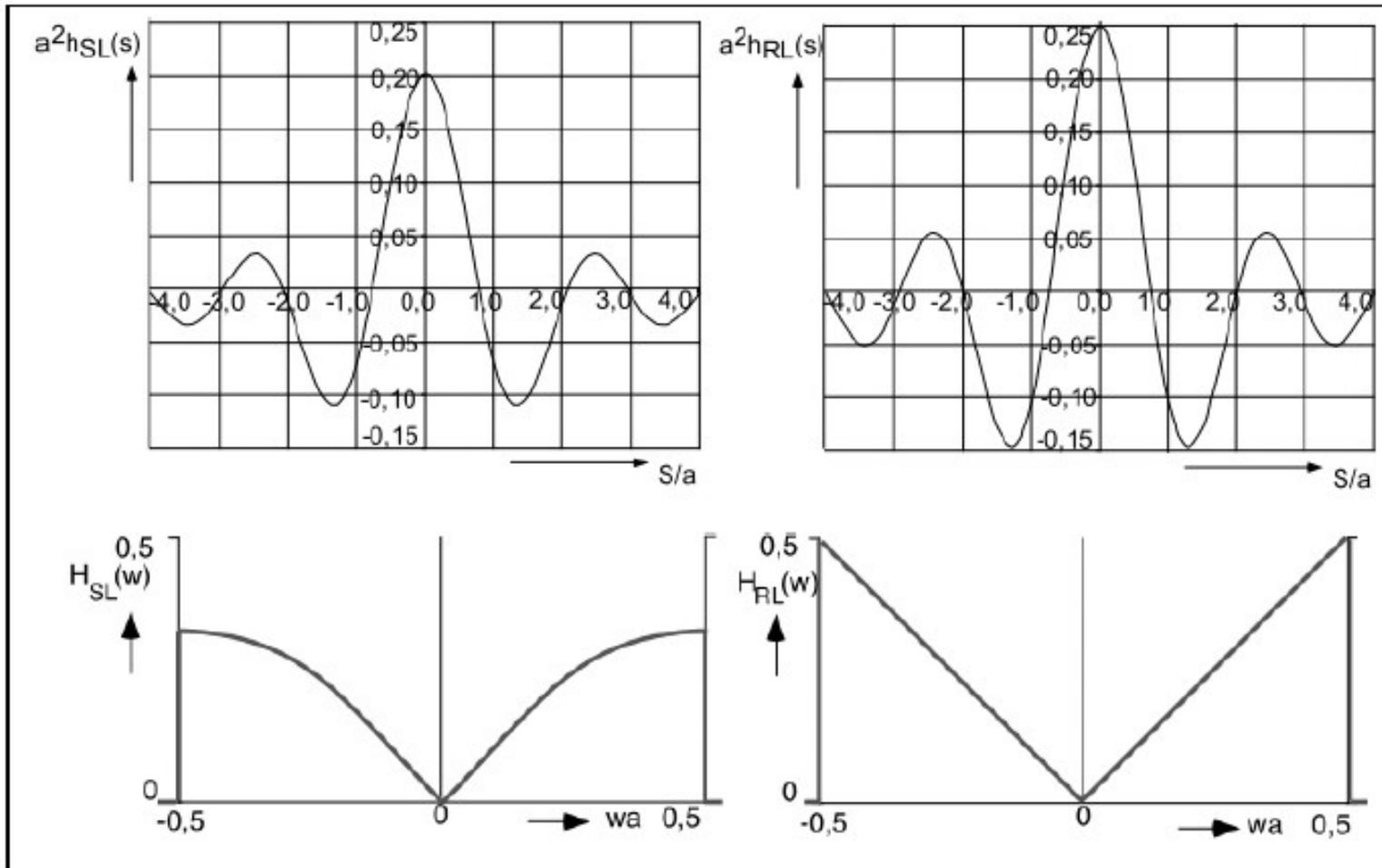
# x-ray computed tomography (CT)

## image reconstruction with filtered back projection (IX-a) alternative filter functions according to Shepp & Logan and Ramachandran & Lakshminarayan

	convolution kernel	
	Shepp und Logan	Ramachandran u. Lakshminarayan
hk	$-\frac{2}{\pi^2 a^2} \frac{1}{4k^2 - 1}$	$\frac{1}{4a^2} \quad k = 0$ $0 \quad k = \text{even}, \neq 0$ $-\frac{1}{\pi^2 a^2 k^2} \quad k = \text{odd}$
h(s)	$-\frac{2}{\pi^2 a^2} \frac{1 - 2\frac{s}{a} \sin \pi \frac{s}{a}}{4\left(\frac{s}{a}\right)^2 - 1}$	$\frac{1}{2a^2} \left\{ \frac{\sin \pi \frac{s}{a}}{\pi \frac{s}{a}} + \frac{\cos \pi \frac{s}{a} - 1}{\left(\pi \frac{s}{a}\right)^2} \right\}$
H( w )	$ w  \cdot \left  \frac{\sin \pi w a}{\pi w a} \right  \text{rect}(2aw)$	$ w  \cdot \text{rect}(2aw)$

# *x-ray computed tomography (CT)*

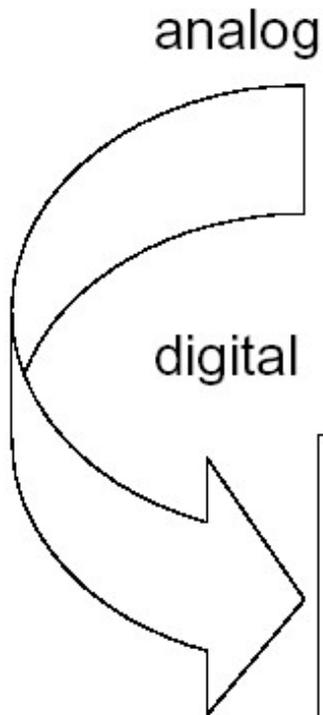
## **image reconstruction with filtered back projection (IX-b)** alternative filter functions according to Shepp & Logan and Ramachandran & Lakshminarayan



# *x-ray computed tomography (CT)*

## **image reconstruction with filtered back projection (X)**

analog and digital filtering:



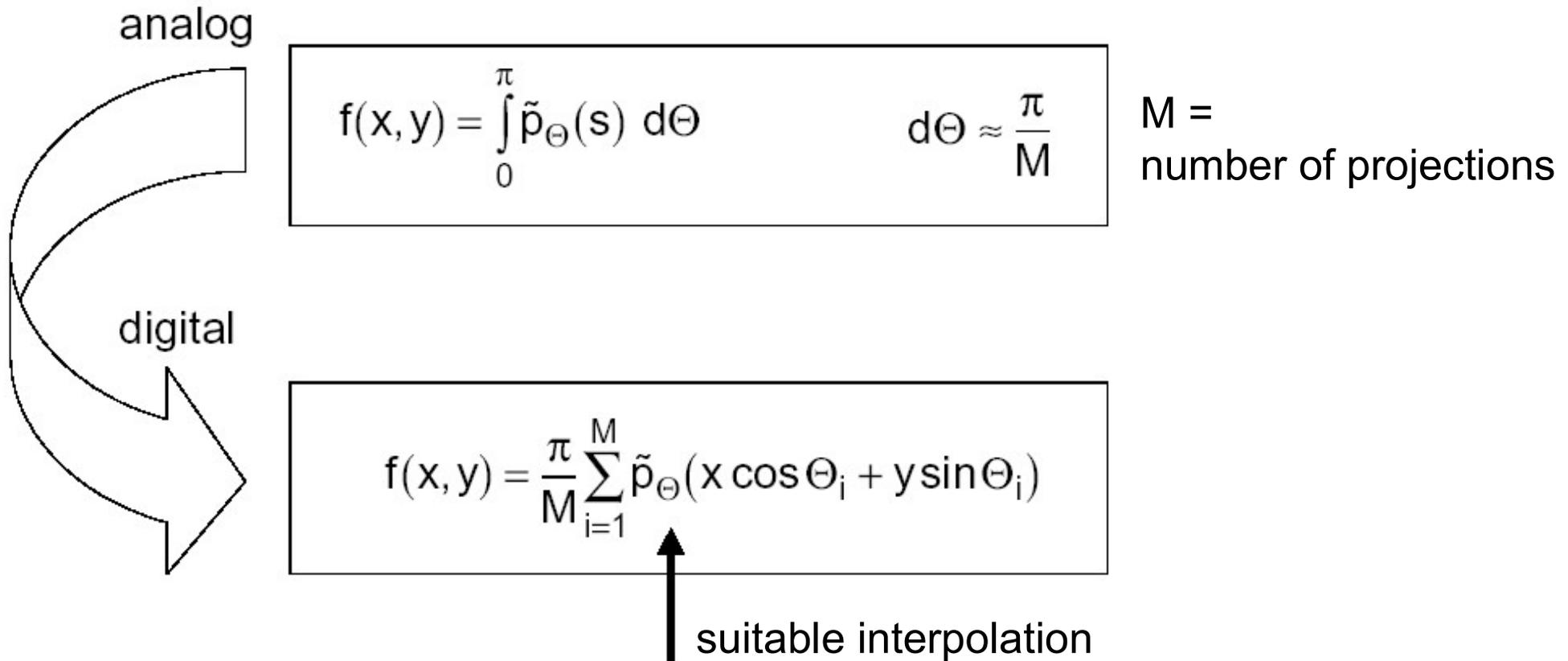
$$\tilde{p}_{\Theta}(s) = \int_{-\infty}^{+\infty} p_{\Theta}(s) \cdot h(s - s') ds'$$

$$\begin{aligned} \tilde{p}_{\Theta}(n \cdot \Delta s) &= \Delta s \cdot \sum p_{\Theta}(k \cdot \Delta s) \cdot h(n \cdot \Delta s - k \cdot \Delta s) \\ &= \Delta s \cdot \sum_{k=-K}^{k=+K} p_{\Theta}(n \cdot \Delta s - k \cdot \Delta s) \cdot h(k \cdot \Delta s). \end{aligned}$$

# *x-ray computed tomography (CT)*

## **image reconstruction with filtered back projection (X)**

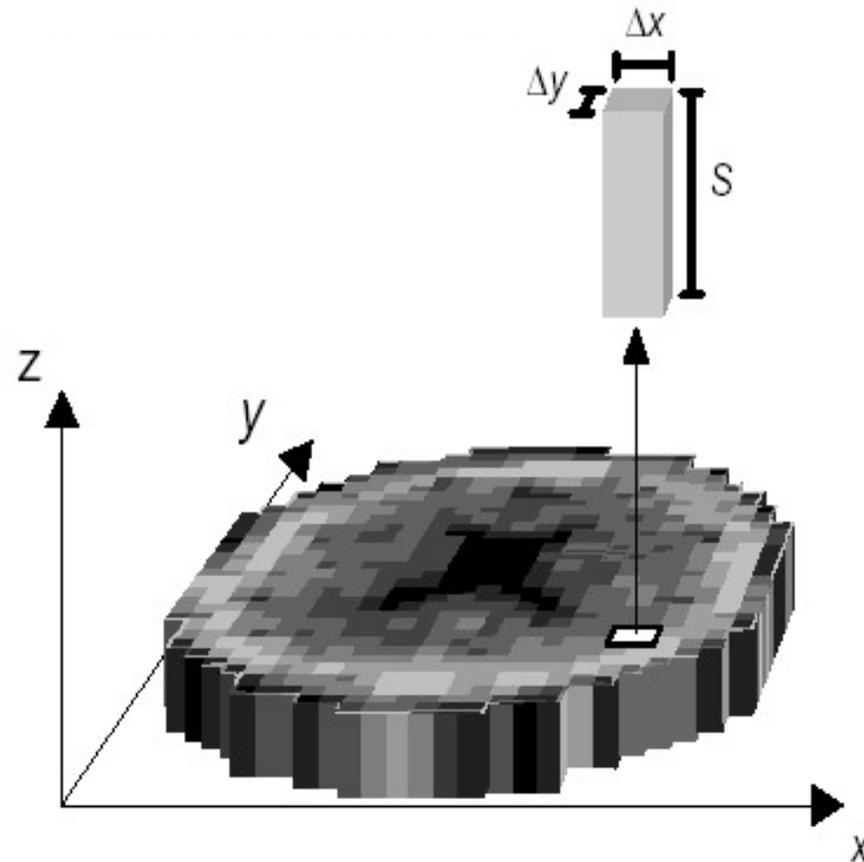
analog and digital filtering:



# *x-ray computed tomography (CT)*

## what does a CT image represent?

average of all linear attenuation coefficients in a given volume element (voxel) in Hounsfield units



## *x-ray computed tomography (CT)*

### **Hounsfield scale**

absorption- or attenuation coefficient

in nuclear physics:

$$[\mu] = \text{cm}^{-1} \quad (\mu \text{ depends on energy of x-rays!})$$

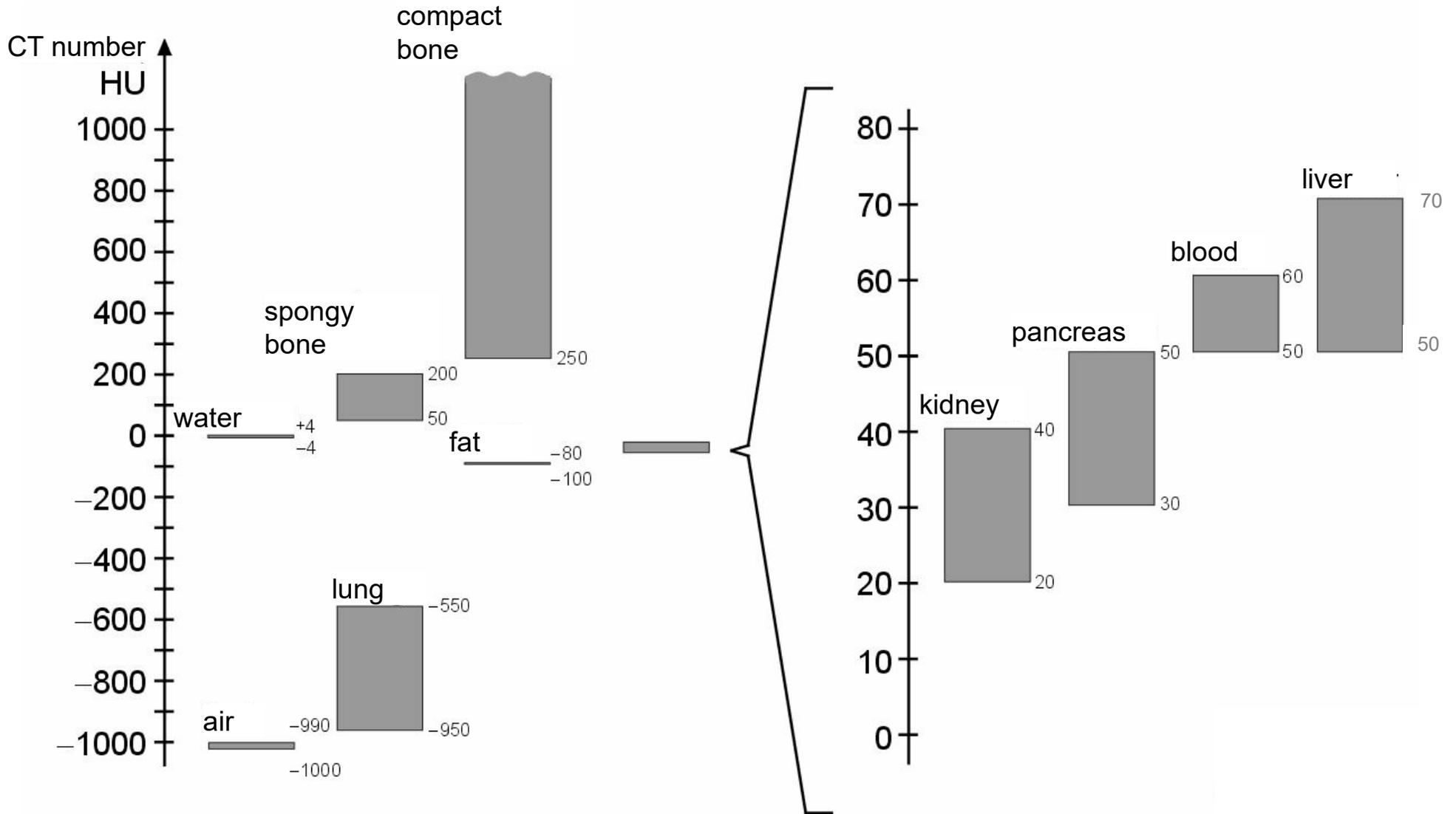
in medical imaging: ***relative Hounsfield unit (HU; CT number)***

$$\mu_{rel} = \frac{\mu - \mu_{\text{water}}}{\mu_{\text{water}}} \cdot 1000$$

- human body mostly consists of water (~60 %)
- representation in *per mille* since most soft tissues differ only very little from  $\mu_{\text{water}}$

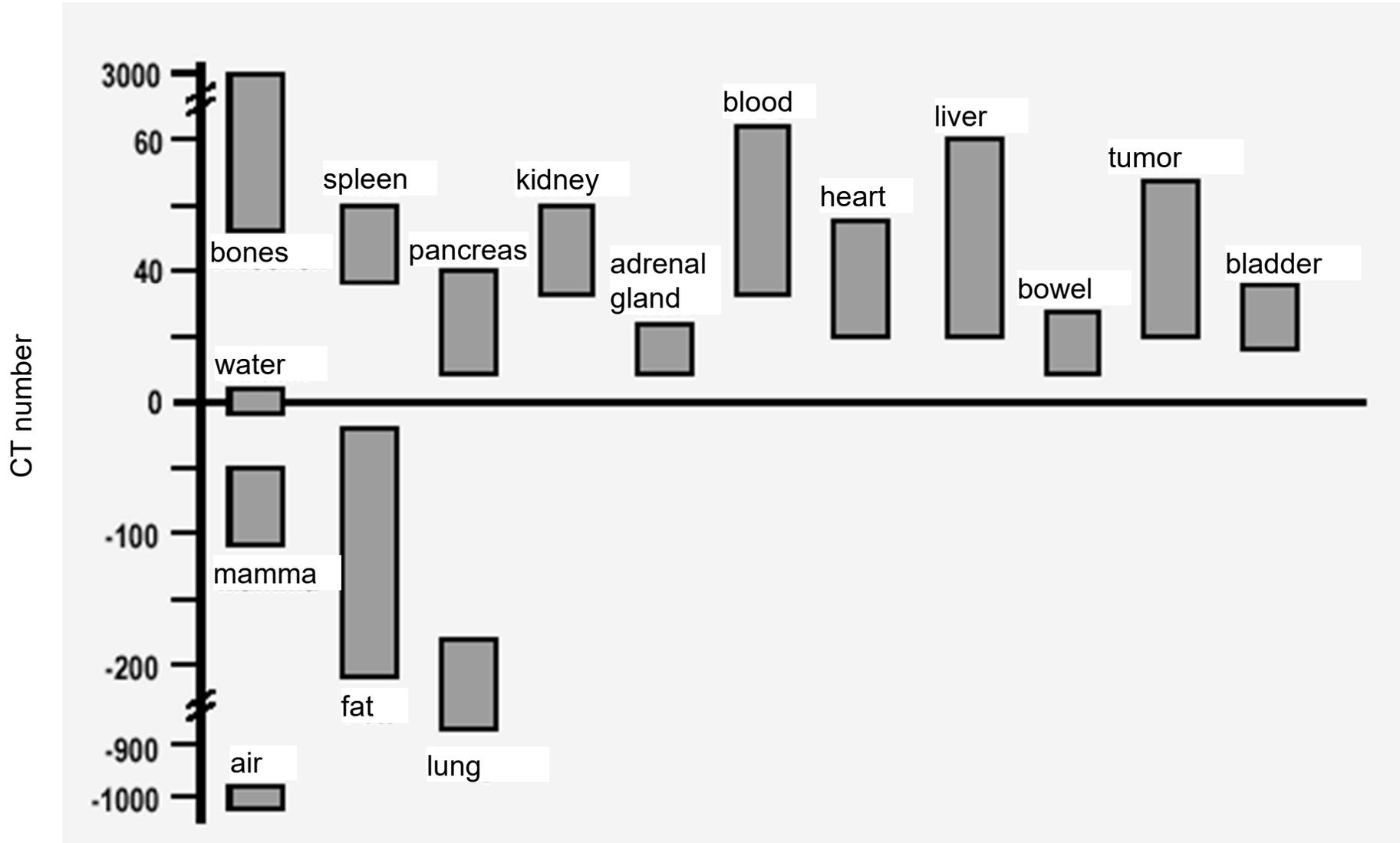
# *x-ray computed tomography (CT)*

## Hounsfield scale



# *x-ray computed tomography (CT)*

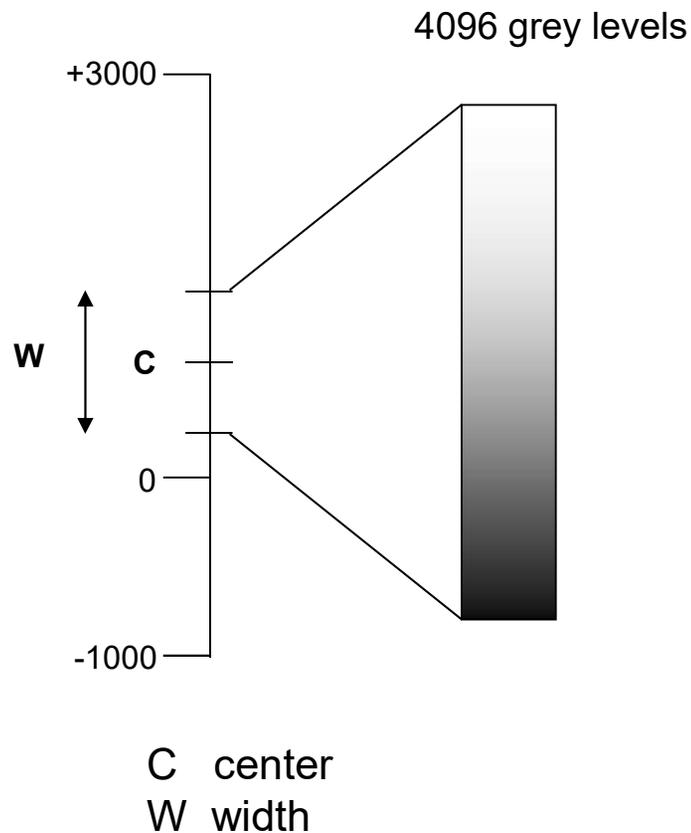
## Hounsfield scale



# *x-ray computed tomography (CT)*

## **Hounsfield scale and window techniques**

improve differentiability of soft tissues with window technique



improved interpretability of CT images

e.g.

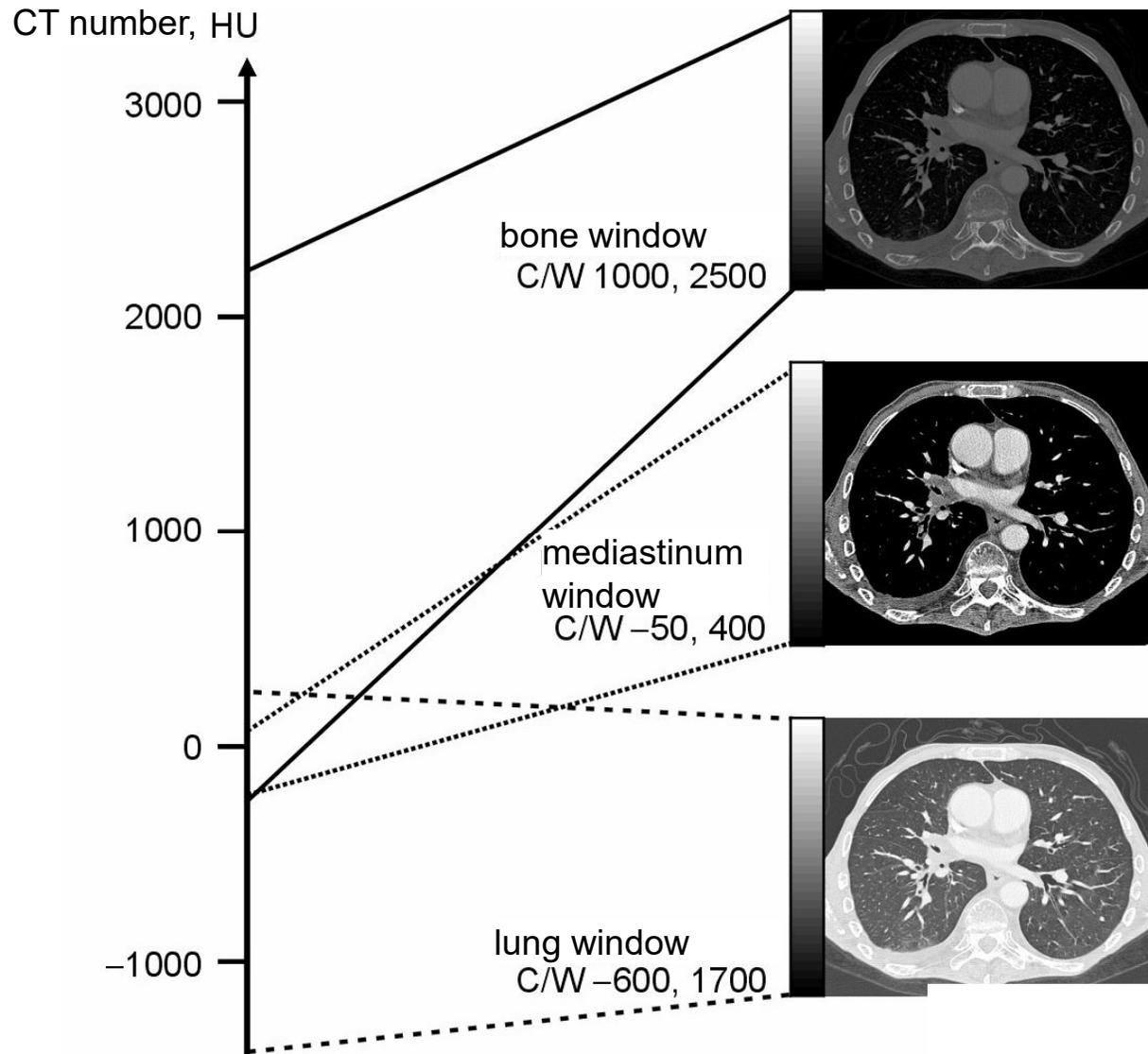
bone window:  $C/W = 1000/2500$

mediastinum window:  $C/W = -50/400$

lung window:  $C/W = -625/1250$

# *x-ray computed tomography (CT)*

## Hounsfield scale and window techniques



## *x-ray computed tomography (CT)*

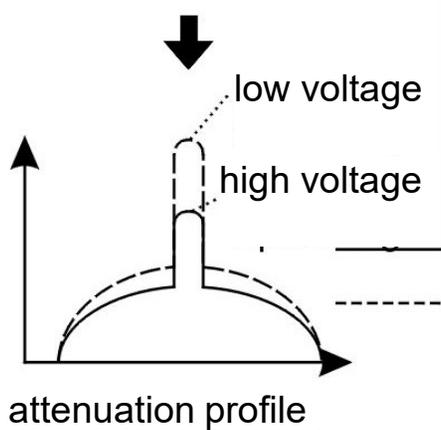
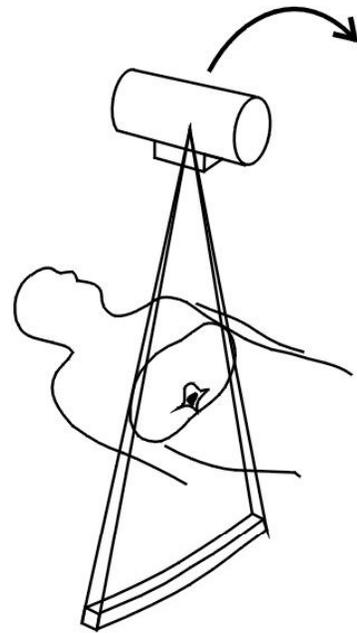
### **Hounsfield scale and dual-energy CT**

- CT number hard to interpret unequivocally  
(note: average of attenuation coefficients of all chemical elements in a given voxel!)
- ambiguous diagnostic finding (example):
  - observation: area with increased attenuation in soft tissue
  - question: recent (bleeding) or existent process (calcium deposit)
- use order-number-dependent energy-dependence of  $\mu = f(E,Z)$
- dual energy CT:
  - two recordings with different energies of x-rays
  - subtraction of images yields material-selective image

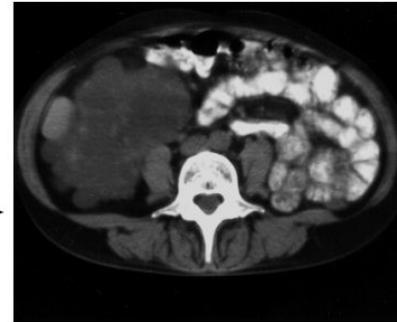
# *x-ray computed tomography (CT)*

## **Hounsfield scale and dual-energy CT**

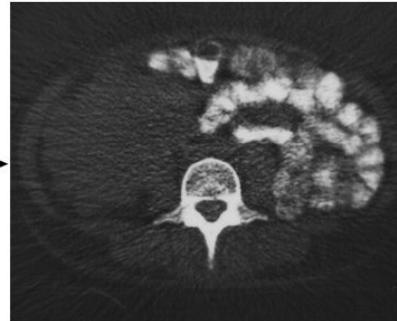
rapid switching of tube voltage



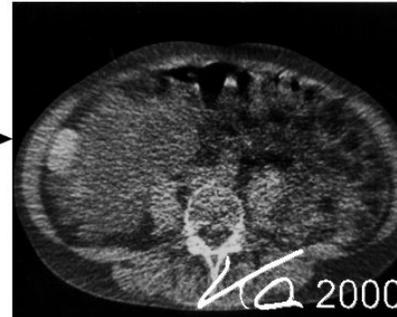
normal image



calcium image



soft tissue image

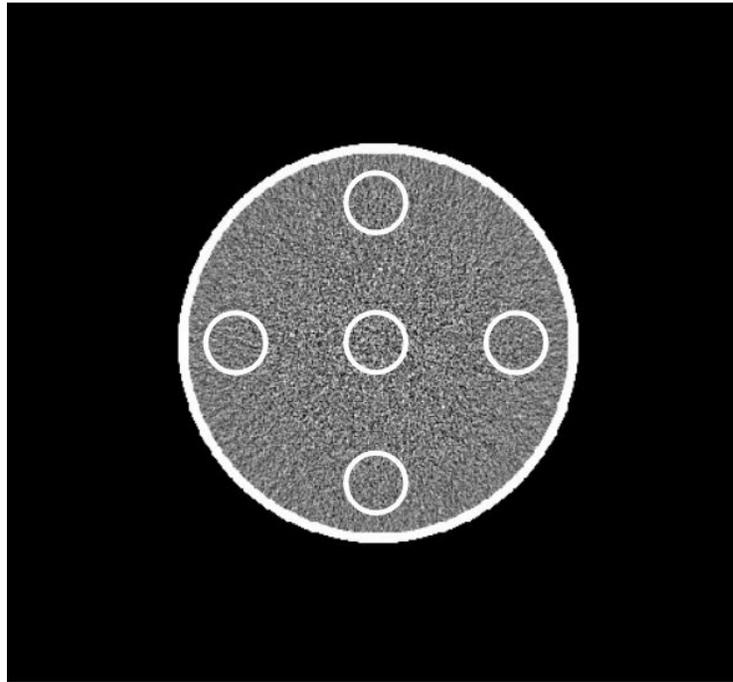


base-material decomposition

# *x-ray computed tomography (CT)*

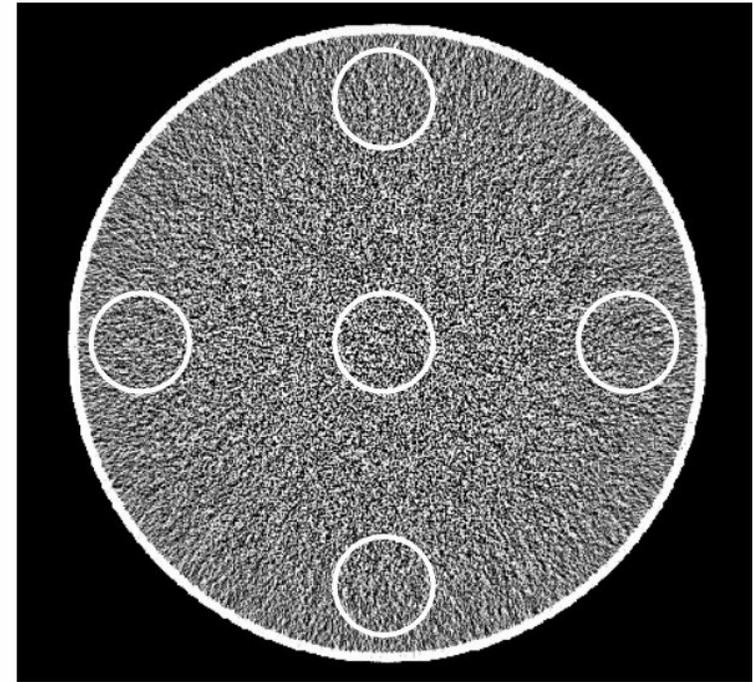
## estimating homogeneity

tolerance:  
 $\pm 4\text{HU}$   
from set value  
(water: 0 HU)



20 cm water phantom

	mean	$\sigma$
middle	-1,6 HU	21,3 HU
top	-0,9 HU	14,8 HU
right	-1,3 HU	14,7 HU
bottom	-0,9 HU	14,6 HU
left	-1,3 HU	14,9 HU



32 cm water phantom

	mean	$\sigma$
middle	3,0 HU	68,5 HU
top	-1,6 HU	34,8 HU
right	-0,9 HU	34,2 HU
bottom	-0,9 HU	35,1 HU
left	-0,1 HU	35,3 HU

## *x-ray computed tomography (CT)*

### **comparing projection radiography and computed tomography**

both techniques:

- imaging with x-rays
- require comparable dose  
(novel CT-systems even lower dose)

projection radiography:

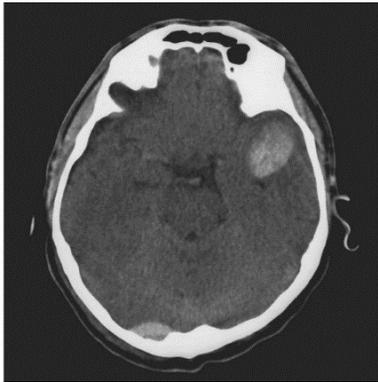
- contrast = sum of signals ( $\mu$ ) along transmission trajectory
- contrast depends on atomic number  $Z$  and dose

computed tomography

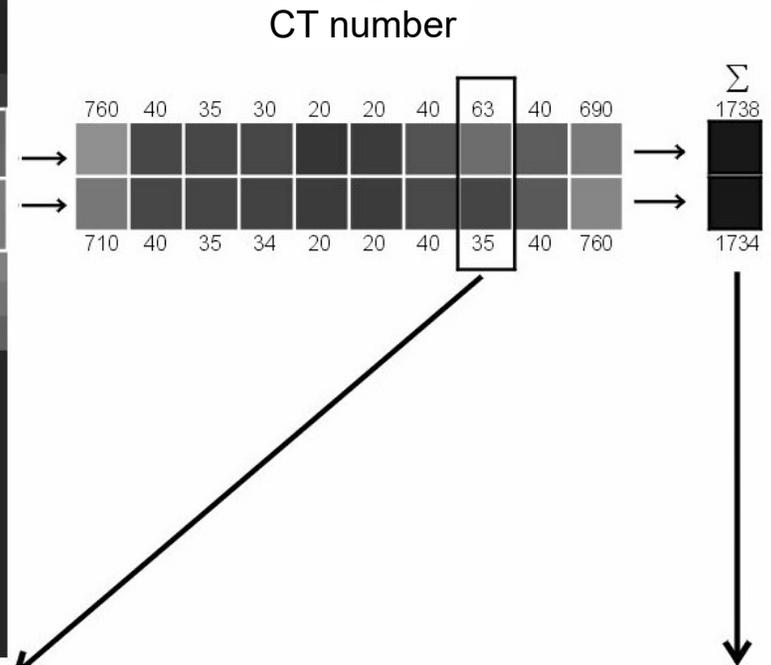
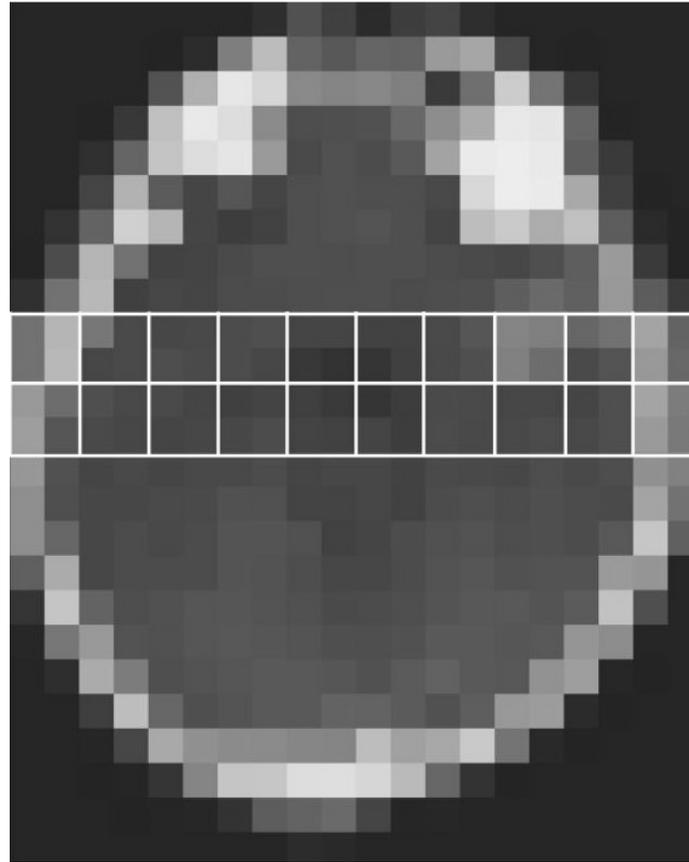
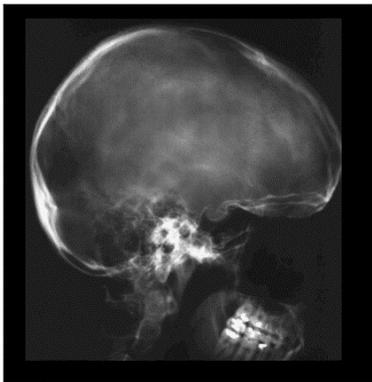
- contrast =  $\mu$ -values from adjacent voxel (not due to summation or line integrals); local composition of tissue
- no influence from adjacent or overlapping structures

# x-ray computed tomography (CT)

CT image



Röntgen image



CT image:  
 high local contrast  $K$   
 $K = \Delta CT = J_1 - J_2$   
 $\sim 50 \%$

Röntgen image:  
 low soft-tissue contrast  
 $K = (J_1 - J_2) / ((J_1 + J_2) / 2)$   
 $\sim 0,23 \%$

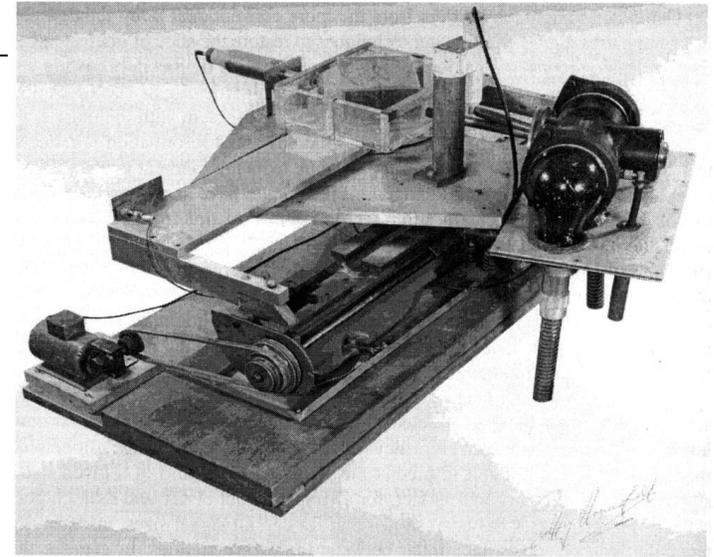
*x-ray computed tomography (CT)*

measurement devices  
for  
x-ray computed tomography

# *x-ray computed tomography (CT)*

## **data acquisition**

### **1. generation CT scanner**



Hounsfield 1969 (phantom measurements)

*(A method of and apparatus for examination of a body by radiation such as x-ray or gamma radiation, US patent 1970)*

technique: *pencil beam* (single x-ray needle-like beam)

principle: translation-rotation

detectors: 1

x-ray source: americium 95

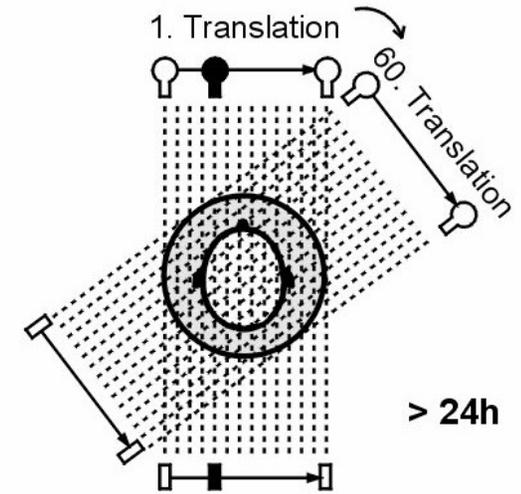
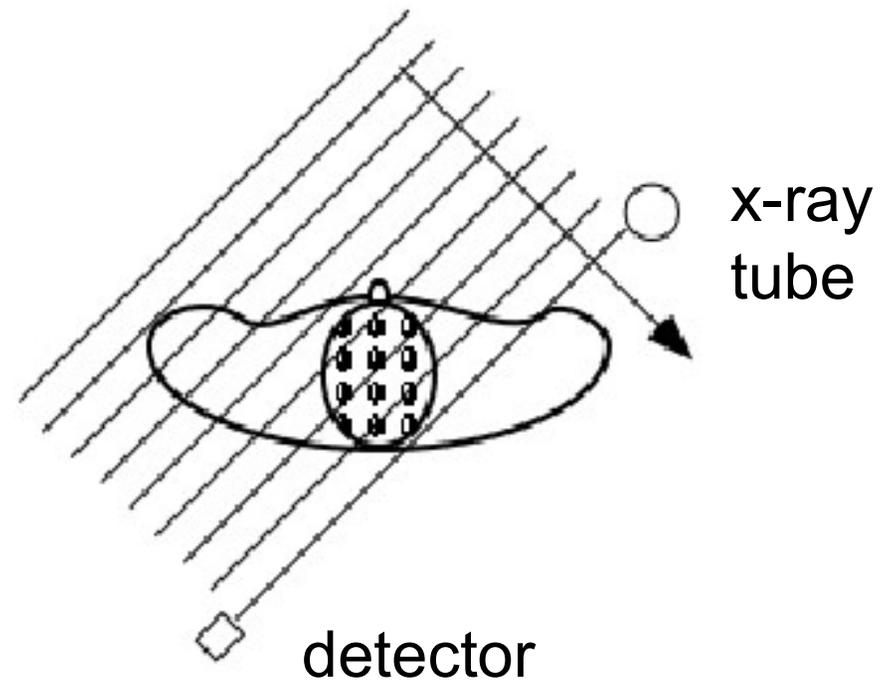
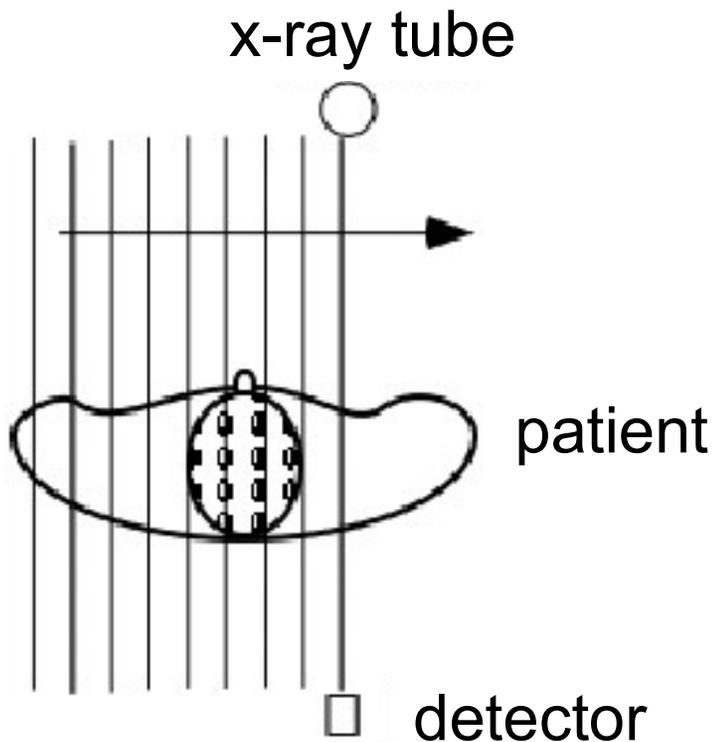
recording duration: 9 days

(image reconstruction: 2.5 hrs; computing center EMI)

# *x-ray computed tomography (CT)*

## **data acquisition**

### **scheme of recording with 1. generation CT scanner**



# *x-ray computed tomography (CT)*

## **data acquisition**

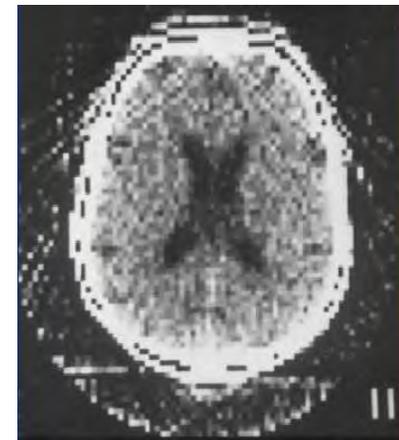
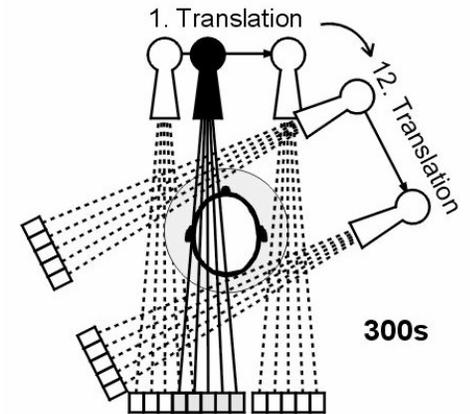
### **2. generation CT scanner (first commercial system)**

Hounsfield 1972-1975

technique: *partial fan beam*  
beam width:  $10^\circ$   
principle: translation-rotation  
detectors: array ( $>30$ )  
x-ray source: high performance tube  
recording duration: 300 s

matrix size:  $80 \times 80 = 6400$  pixel

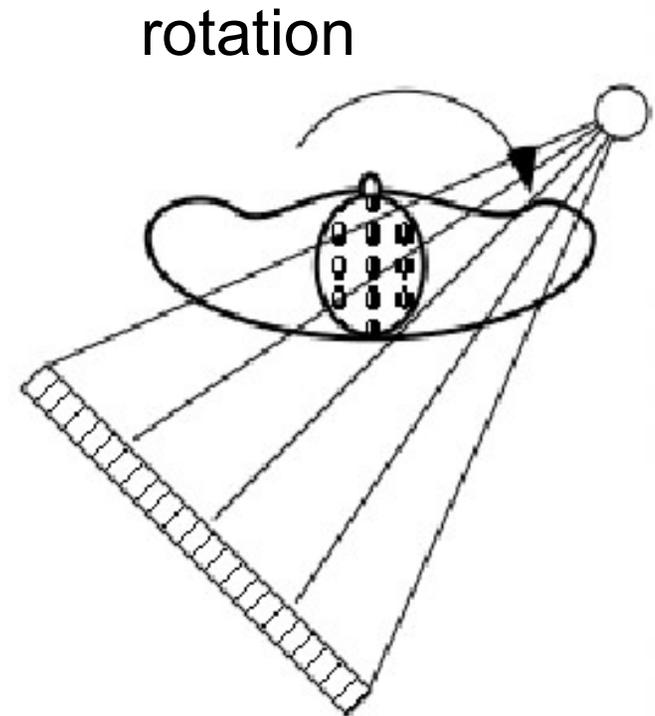
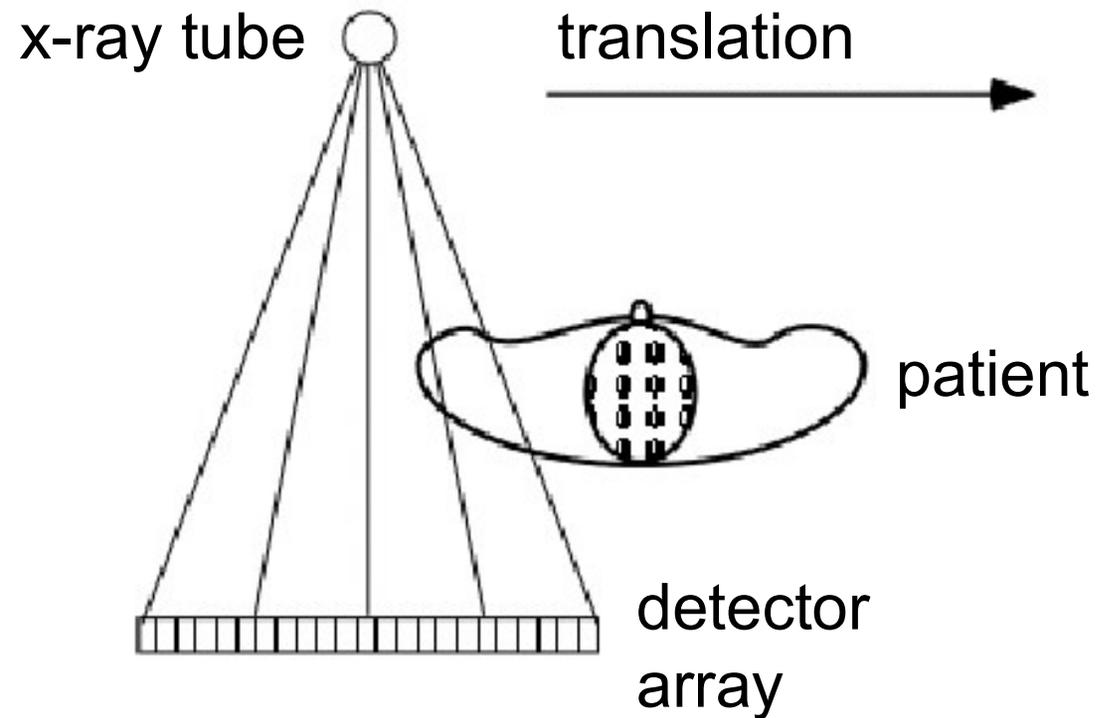
estimated from 180 projections ( $1^\circ$ -steps) with 160 data each = 28.800 data/scan



# *x-ray computed tomography (CT)*

## **data acquisition**

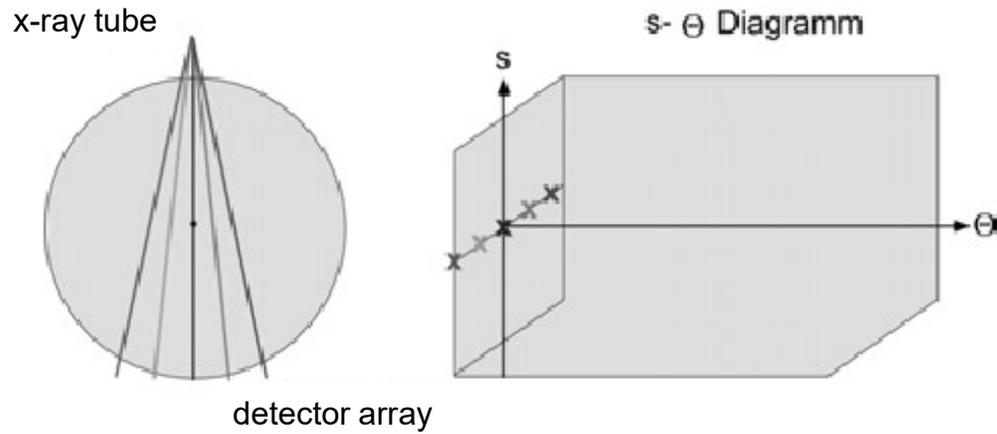
### **scheme of recording with 2. generation CT scanner**



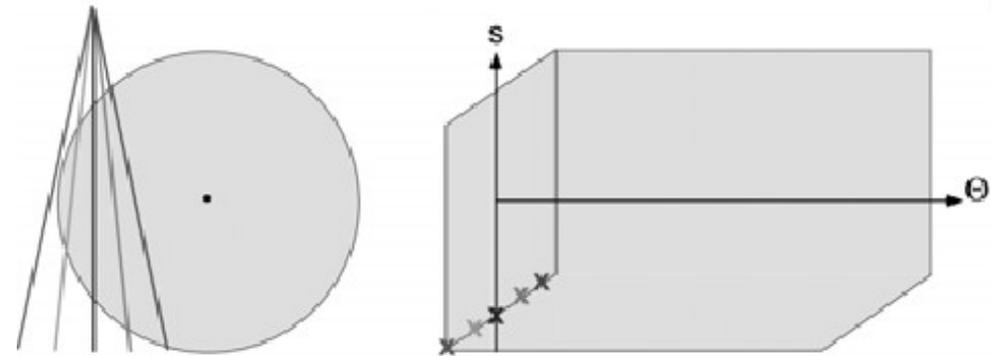
# *x-ray computed tomography (CT)*

## **data acquisition in Radon space (2. generation CT scanner)**

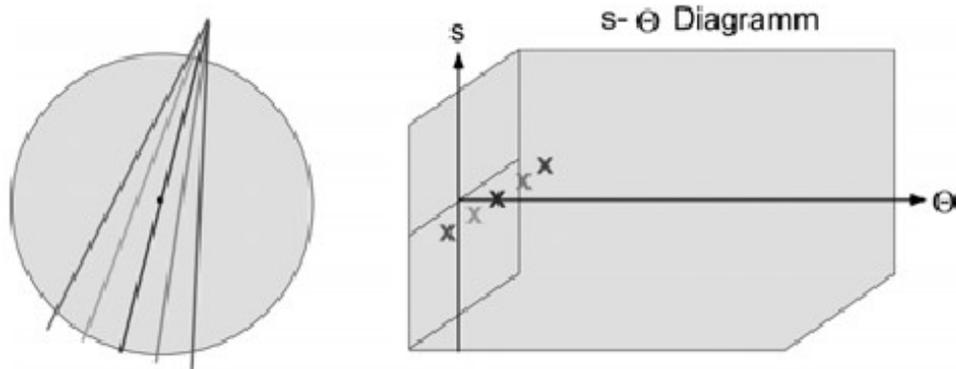
$$s = 0, \Theta = 0$$



$$s \neq 0, \Theta = 0$$



$$s \neq 0, \Theta \neq 0$$



# x-ray computed tomography (CT)

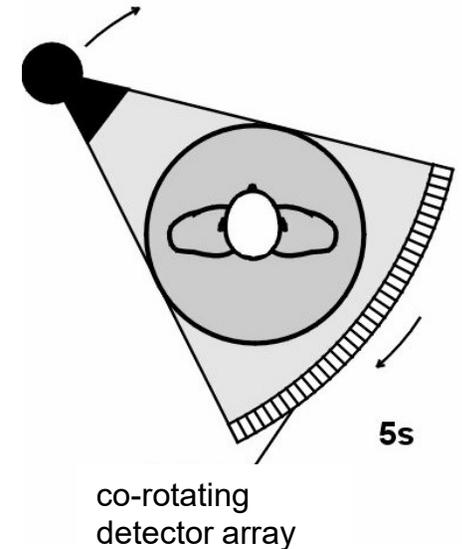
## data acquisition

### 3. generation CT scanner

1976

- improved utilization of available dose
- enables whole-body scanning

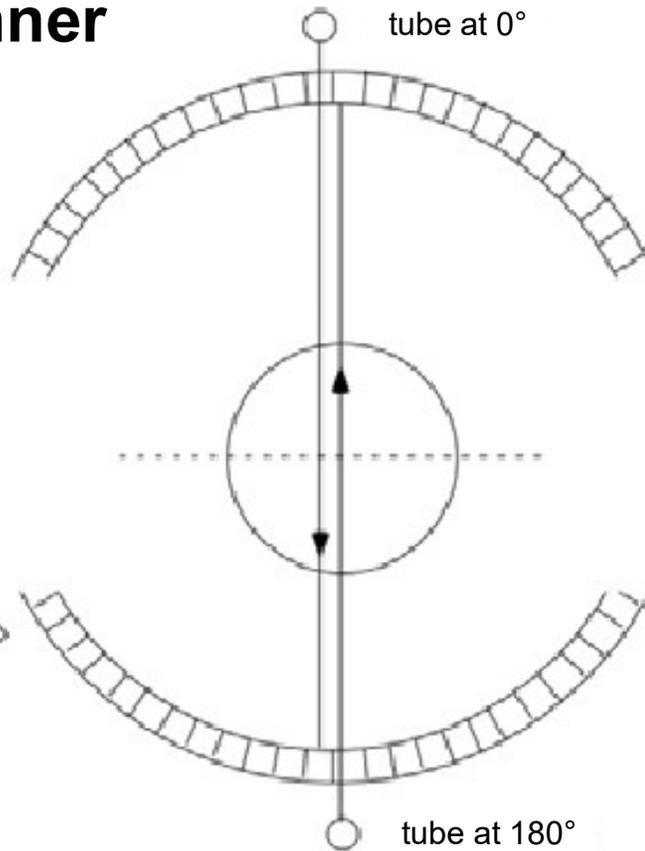
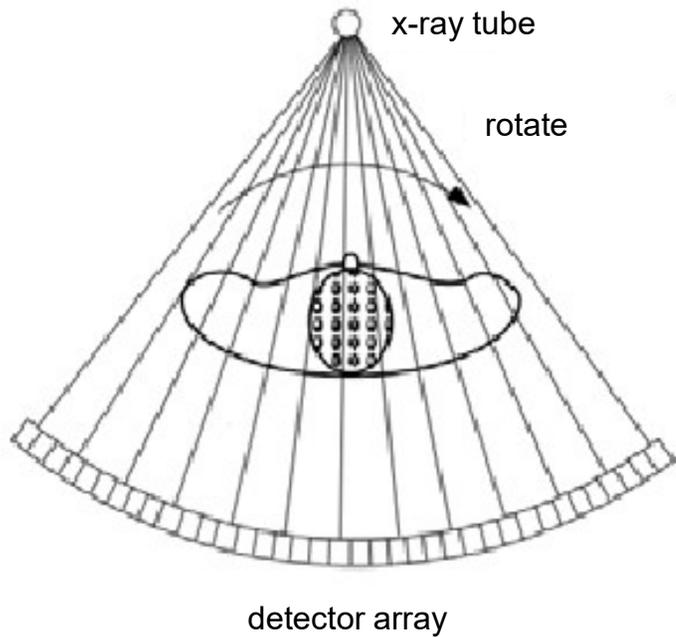
technique: *full fan beam*  
beam width:  $40^\circ - 60^\circ$   
principle: continuous rotation  
(tube and detector array rotate around patient)  
detectors: array (500-800)  
x-ray source: high-performance tube (1-2 ms pulses every 13 ms)  
recording duration: 5 s



# x-ray computed tomography (CT)

## data acquisition

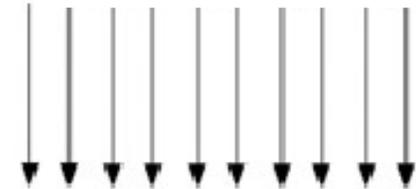
### scheme of recording with 3. generation CT scanner



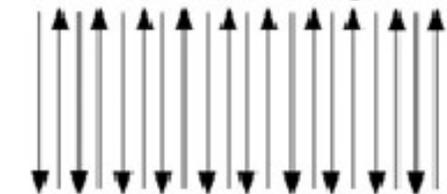
rapidly "bouncing" focus



direction of beams at tube at 180°



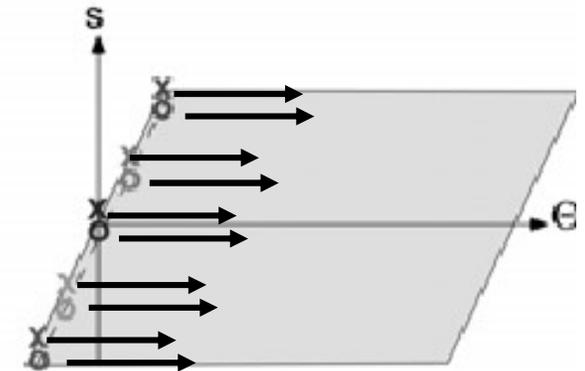
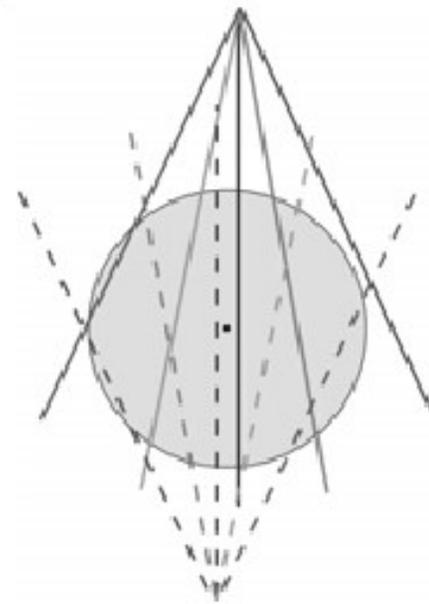
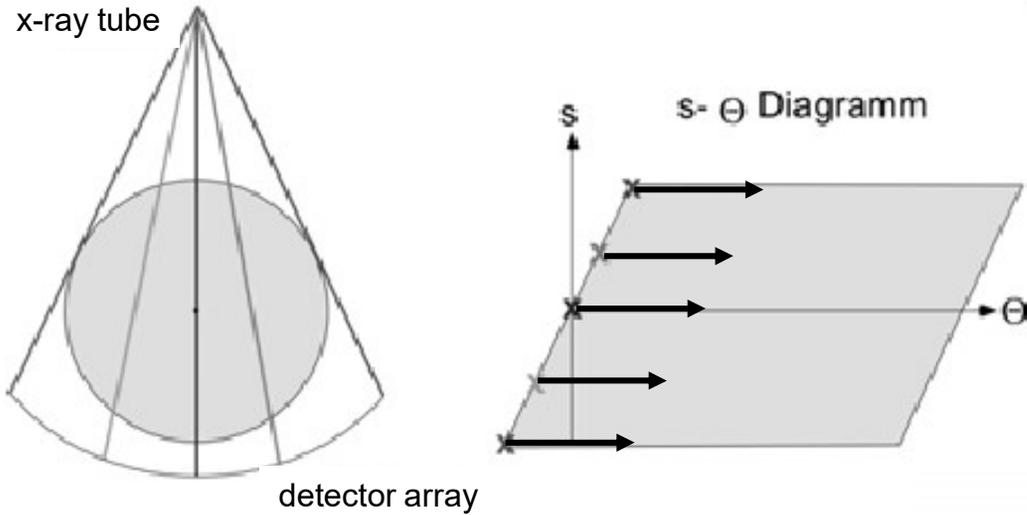
direction of beams at tube at 0°



anti-parallel direction of beams at tube at 0° and 180°

# x-ray computed tomography (CT)

## data acquisition in Radon space (3. generation CT scanner)



- rapidly "bouncing" focus:
- finer sampling of Radon space
  - higher resolution

# *x-ray computed tomography (CT)*

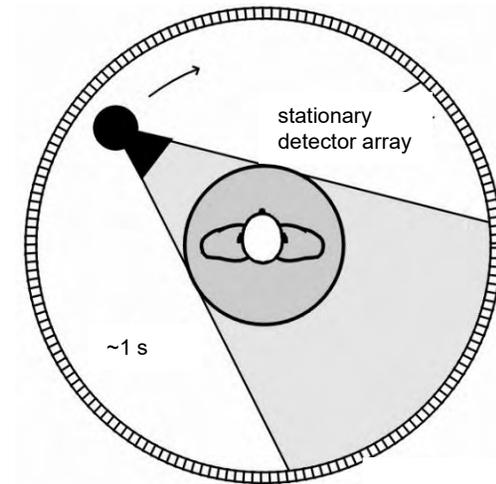
## **data acquisition**

### **4. generation CT scanner**

1978

- comparable to 3. generation scanner
- no commercialization (logistics, costs)

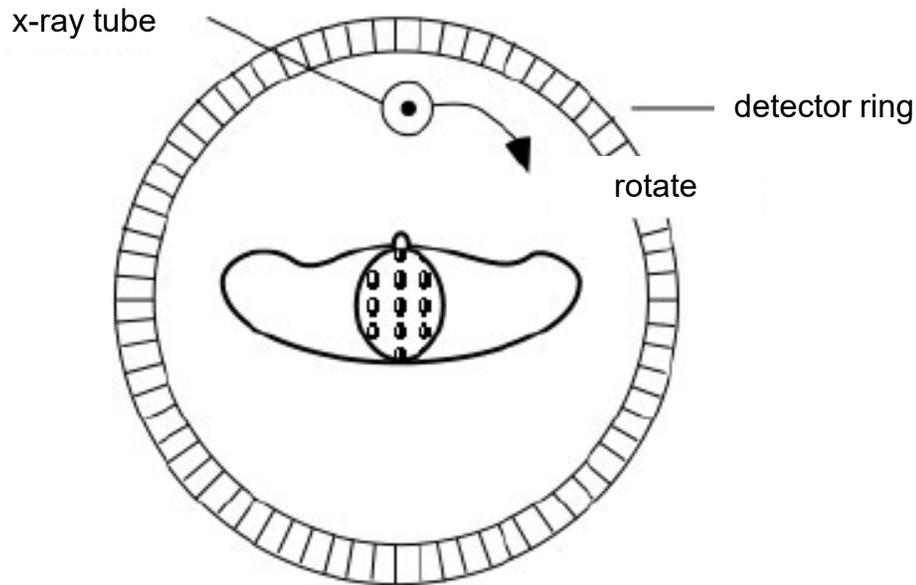
technique:	<i>full fan beam</i>
	beam width: 40° - 60°
principle:	tube rotates continuously around patient
detectors:	stationary array (~5000)
x-ray source:	continuously emitting high-performance tube
recording duration:	~ 1 s



# *x-ray computed tomography (CT)*

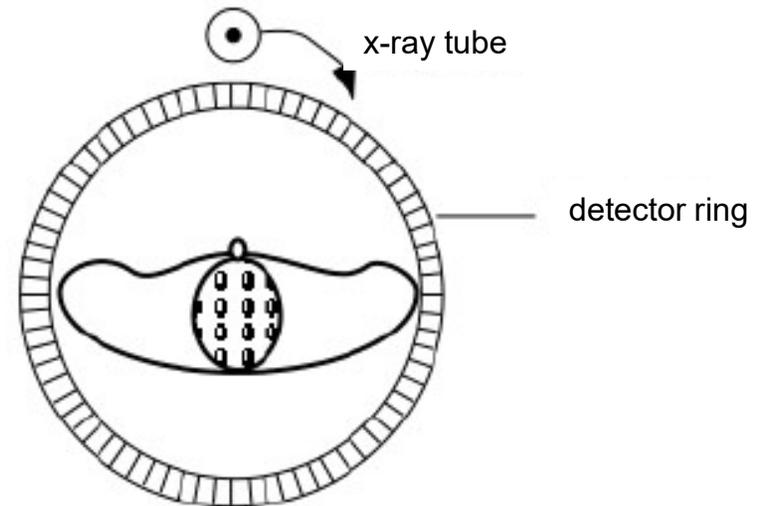
## **data acquisition**

### **scheme of recording with 4. generation CT scanner**



"inside" detector ring

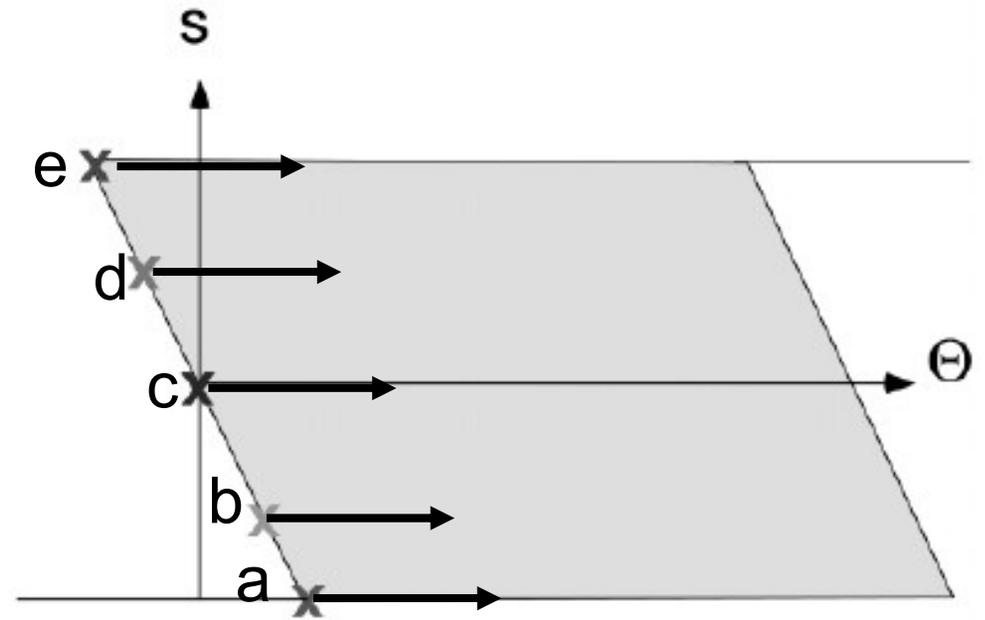
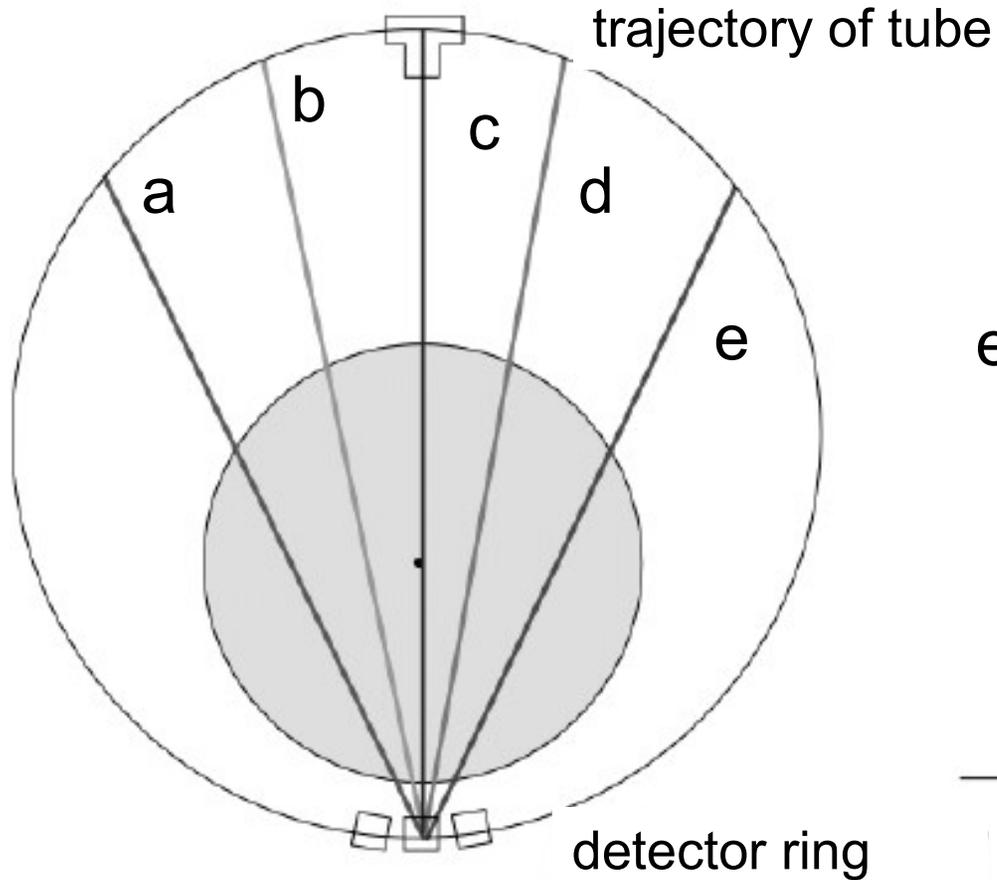
"outside" detector ring



detector ring needs to be tilted  
wrt rotation-axis of x-ray tube

*x-ray computed tomography (CT)*

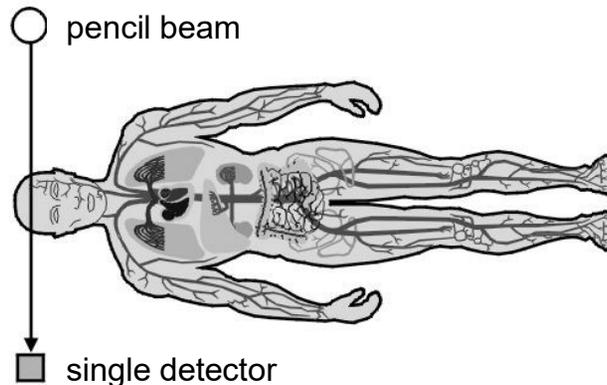
**data acquisition in Radon space (4. generation CT scanner)**



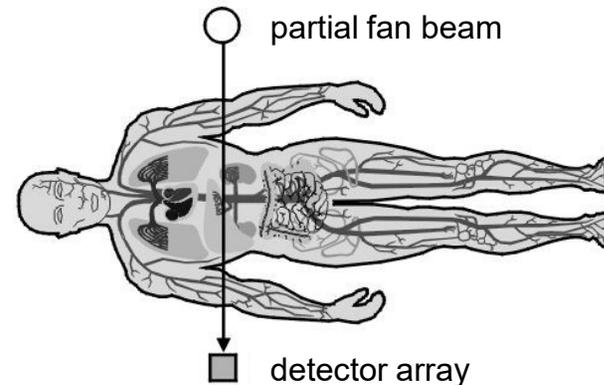
# *x-ray computed tomography (CT)*

## **data acquisition with 1. to 4. generation CT scanner**

1970



1978

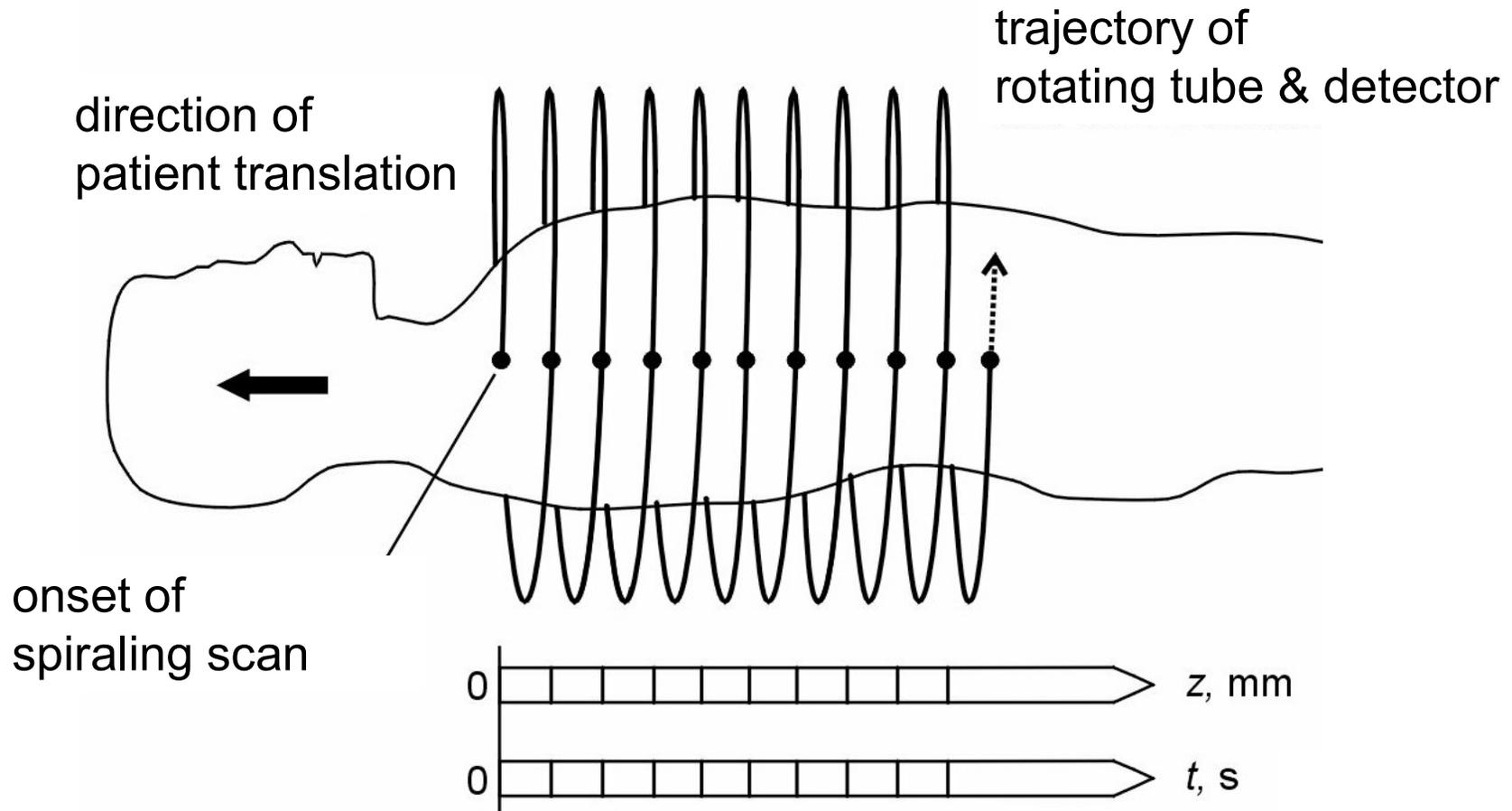


- 1. and 2. generation: head recording only
- image reconstruction of a single slice only (2-5 mm thickness)
- not suitable for large regions or whole-body imaging:
  - record, shift patient (e.g. by 2 mm), record, ...
  - duration, high radiation exposure, artifacts

# *x-ray computed tomography (CT)*

## **data acquisition with spiraling CT (W. Kalender, 1989)**

idea: slow but continuous translation of patient inside scanner while tube rotates around the center



# *x-ray computed tomography (CT)*

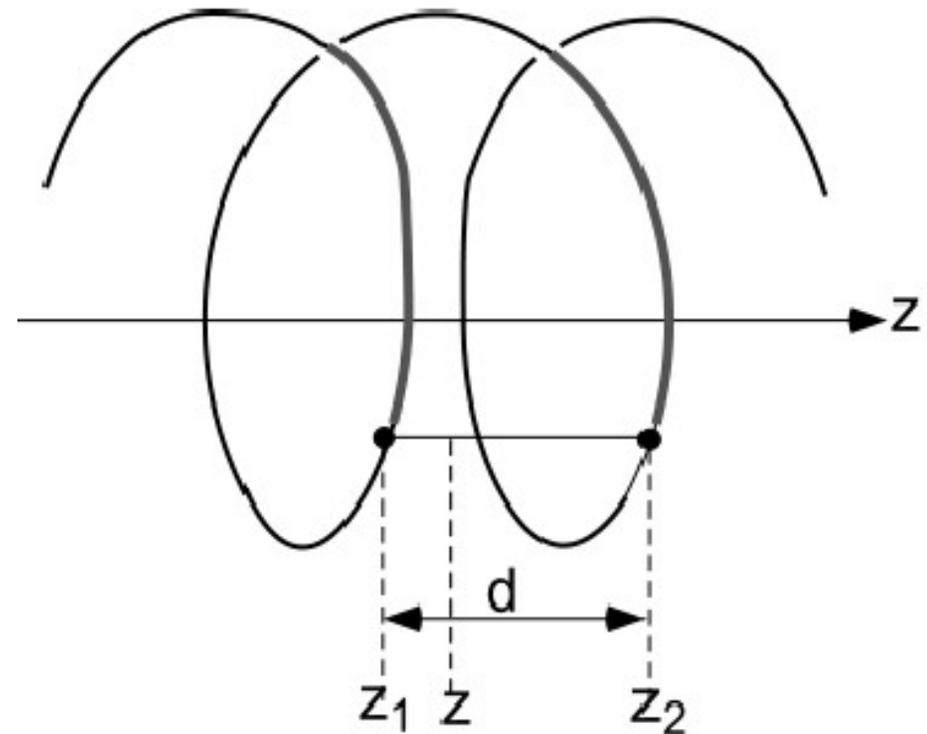
## **data acquisition with spiraling CT**

### **problem:**

- which data to use for image reconstruction ?
- projections under different angles  $\Theta$  do not fit each other !

### **ansatz:**

- for each  $\Theta$ , there are several data sets shifted rel. to each other by  $d$  ( $d$  = patient advance)
- estimate “missing” projections at each intermediate step  $z_1 < z < z_1 + d$  by interpolation (not exact but sufficiently accurate)

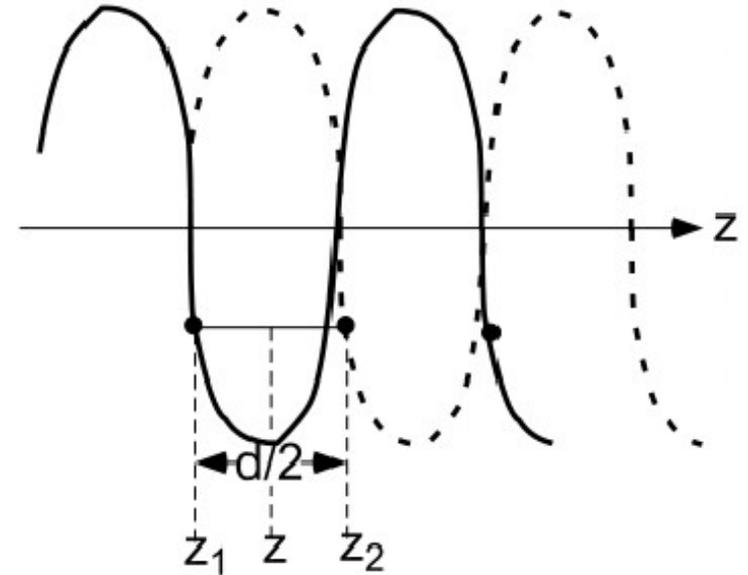


## *x-ray computed tomography (CT)*

### **data acquisition with spiraling CT**

continuously rotating tube:

- for  $d=0$ : projections with  $180^\circ < \Theta < 360^\circ$  are redundant
- for  $d \neq 0$ : projections with  $180^\circ < \Theta < 360^\circ$  provide data from intermediate slices
- these can be used for interpolation



⇒ effectively, interpolation with intermediate slices  $0 < z < d/2$  only (corresponding to rotation around  $180^\circ$ )

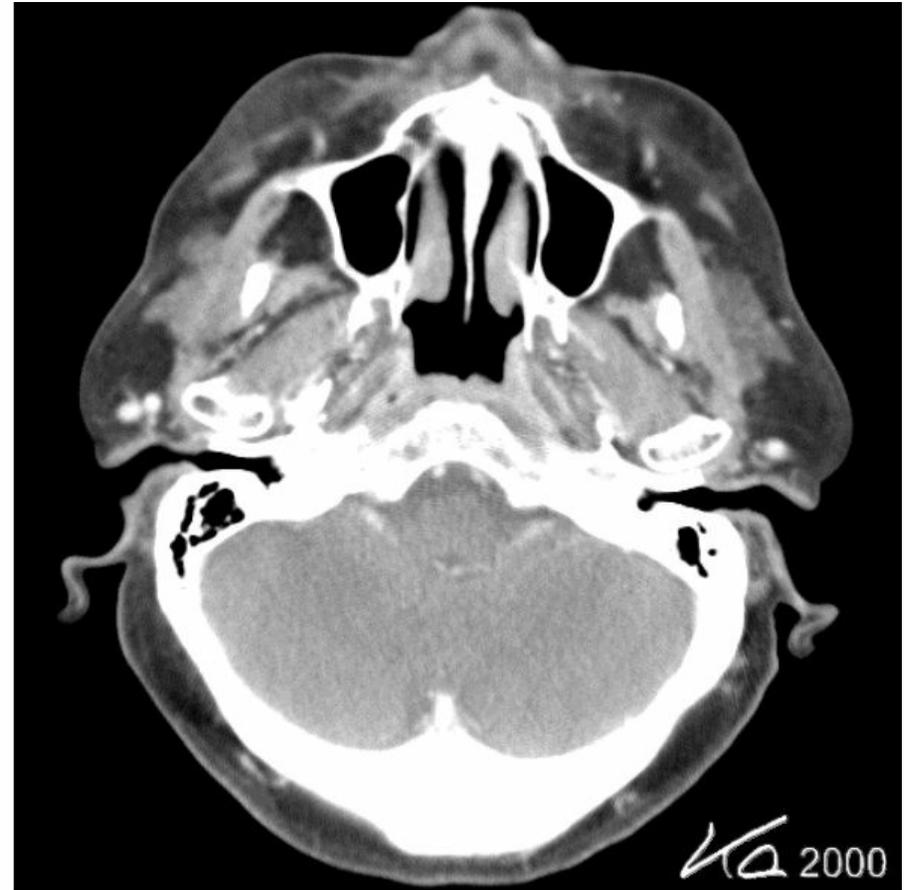
⇒ **fast 3D-acquisition of body region**

*x-ray computed tomography (CT)*

**data acquisition with spiraling CT**



a) without interpolation



b) with interpolation

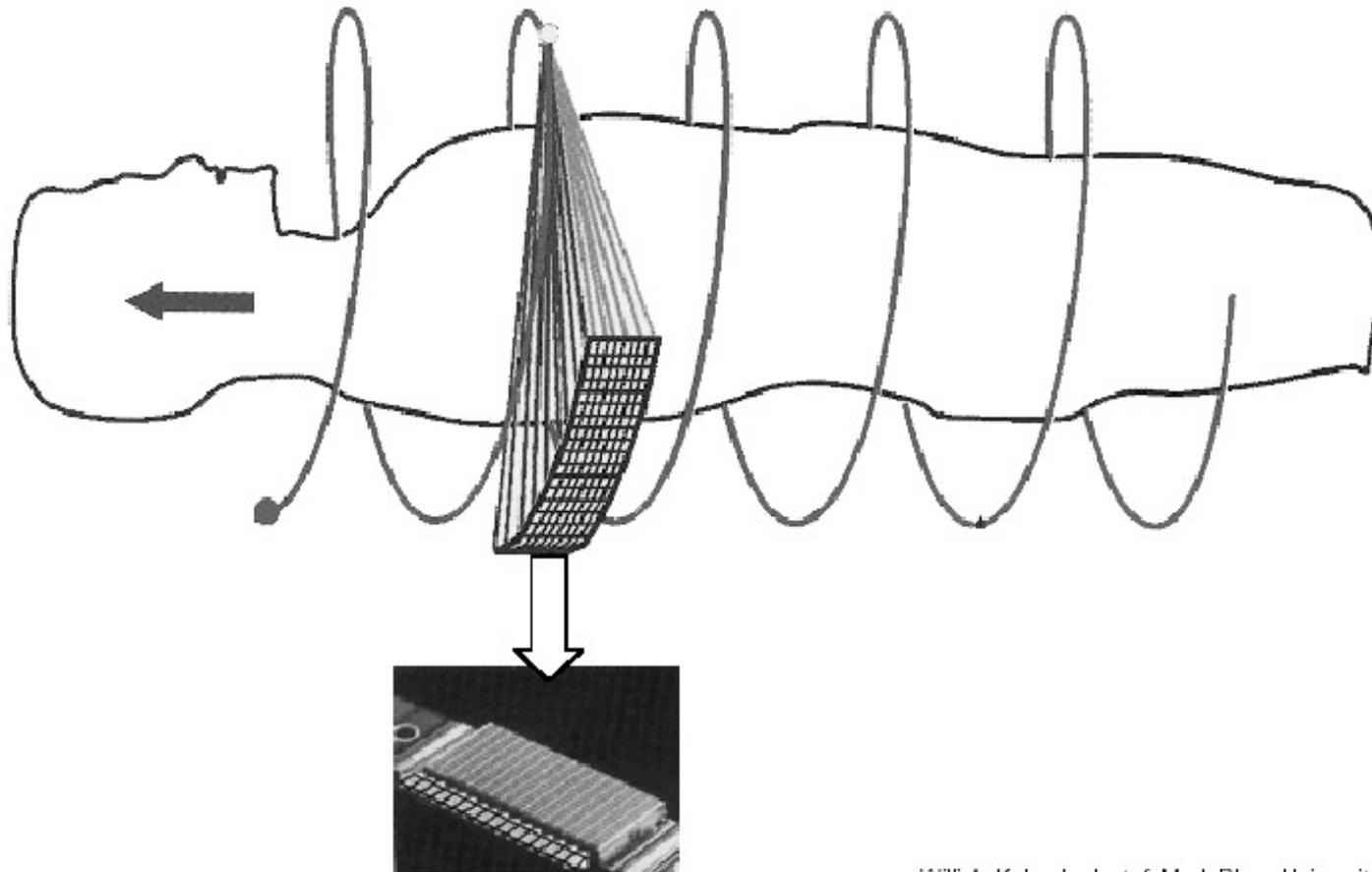
# *x-ray computed tomography (CT)*

## **data acquisition with spiraling CT**

	<b>conventional CT</b>	<b>spiraling CT</b>
acquisition	$n$ Scans each over $360^\circ$ at positions $z_1 - z_n$	1 Scan over $n \cdot 360^\circ$ at positions $z_1 - z_n$
pre-processing	data correction	data correction
intermediate steps	--	z-interpolation
image reconstruction	convolution and back projection	convolution and back projection
result	$n$ images from fixed positions $z_1 - z_n$	$>n$ images at arbitrary positions $z_1 - z_n$

# *x-ray computed tomography (CT)*

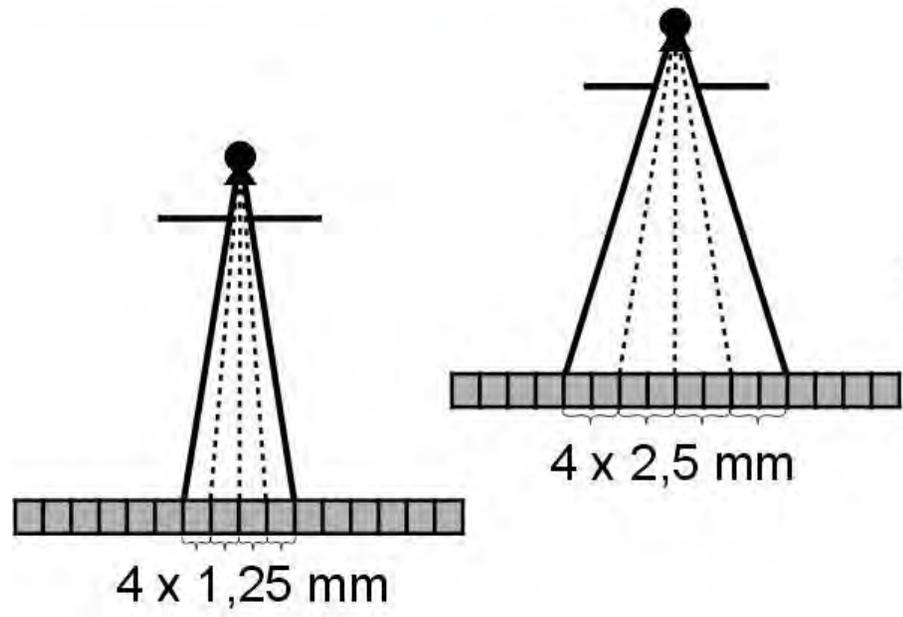
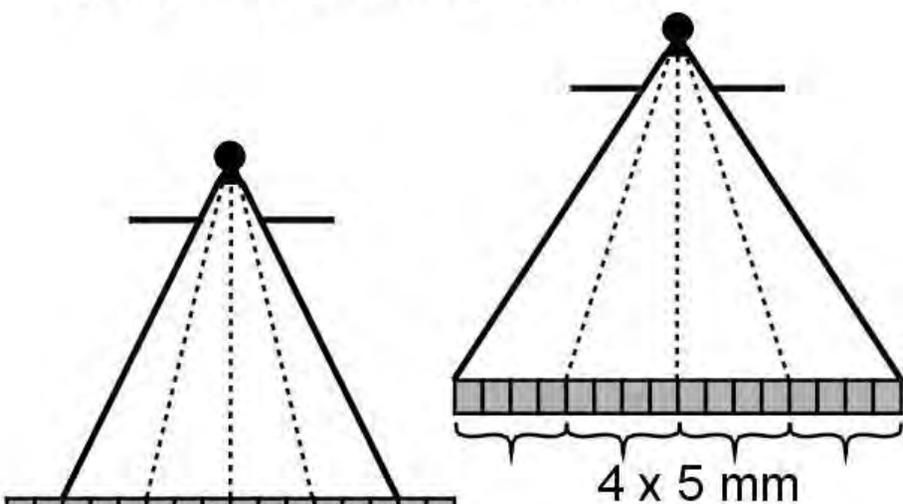
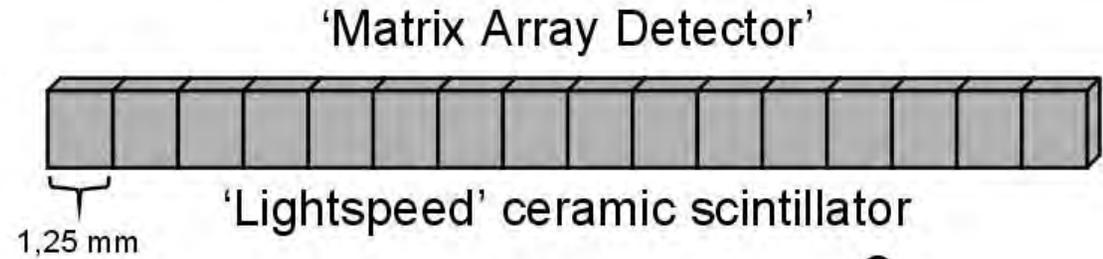
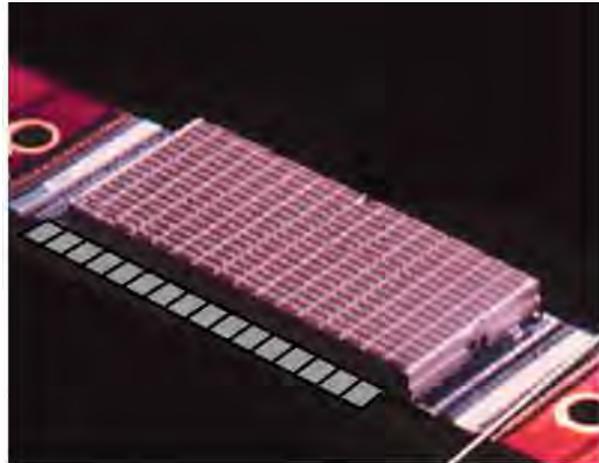
## **data acquisition with spiraling multi-slice CT**



Willi A. Kalender Inst. f. Med. Phys. Universität Erlangen

# *x-ray computed tomography (CT)*

## **data acquisition with spiraling multi-slice CT**



adjustable collimation of slices

# x-ray computed tomography (CT)

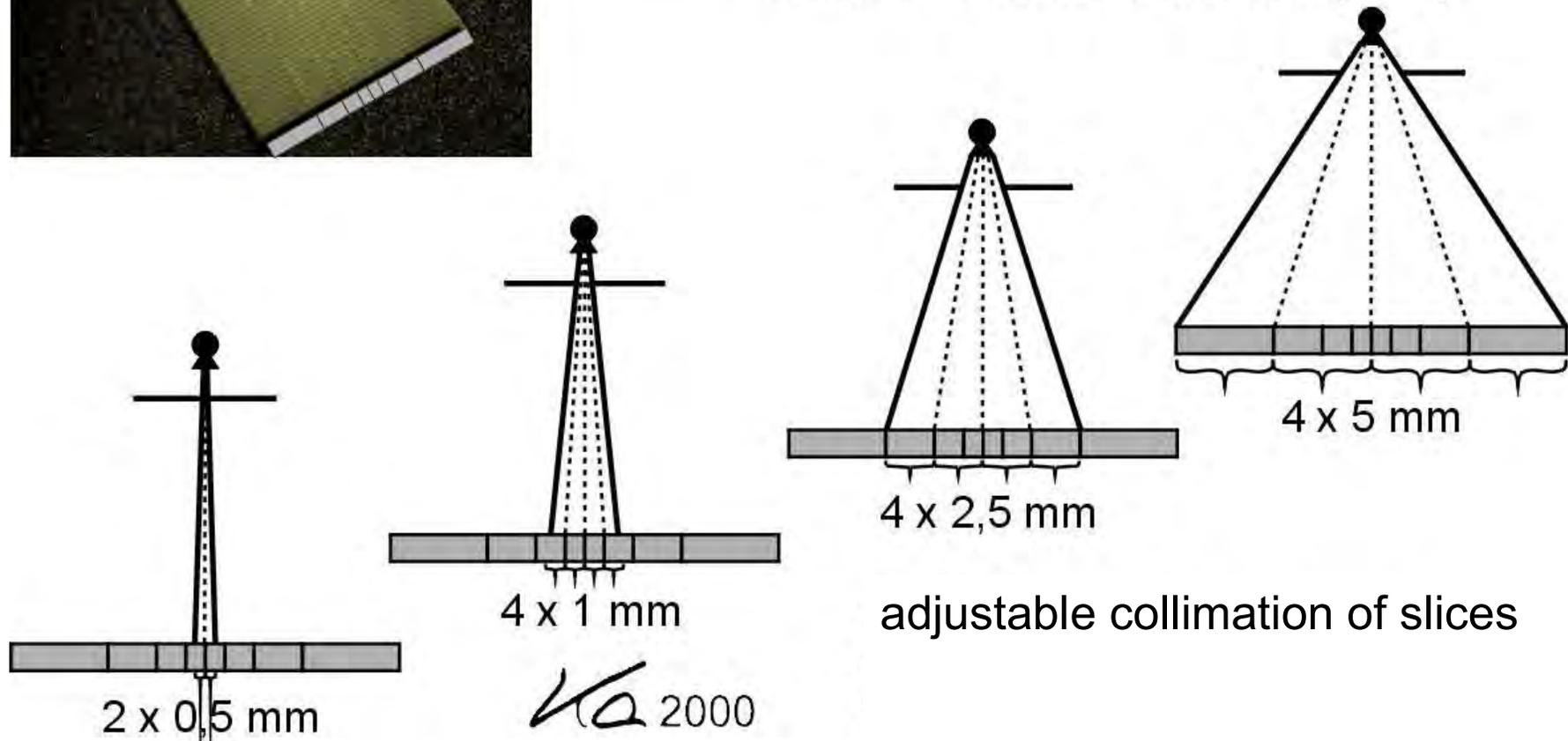
## data acquisition with spiraling multi-slice CT



'Adaptive Array Detector'

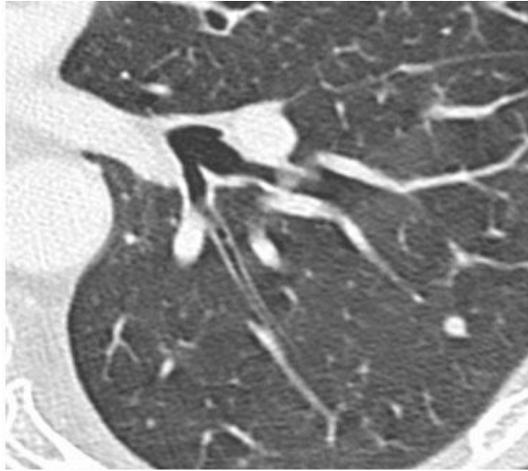


'Ultrafast' ceramic scintillator (UFC)

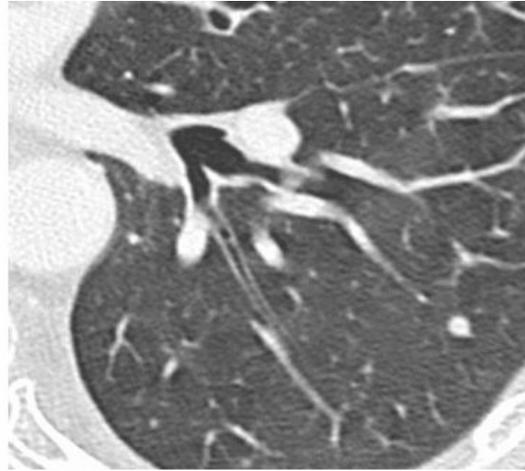


*x-ray computed tomography (CT)*

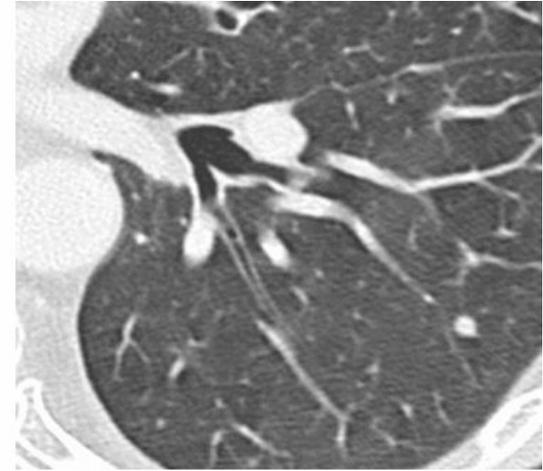
**data acquisition with spiraling multi-slice CT**  
**impact of effective slice thickness**



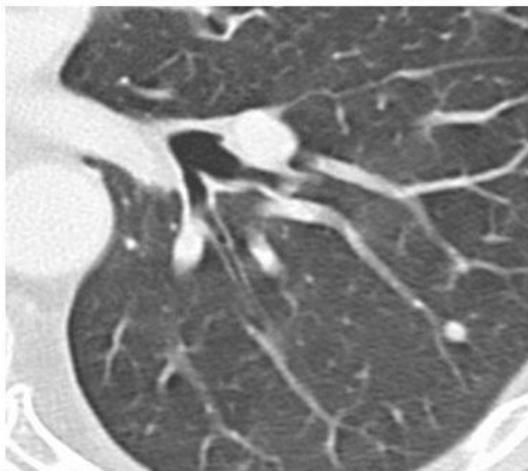
1,25 mm



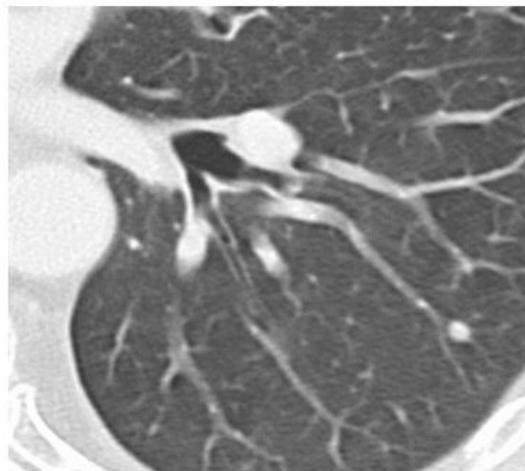
1,5 mm



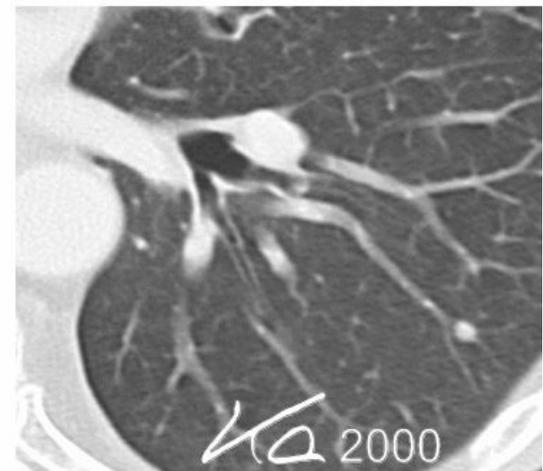
2,0 mm



3,0 mm



4,0 mm



5,0 mm

## *x-ray computed tomography (CT)*

### **alternative concepts for data acquisition: electron beam CT**

aim: shorten scan time

idea: scanning without mechanical movements (tube, detector)

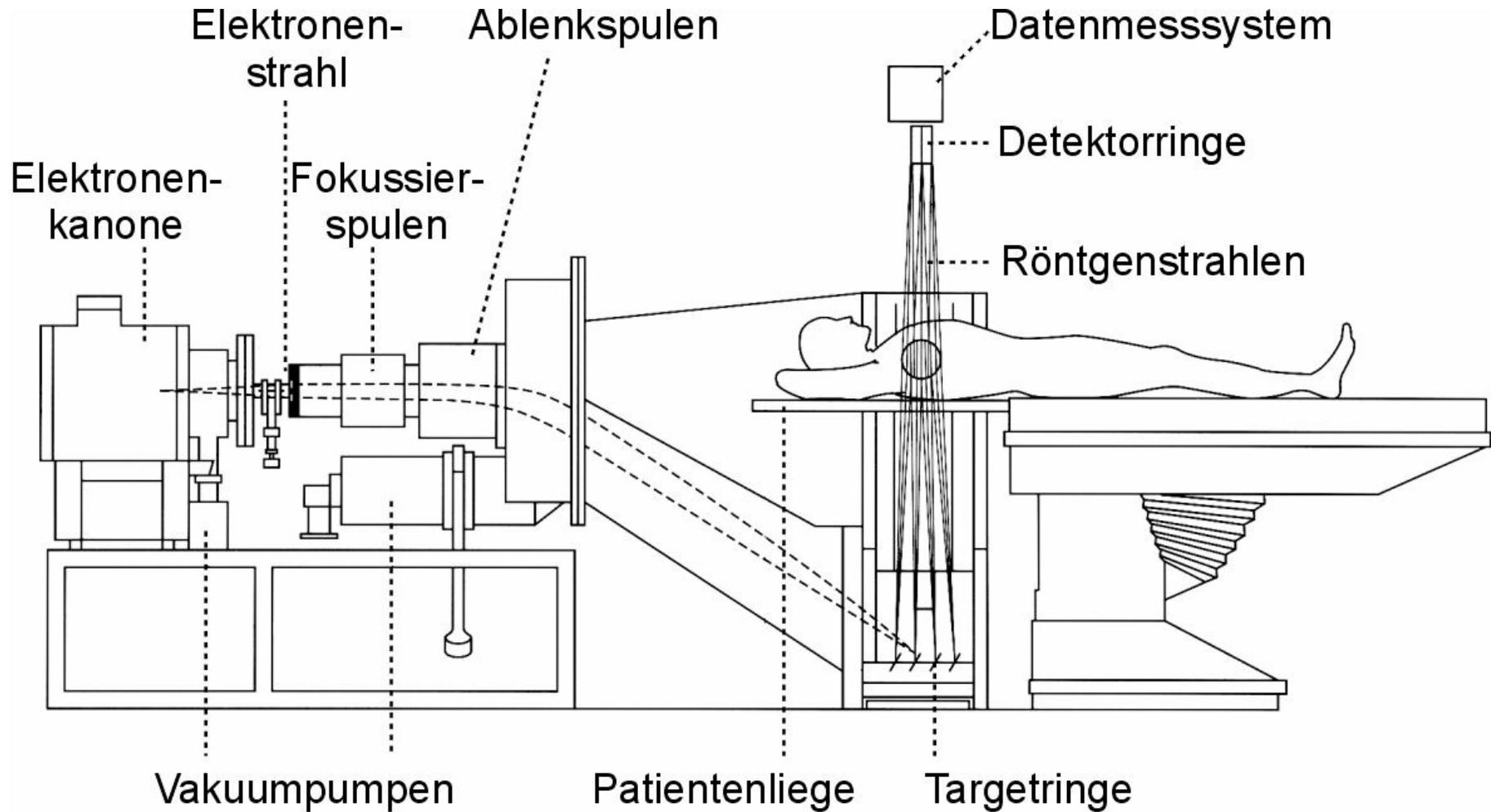
ansatz: generate electron beam, acceleration and focusing onto anode (ring-like target that encompasses the patient)

advantage: 50 -100 ms scanning time @ slice thickness 1.5 mm

disadvantage: expensive, bad image quality, limited use (e.g. arteries, bypass), rare

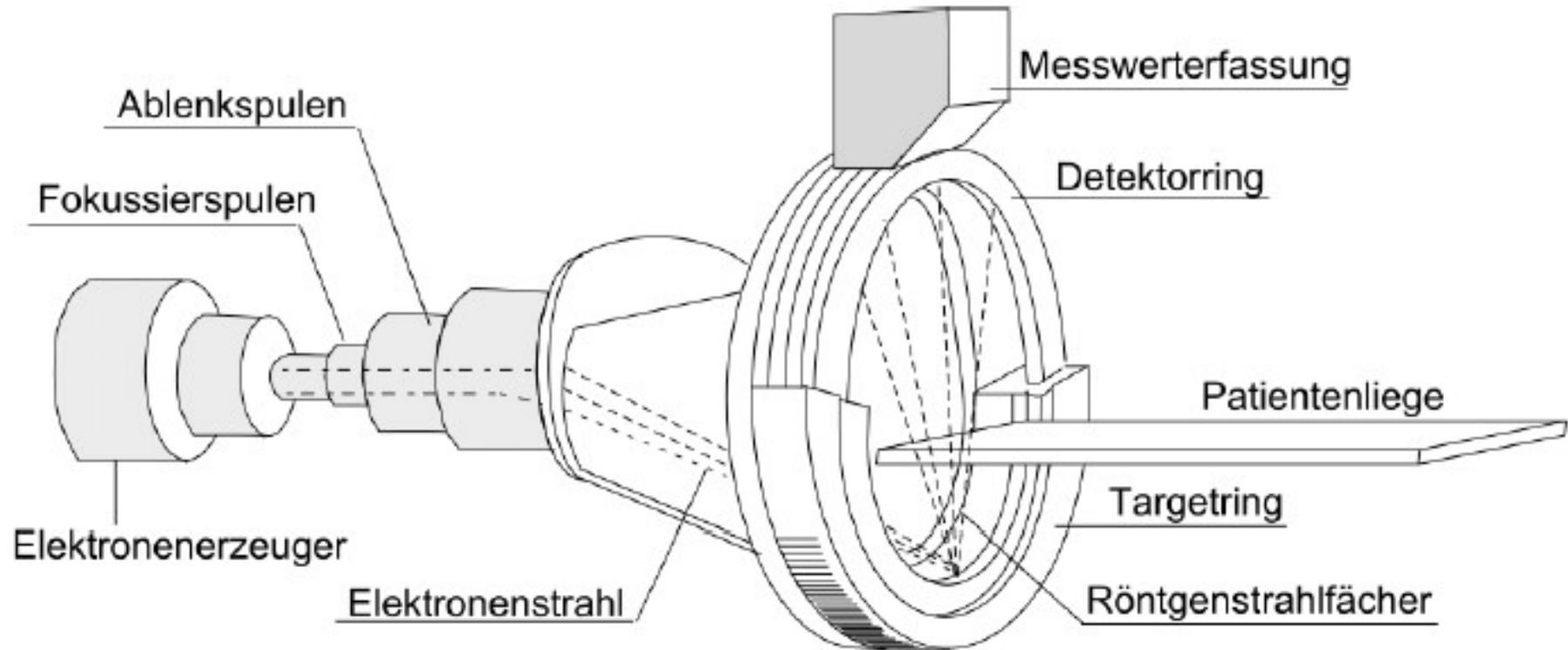
# *x-ray computed tomography (CT)*

## **alternative concepts for data acquisition: electron beam CT**



# *x-ray computed tomography (CT)*

## **alternative concepts for data acquisition: electron beam CT**



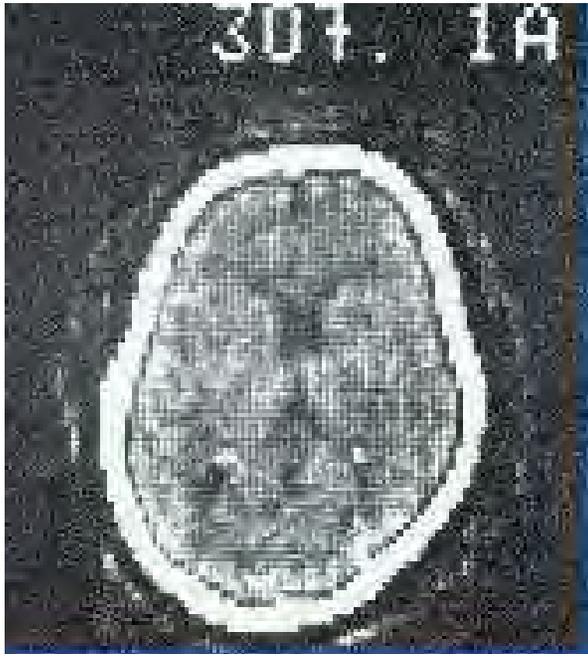
# *x-ray computed tomography (CT)*

## **development of CT performance characteristics**

	<b>1972</b>	<b>1980</b>	<b>1990</b>	<b>2000</b>
<b>min. scan time</b>	300 s	5-10 s	1-2 s	0,3-1 s
<b>data/360° scan</b>	57,6 kB	1 MB	2 MB	4x2 MB
<b>data/spiraling scan</b>	-	-	24-48 MB	200-500 MB
<b>image matrix</b>	80x80	256x256	512x512	512x512
<b>power</b>	2 kW	10 kW	40 kW	60 kW
<b>slice thickness</b>	13 mm	2-10 mm	1-10 mm	0,5 - 5 mm
<b>spatial resolution</b>	3 Lp/cm	8-12 Lp/cm	10-15 Lp/cm	12-25 Lp/cm
<b>contrast resolution</b>	5 mm(50 mGy)	3 mm (30 mGy)	3 mm (30 mGy)	3 mm (30 mGy)

apparent stagnation of contrast resolution due to early use of efficient detector systems

# *x-ray computed tomography (CT)*



**1972**

rotation in 4 min  
slice thickness: 8-13 mm  
~10 cm in >30 min



**2001**

rotation in 0.5 s  
slice thickness: 1 mm  
1 m in 1 min

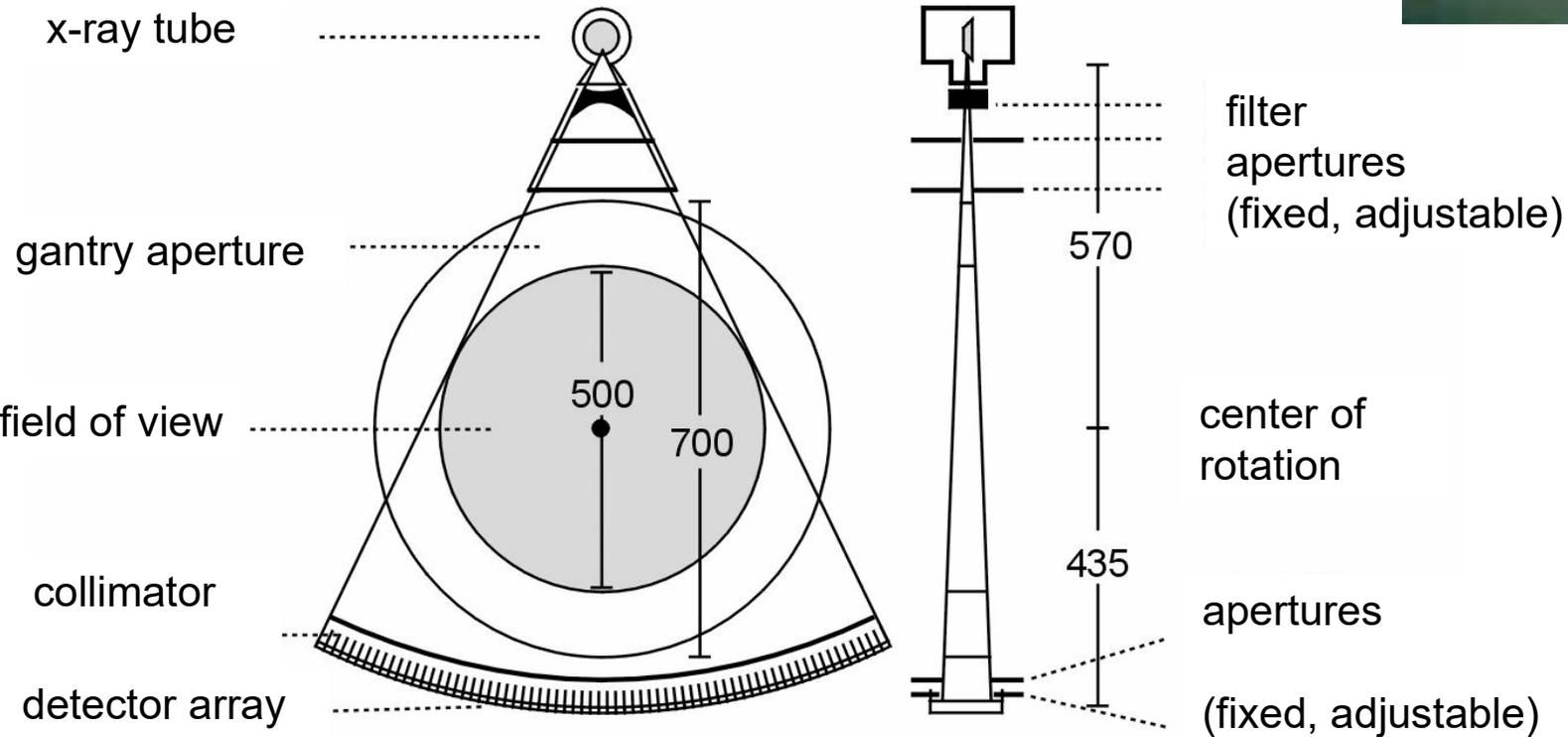
# *x-ray computed tomography (CT)*

## **system components**



# x-ray computed tomography (CT)

## system components gantry



# *x-ray computed tomography (CT)*

## **system components**

## **gantry**

total weight: 400 - 1000 kg

weight x-ray tube: ~ 100 kg

rotations: 1-2 per s

estimation of centrifugal force:

distance tube – center of rotation: ~ 600 mm

rotation time: 0.5 s / turn

⇒ acceleration:  $9.6 \text{ g} = 9.6 \text{ N/kg}$

⇒ centrifugal force acting on mounting: ~ 10.000 N

## *x-ray computed tomography (CT)*

### **system components**

### **x-ray tube**

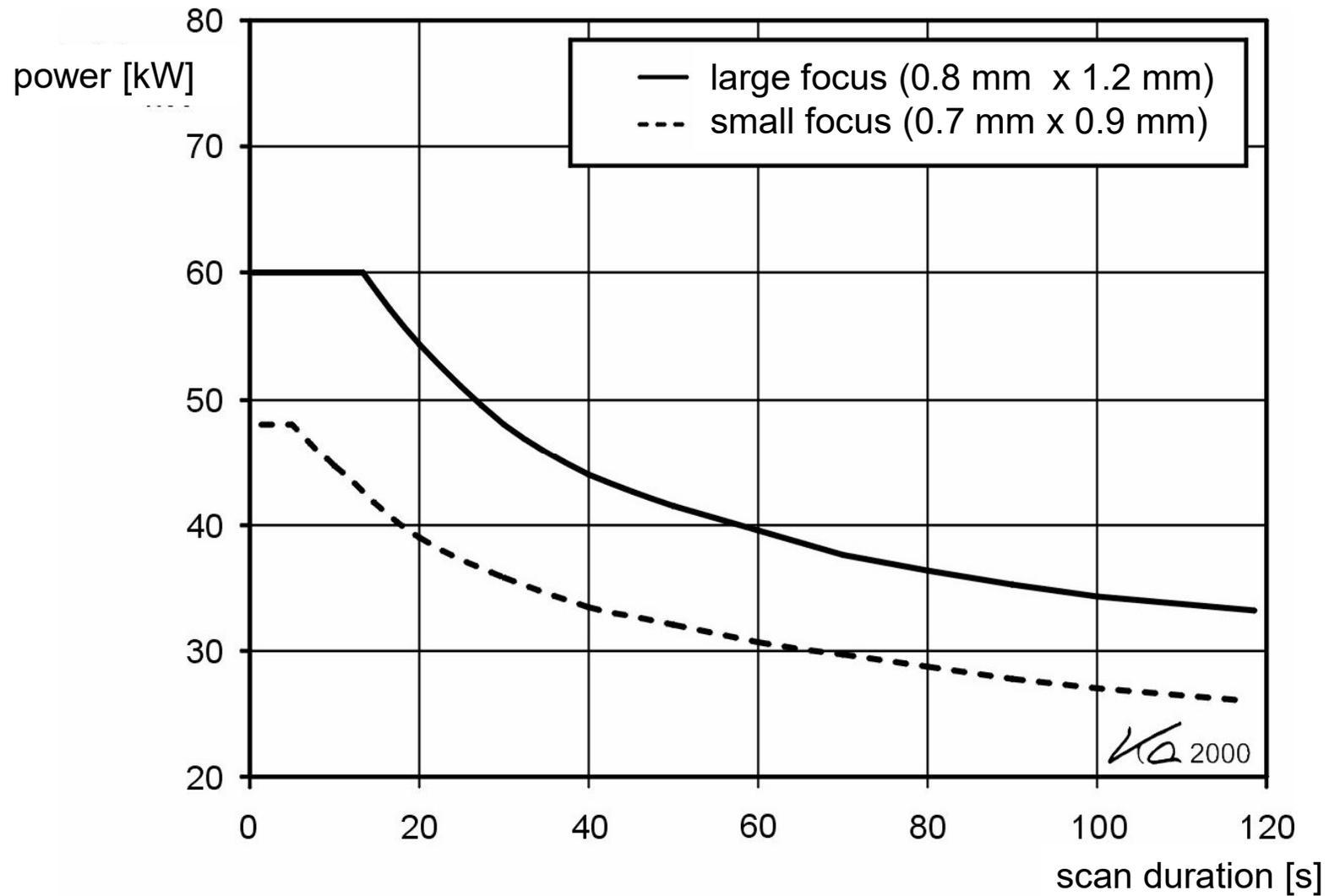
#### characteristics:

- typical power values: 20 - 60 kW @tube voltage 80 - 140 kV
- size of focus: 0.5 – 2.0 mm  
application dependent:  
e.g.: small focus: thin slices, high resolution
- heat storage capacity of anode
- scan duration

# x-ray computed tomography (CT)

system components

x-ray tube

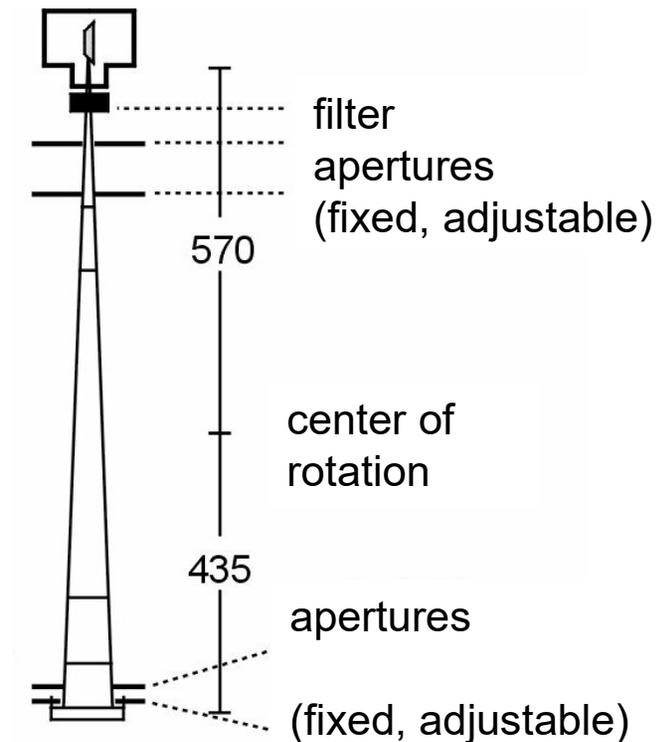


# *x-ray computed tomography (CT)*

## **system components**

- filtering x-ray energy spectrum
- definition of slice
- shielding of detector against scattering
- radiation protection

## **filter, apertures, collimation**

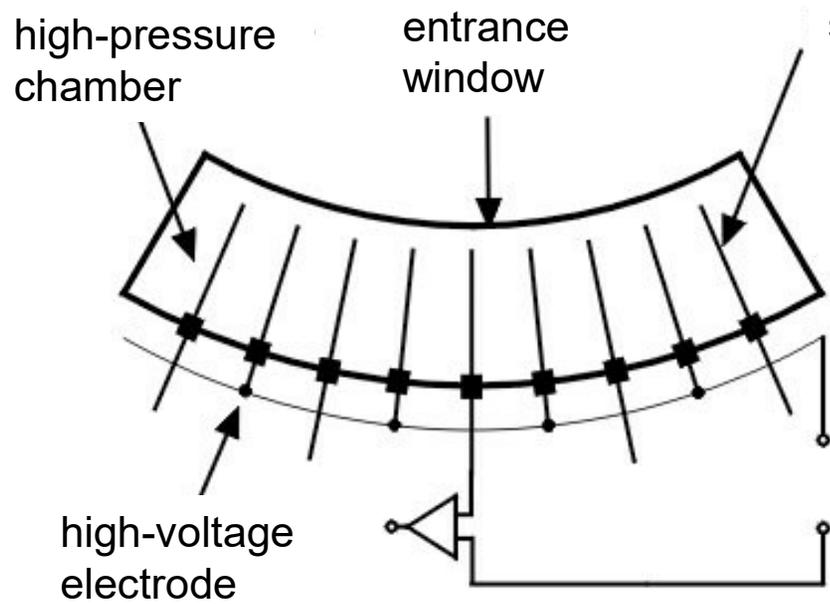


# x-ray computed tomography (CT)

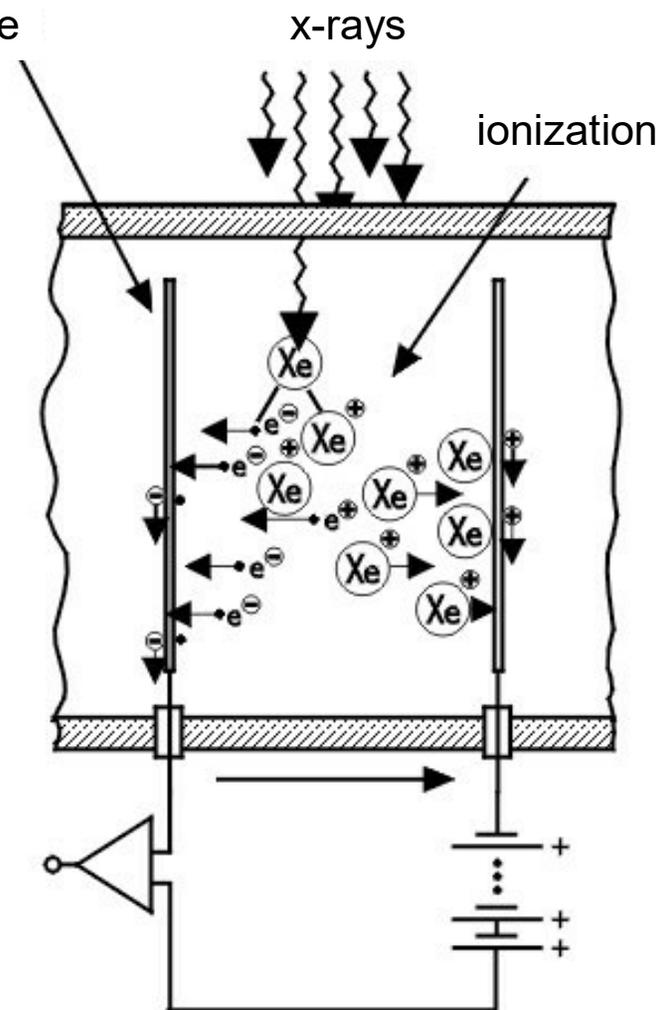
## system components

### xenon high-pressure ionization chamber

## detectors



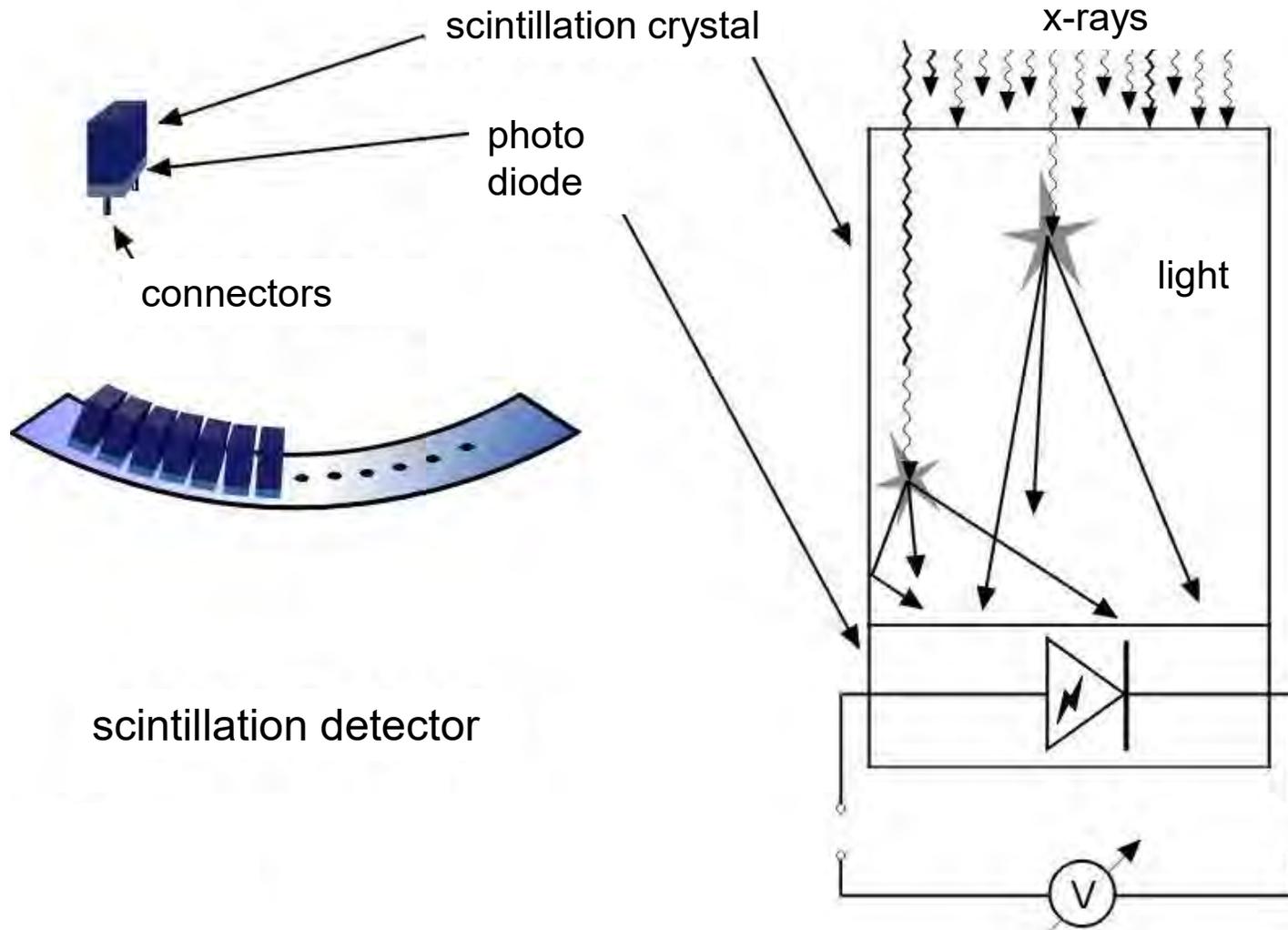
Xenon detector



# x-ray computed tomography (CT)

## system components solid-state scintillation detector

## detectors



# *x-ray computed tomography (CT)*

## **system components**

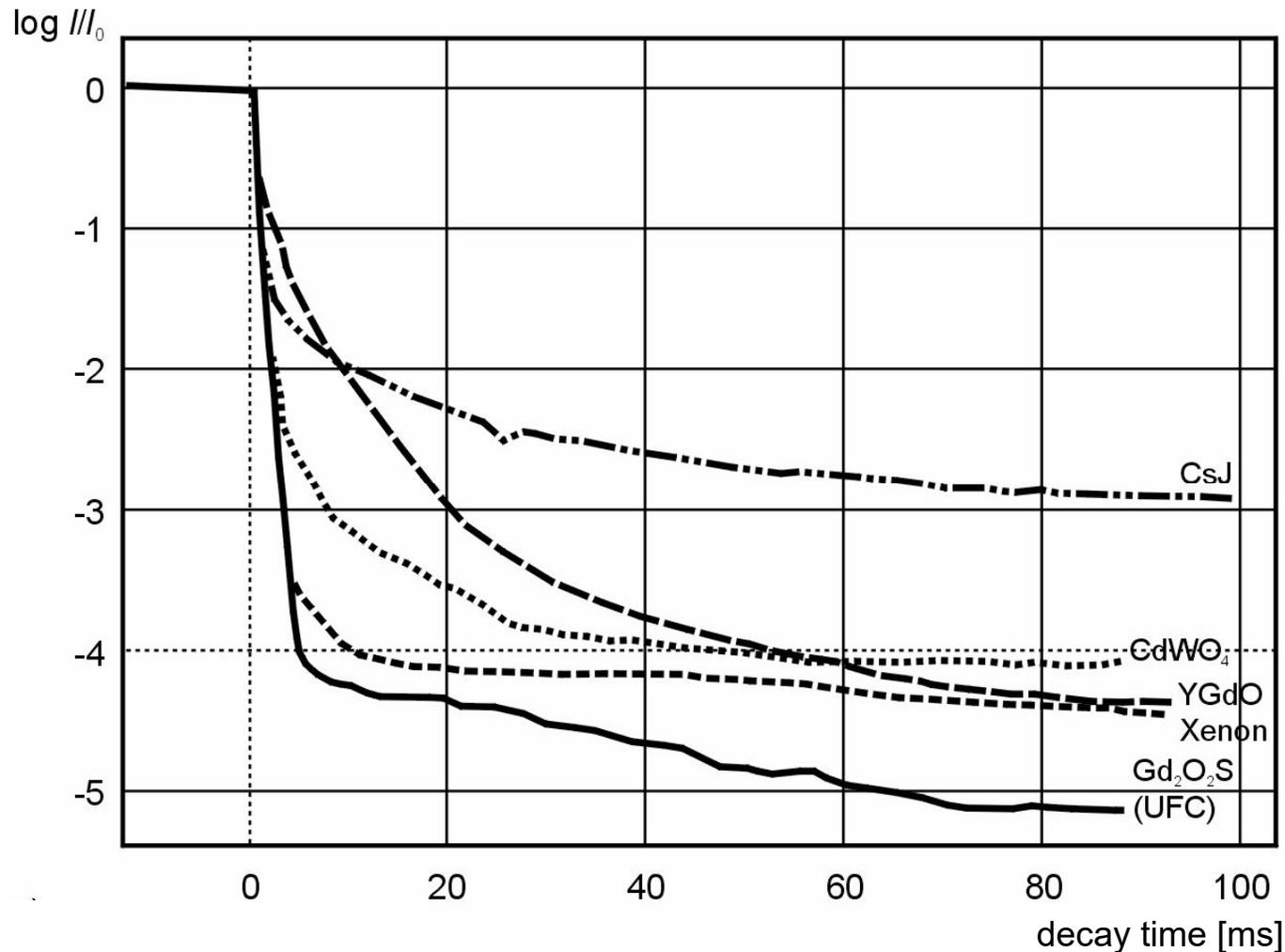
## **detection sensitivity of detectors**

	20 cm H2O	20 cm H2O + 2 cm bone	40 cm H2O + 4 cm bone
<b>detector</b>			
		<b>120 kV</b>	
Xenon (10 bar, 3cm)	42.8%	39.2%	32.9%
Xenon (25 bar, 6cm)	73,8%	74.0%	72.7%
Gadolinium- oxysulfide (1.4 mm)	89.9%	88.1%	84.5%
		<b>140 kV</b>	
Xenon (10 bar, 3cm)	38.4%	34.3%	27,1%
Xenon (25 bar, 6cm)	71.0%	70.3%	67.0%
Gadolinium- oxysulfide (1.4 mm)	85.3%	83.0%	78.2%

# x-ray computed tomography (CT)

## system components

## decay behavior of detectors



decay behavior can be approximated by two exponentials

UFC: ultra fast ceramic  
decay time:  $10^{-6}$  s

short x-ray pulse at  $T=0$

# *x-ray computed tomography (CT)*

**system components**

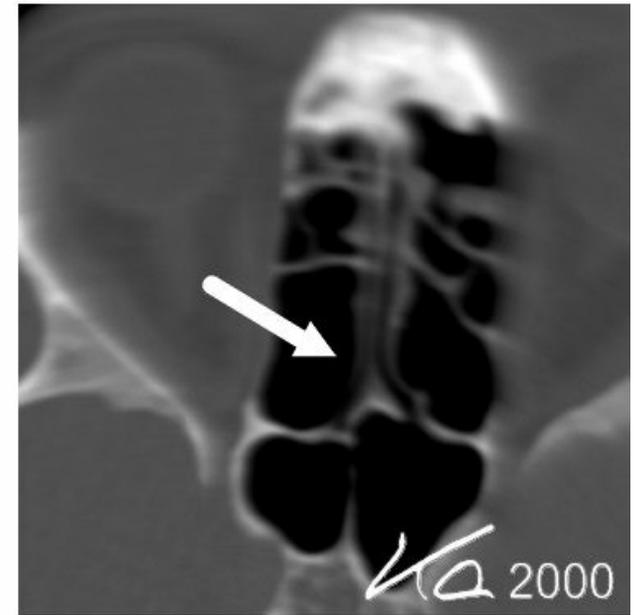
**decay behavior of detectors**



$\tau = 0.0 \text{ ms}$



$\tau = 1.0 \text{ ms}$



$\tau = 2.5 \text{ ms}$

**a too long decay time  $\tau$  (afterglow) can deteriorate spatial resolution and image quality!**

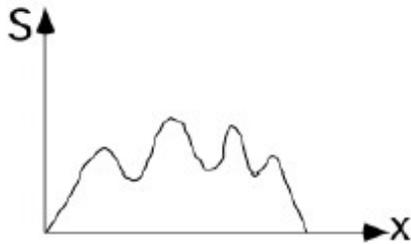
# x-ray computed tomography (CT)

## system components

## detectors and sampling theorem

D = size of detector;  $\Delta s$  = center-to-center distance between detectors

signal at detector array



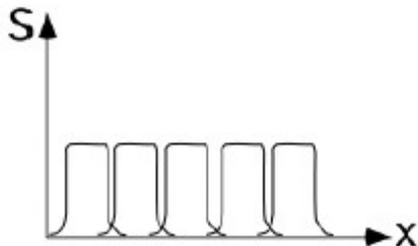
convolution  
with  
rectangular  
function



multiplication in  
Fourier domain  
with:

$$\frac{\sin(\pi \cdot D \cdot w)}{\pi \cdot D \cdot w}$$

sensitivity profile  
of detectors



max. frequency:

$$\pi \cdot D \cdot w_{\max} = \pi \Rightarrow w_{\max} = \frac{1}{D}$$

sampling theorem requires:

$$\Delta s \leq \frac{1}{2w_{\max}} = \frac{D}{2}$$



**aliasing !**

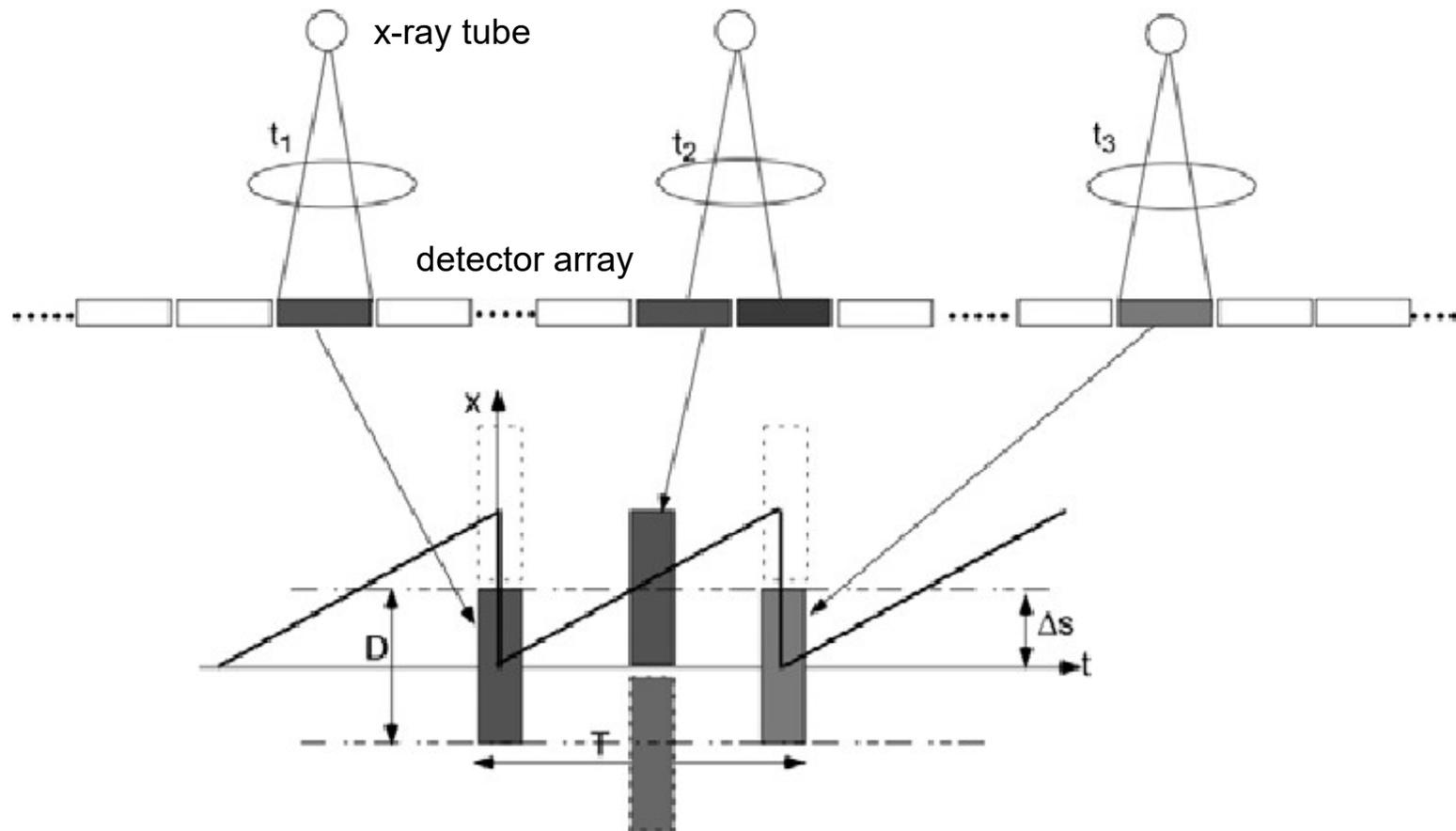
# *x-ray computed tomography (CT)*

## **system components**

## **detectors and sampling theorem**

solution: “bouncing” focus (cf. 3. generation CT scanner)

→ sampling with half the width of detector



## *x-ray computed tomography (CT)*

### **resolution of CT (I)**

criterion: modulation transfer function (MTF)

derivation of MTF for CT (restriction to center of scanner):

uncertainties:

(1) deviation of x-ray beam from ideal needle-like beam

(2) reconstruction algorithm

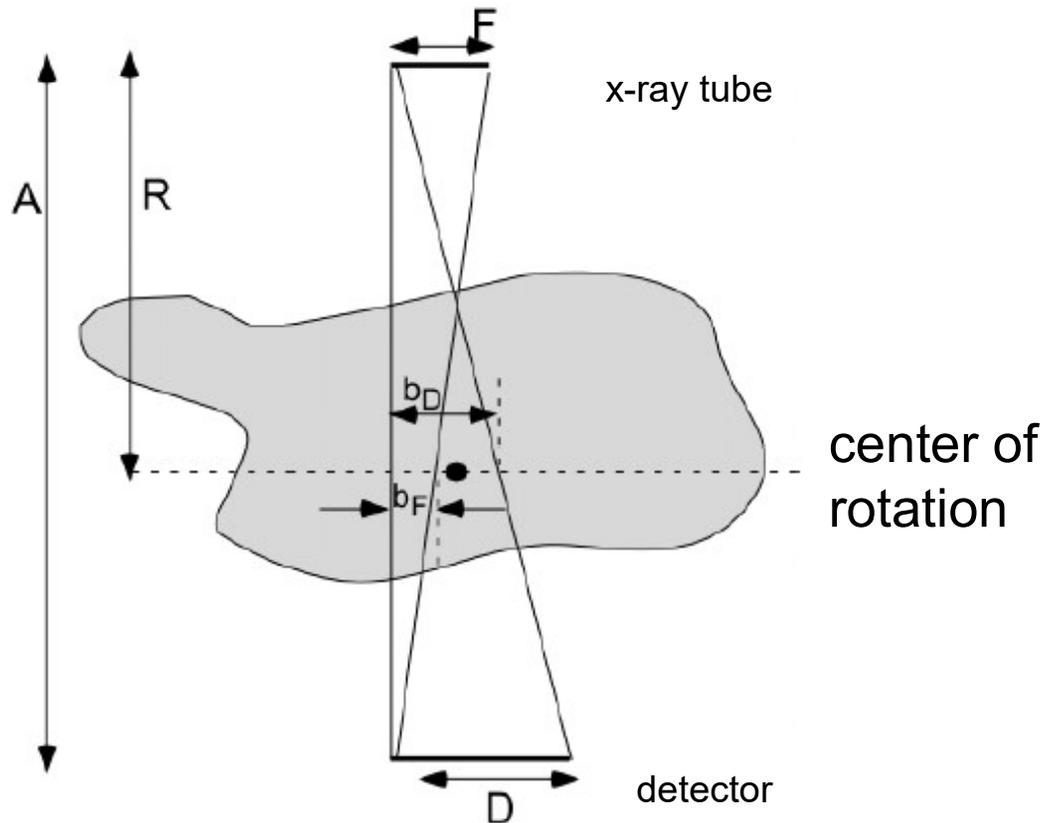
⇒

$$\mathbf{MTF}_{\text{CT}} = \mathbf{MTF}_{\text{beam}} \cdot \mathbf{MTF}_{\text{algorithm}}$$

# *x-ray computed tomography (CT)*

## **resolution of CT (II)**

trajectories of x-rays in scanner and definition of some geometric quantities



$A$  = distance tube - detector

$R$  = distance tube - center of rotation

$F$  = size of focus in tube

$D$  = size of detector

$b_F = F \cdot \frac{A-R}{A}$  effective size of focus  
in center

$b_D = D \cdot \frac{R}{A}$  effective size of detector  
in center

## resolution of CT (III)

**MTF<sub>beam</sub>**

- assumption 1: point-like detector; extended focus of x-ray tube

⇒ point spread function = rectangular function with width  $b_F$

⇒ associated MTF in Fourier domain =  $|\sin(u)/u|$

- assumption 2: point-like focus of x-ray tube, extended detector

⇒ point spread function = rectangular function with width  $b_D$

⇒ associated MTF in Fourier domain =  $|\sin(u)/u|$

(with cylindrical coordinates  $u=w \cdot \cos\Theta$  und  $v=w \cdot \sin\Theta$  in Fourier domain)

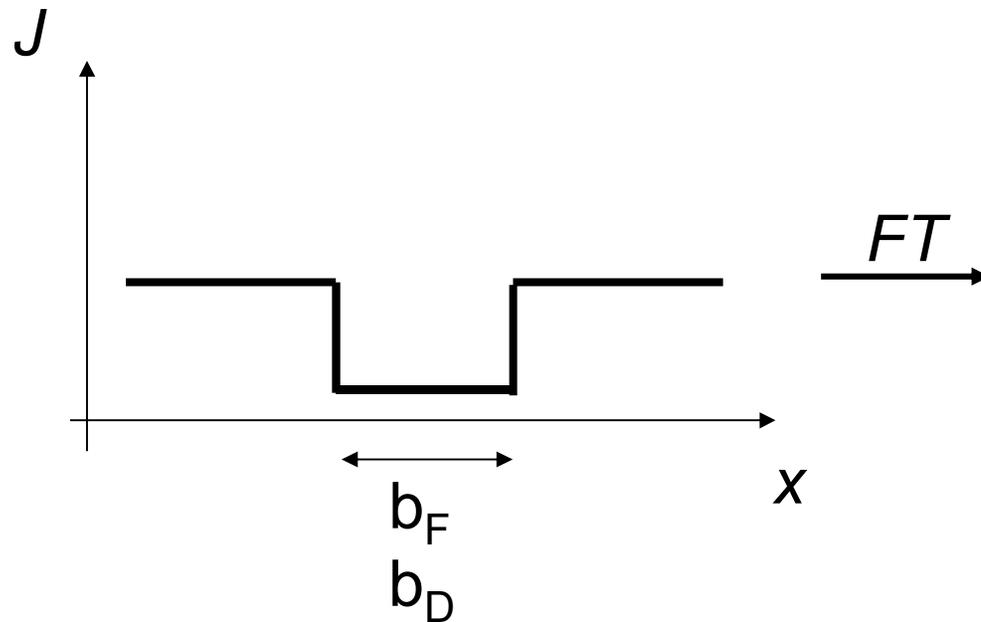
$$\Rightarrow MTF_{beam}(w) = \left| \frac{\sin(\pi \cdot b_F \cdot w)}{\pi \cdot b_F \cdot w} \right| \cdot \left| \frac{\sin(\pi \cdot b_D \cdot w)}{\pi \cdot b_D \cdot w} \right|$$

# x-ray computed tomography (CT)

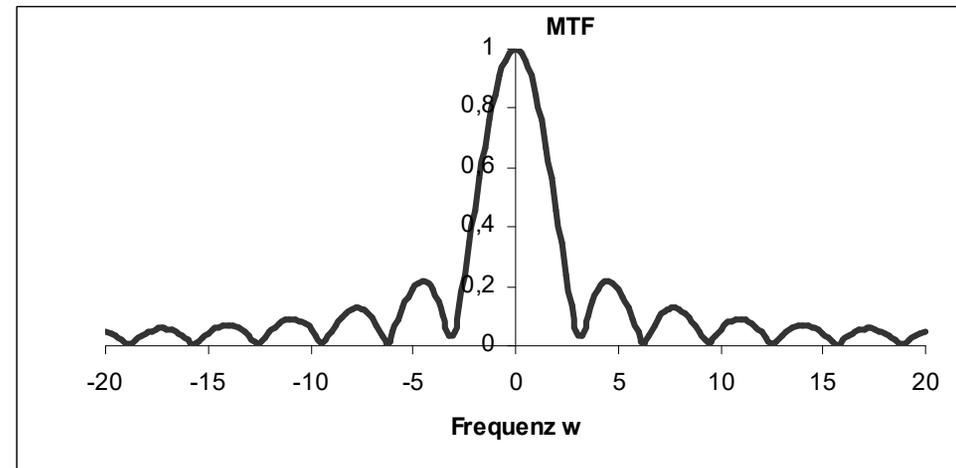
## resolution of CT (III)

**MTF<sub>beam</sub>**

point spread function



modulation transfer function



either extended focus in tube (width  $b_F$ )  
or extended detector (width  $b_D$ )

## resolution of CT (III)

**MTF<sub>beam</sub>**

MTF<sub>beam</sub> is the better the smaller  $b_F$  **and**  $b_D$

if patient is lying in the center of scanner, we have with theorem of intersecting lines:

$$b_F = 1/2 \cdot F \text{ and } b_D = 1/2 \cdot D$$

example:

size of focus and detector: 1 mm  
⇒ resolution: 0.5 mm (typical value !)

## resolution of CT (IV)

**MTF** algorithm

assumption: image reconstruction with filtered back projection

⇒ influencing factors:

(1)  $H(w)$  = filter function = FT of convolution kernel  
(application-dependent)

(2)  $G(w)$  = FT of interpolation function

$$G(w) = \left( \frac{\sin(\pi \cdot \Delta s \cdot w)}{\pi \cdot \Delta s \cdot w} \right)^2$$

$\Delta s$  = center-to-center distance  
between detectors

coarse-grained sampling → additional interpolation → bad resolution

## *x-ray computed tomography (CT)*

### **resolution of CT (V)**

$$MTF_{CT}(w) = \left| \frac{\sin(\pi \cdot b_F \cdot w)}{\pi \cdot b_F \cdot w} \right| \cdot \left| \frac{\sin(\pi \cdot b_D \cdot w)}{\pi \cdot b_D \cdot w} \right| \cdot \left| \frac{\sin(\pi \cdot \Delta s \cdot w)}{\pi \cdot \Delta s \cdot w} \right|^2 \cdot \frac{|H(w)|}{|w|}$$

consider frequency  $w$ , where MTF is reduced to 50 % :

CT	up to 1.2 lp/mm (~ 0.5 mm)
x-ray image amplifier	up to 5 lp/mm (~ 0.1 mm)
x-ray film	up to 10 lp/mm (~ 0.05 mm)

CT has inferior resolution than other x-ray-based techniques  
BUT: CT provides tomographic images!

# *x-ray computed tomography (CT)*

## **noise and CT**

### ***noise sources in CT***

noisy data (detected quanta)



recording

noisy projections



reconstruction algorithm

pixel noise

## *x-ray computed tomography (CT)*

### **noise and CT**

#### ***noisy data***

consider number  $N$  of quanta in detector:  $N_{\Theta}(s) = N_0 \cdot e^{-\int \mu(x,y) dl}$

where  $N_0$  = detected quanta/detector without patient  
and  $N_{\Theta}(s)$  = detected quanta/detector with patient  
(projection angle  $\Theta$ ; site of detector  $s$ )

for the projections, we have:

$$p_{\Theta}(s) = \ln \frac{N_0}{N_{\Theta}(s)} = \ln N_0 - \ln N_{\Theta}(s)$$

number of detected quanta is Poisson distributed:

$$N_{\Theta}(s) = \bar{N}_{\Theta}(s) \pm \sqrt{\bar{N}_{\Theta}(s)}$$

**noise and CT**

$$\begin{aligned} \Rightarrow \ln N_{\Theta}(s) &= \ln \left\{ \bar{N}_{\Theta}(s) \pm \sqrt{\bar{N}_{\Theta}(s)} \right\} = \ln \left\{ \bar{N}_{\Theta}(s) \left( 1 \pm \frac{\sqrt{\bar{N}_{\Theta}(s)}}{\bar{N}_{\Theta}(s)} \right) \right\} \\ &= \ln \left\{ \bar{N}_{\Theta}(s) \left( 1 \pm \frac{\sqrt{\bar{N}_{\Theta}(s)}}{\bar{N}_{\Theta}(s)} \right) \right\} \approx \ln \bar{N}_{\Theta}(s) \pm \frac{1}{\sqrt{\bar{N}_{\Theta}(s)}} \end{aligned}$$

with  $\frac{1}{\sqrt{\bar{N}_{\Theta}(s)}} \ll 1$

## *x-ray computed tomography (CT)*

### **noise and CT**

#### **⇒ *noisy projections***

assumption: number of quanta  $N_0$  (without patient) can be estimated with arbitrary precision

$$\begin{aligned} p_{\Theta}(s) &= \ln N_0 - \ln N_{\Theta}(s) \\ &= \ln N_0 - \ln \bar{N}_{\Theta}(s) \pm \frac{1}{\sqrt{\bar{N}_{\Theta}(s)}} \\ &= \bar{p}_{\Theta}(s) \pm \frac{1}{\sqrt{\bar{N}_{\Theta}(s)}} \end{aligned}$$

$$\Rightarrow \sigma_P^2 = \frac{1}{\bar{N}_{\Theta}(s)}$$

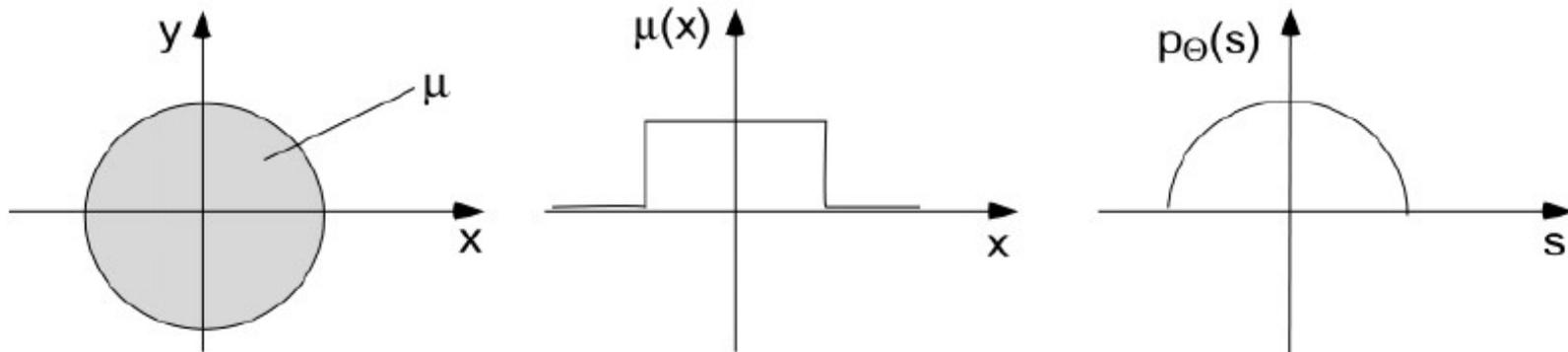
variance of projections

# *x-ray computed tomography (CT)*

## **noise and CT**

### **influence of *noisy projections* on *pixel noise***

assume cylinder with homogeneous  $\mu$  in center of scanner



projections are equal for all angles  $\Theta$ .

now consider pixel noise at  $x=y=0$  ( $p_{\theta}(s)$  are flat)

# *x-ray computed tomography (CT)*

## **noise and CT**

### ***pixel noise***

with

$$\tilde{p}_{\Theta}(n \cdot \Delta s) = \Delta s \cdot \sum_{k=-K}^{+K} p_{\Theta}(n \cdot \Delta s - k \cdot \Delta s) \cdot h(k \cdot \Delta s)$$

and

$$f(x, y) = \frac{\pi}{M} \sum_{i=1}^M \tilde{p}_{\Theta}(x \cos \Theta_i + y \sin \Theta_i)$$

we have:

$$f(0,0) = \frac{\pi}{M} \cdot \Delta s \cdot \sum_{k=-K}^{+K} p_{\Theta}(0) \cdot h(k \cdot \Delta s)$$

cf. image reconstruction with  
filtered back projection  
*analog and digital filtering*

$\Delta s$  = center-to-center distance  
between detectors

M = number of projections

h = filter function

with flat projections:

all data to the left and right of  
 $s = 0$  equal  $p_{\Theta}(0)$  in the range  
-K ... +K

# *x-ray computed tomography (CT)*

## **noise and CT**

### ***pixel noise***

projections  $p_{\Theta}$  are statistically independent  
and distributed around the mean  $\bar{p}_{\Theta}(0)$

$$p_{\Theta}(0) = \bar{p}_{\Theta}(0) \pm \frac{1}{\sqrt{\bar{N}}} \quad \text{where } \bar{N} = \bar{N}_{\Theta}(0)$$

with error propagation :

if  $E(A) = A \pm a$  und  $E(B) = B \pm b$  ( $E(\bullet)$  = expected value)

$$\Rightarrow E(A + B) = A + B \pm \sqrt{a^2 + b^2}$$

$$\Rightarrow f(0,0) = \overline{f(0,0)} \pm \frac{\pi}{M} \cdot \Delta s \cdot \sqrt{\sum_{\Theta_i} \sum_{k=-K}^{+K} \frac{h^2(k \cdot \Delta s)}{\bar{N}}}$$

# x-ray computed tomography (CT)

## noise and CT

### *pixel noise*

$\bar{N}$  is constant; sum over all  $\Theta_i$

$$\Rightarrow \sigma_{Pixel}^2 = \left( \frac{\pi}{M} \cdot \Delta s \right)^2 \cdot M \cdot \frac{1}{\bar{N}} \sum_{k=-K}^{+K} h^2(k \cdot \Delta s)$$

with Parseval's theorem, we have :

$$\Rightarrow \sigma_{pixel}^2 = \frac{\pi^2 \cdot \Delta s}{M} \cdot \frac{1}{\bar{N}} \int_{-\omega_{max}}^{+\omega_{max}} |H(\omega)|^2 d\omega$$

$\Delta s$  = center - to - center distance detectors

$M$  = number of projections

$\bar{N}$  = mean count rate

$H(\omega)$  = filter function for filtered back projection

## *x-ray computed tomography (CT)*

### **noise and CT**

***pixel noise is the smallest, if***

- center-to-center distance between detectors  $\Delta s$  is small***
- number of projections  $M$  is high***
- number of quanta per recording site is high***

***and***

- area under squared filter function  $H(\omega)$  is small***

***BUT:***

***MTF is deteriorated by the same token !***

## *x-ray computed tomography (CT)*

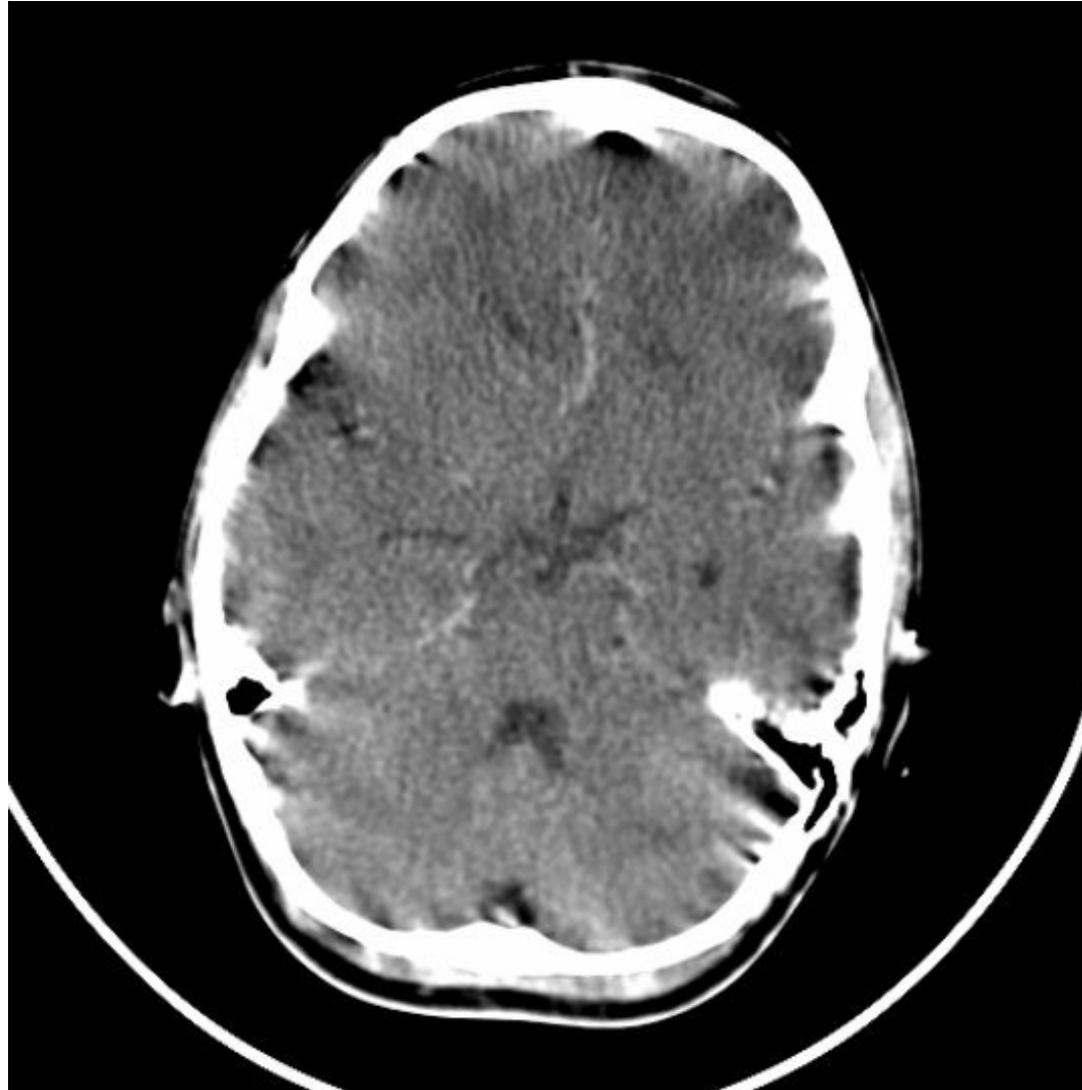
### **artifacts**

- **patient movements**
- **failure of recording electronics**
- **metal implants**
- **violation of limits of field of view**
- **partial volume artifacts**
- **beam hardening**
- **scattering**

*x-ray computed tomography (CT)*

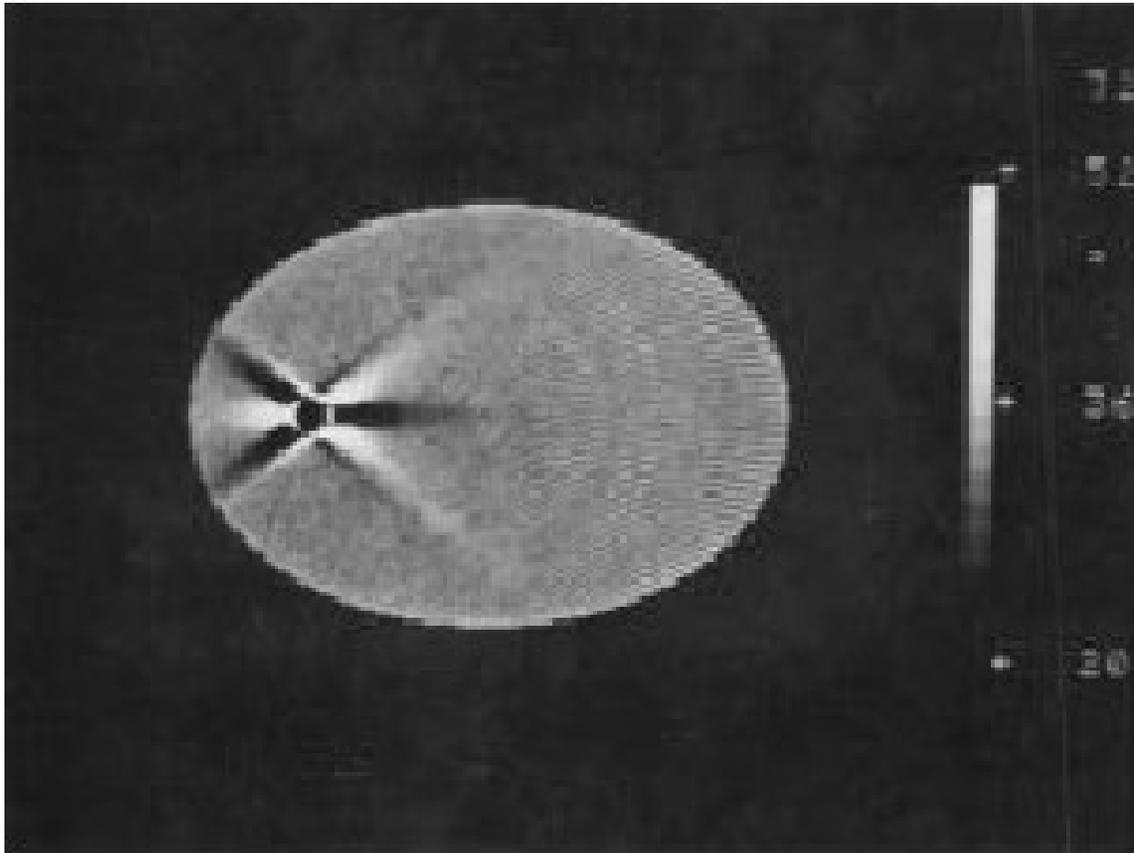
**artifacts**

**patient movements**



# *x-ray computed tomography (CT)*

## **artifacts**



## **patient movements**

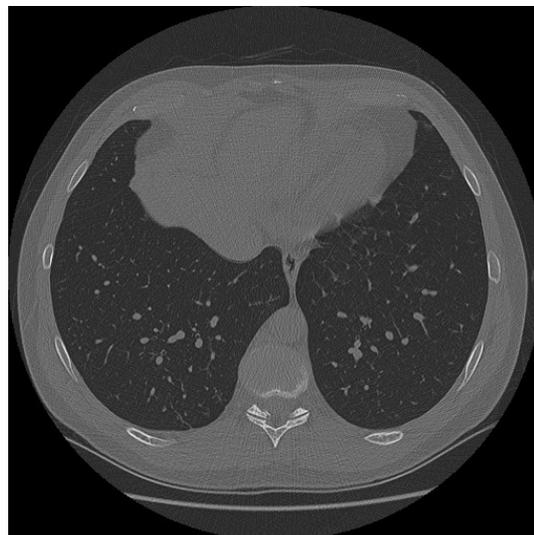
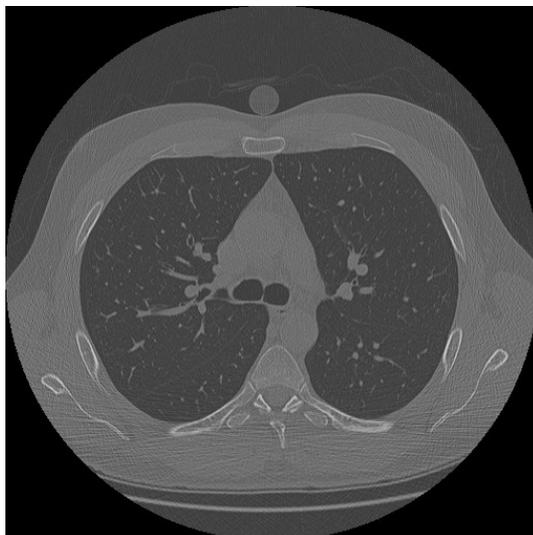
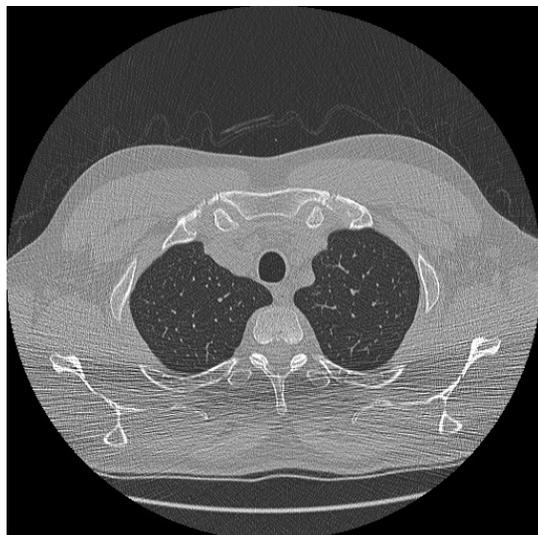
interference structure  
induced by  
movement

# *x-ray computed tomography (CT)*

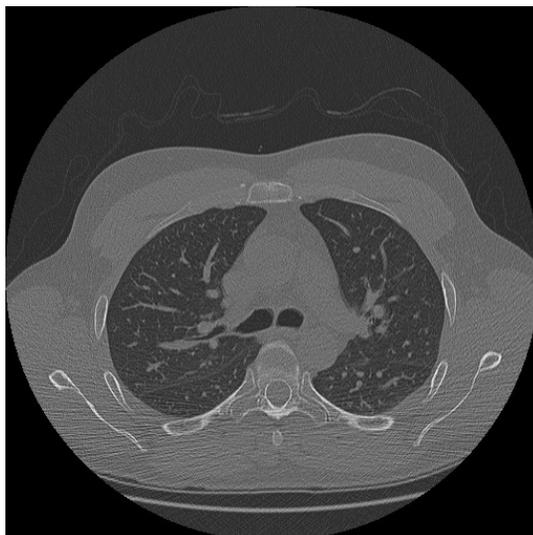
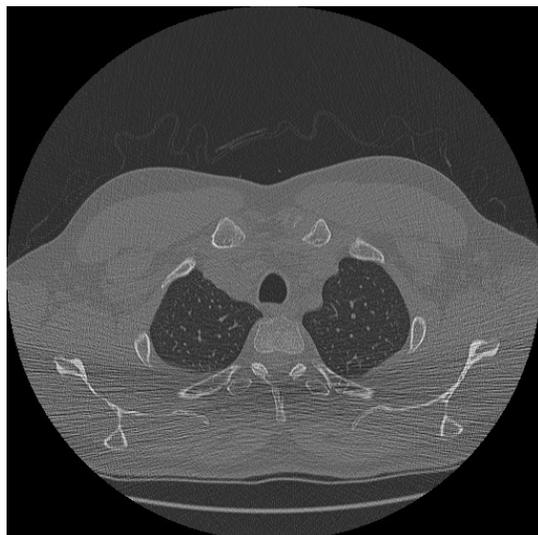
## **artifacts**

## **patient movements**

inhale



exhale



*x-ray computed tomography (CT)*

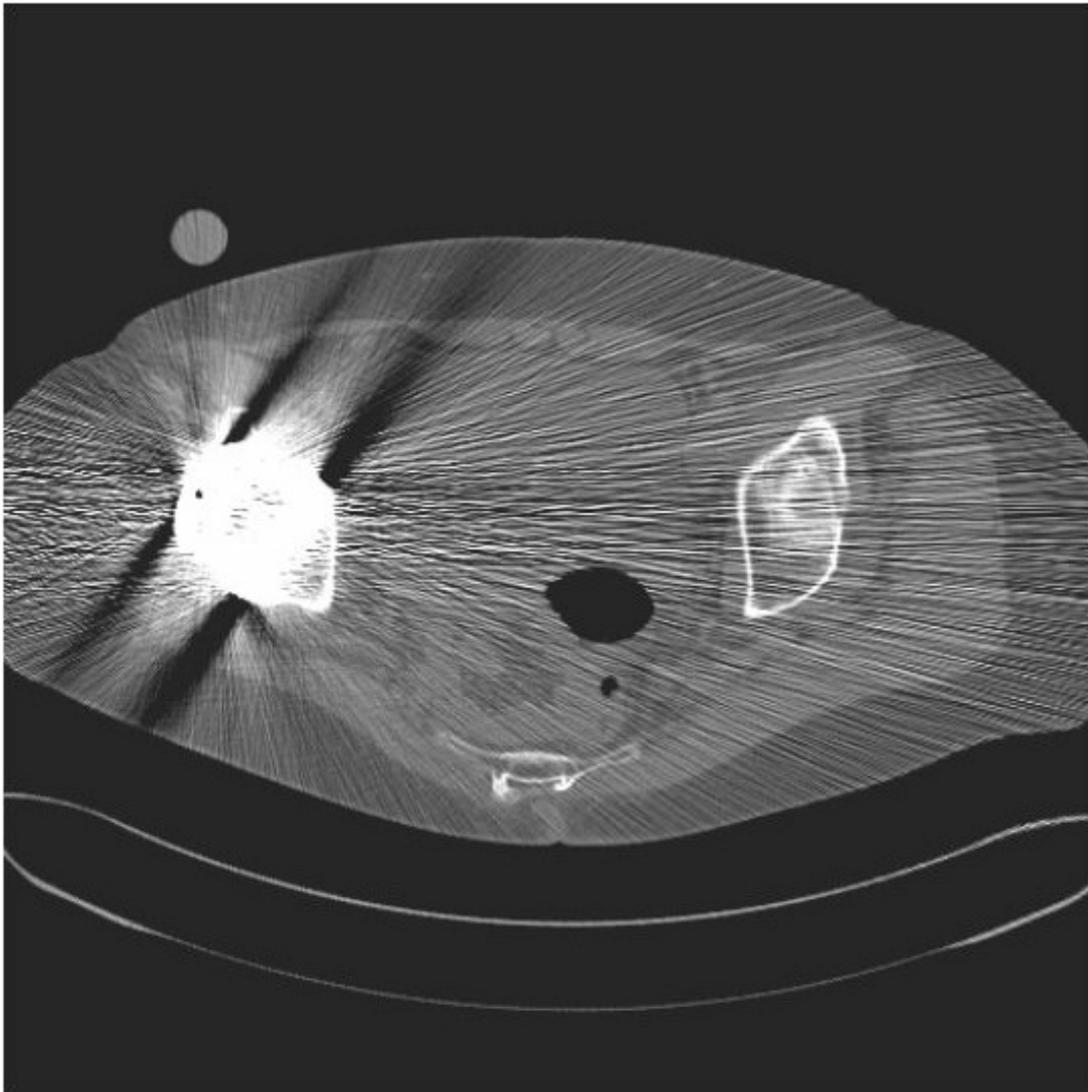
**artifacts**

**failure of recording electronics**



*x-ray computed tomography (CT)*

**artifacts**



**metal implants**

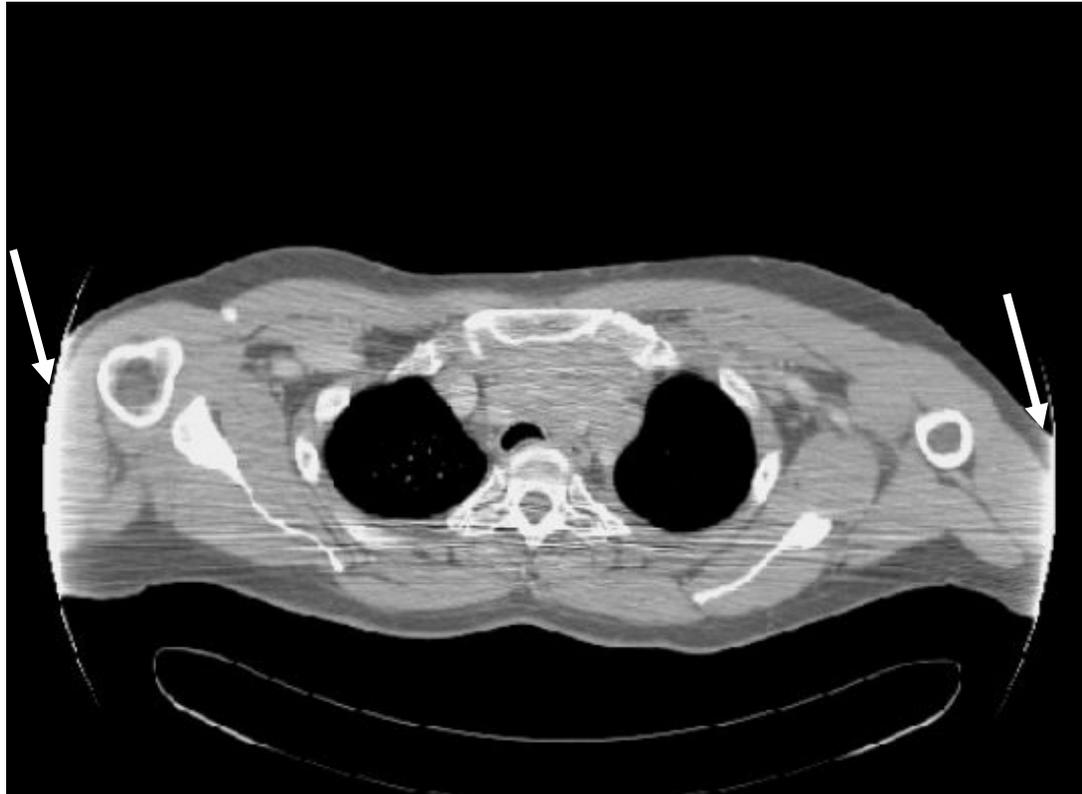


tooth: gold filling

*x-ray computed tomography (CT)*

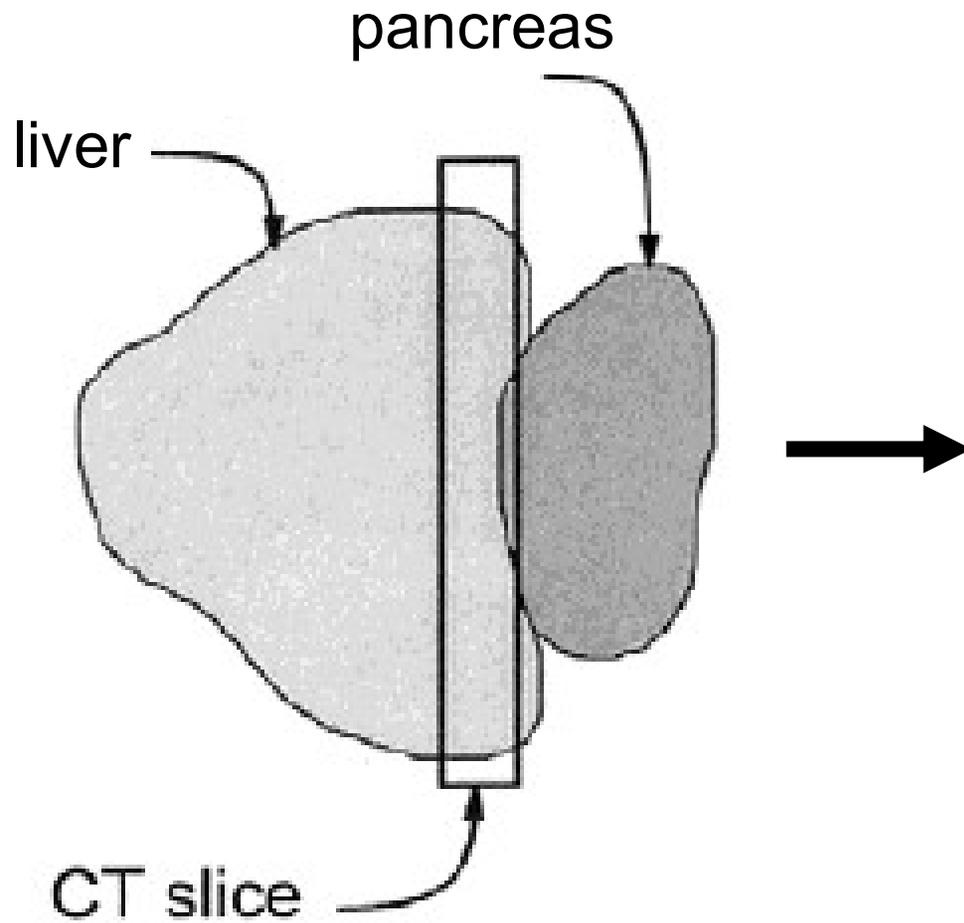
**artifacts**

**violation of limits of field of view**

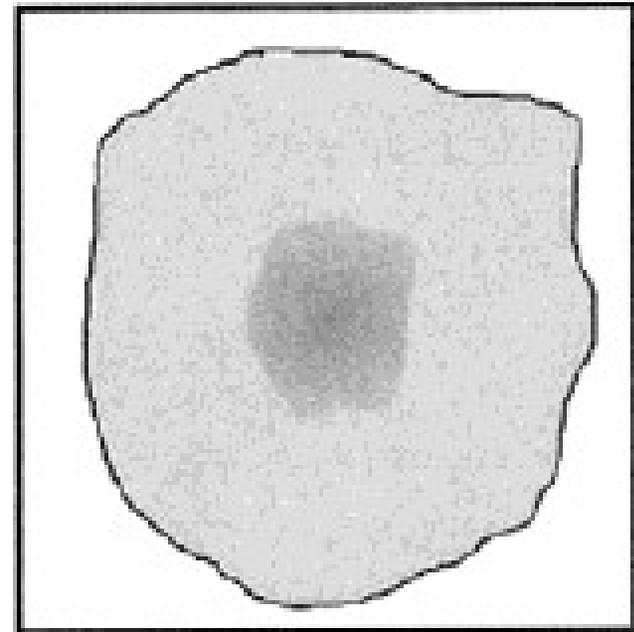


*x-ray computed tomography (CT)*

**artifacts**



**partial volume artifacts**

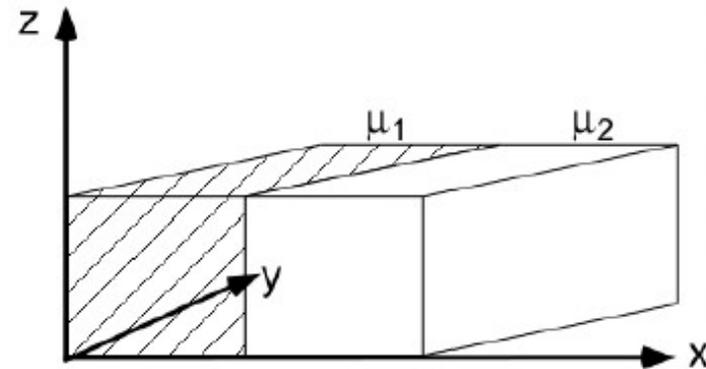
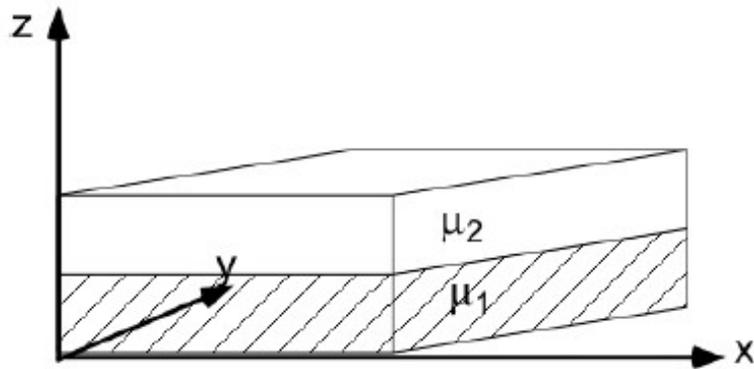


# *x-ray computed tomography (CT)*

## **artifacts**

## **partial volume artifacts**

consider two areas with strongly differing  $\mu$  captured in a single pixel



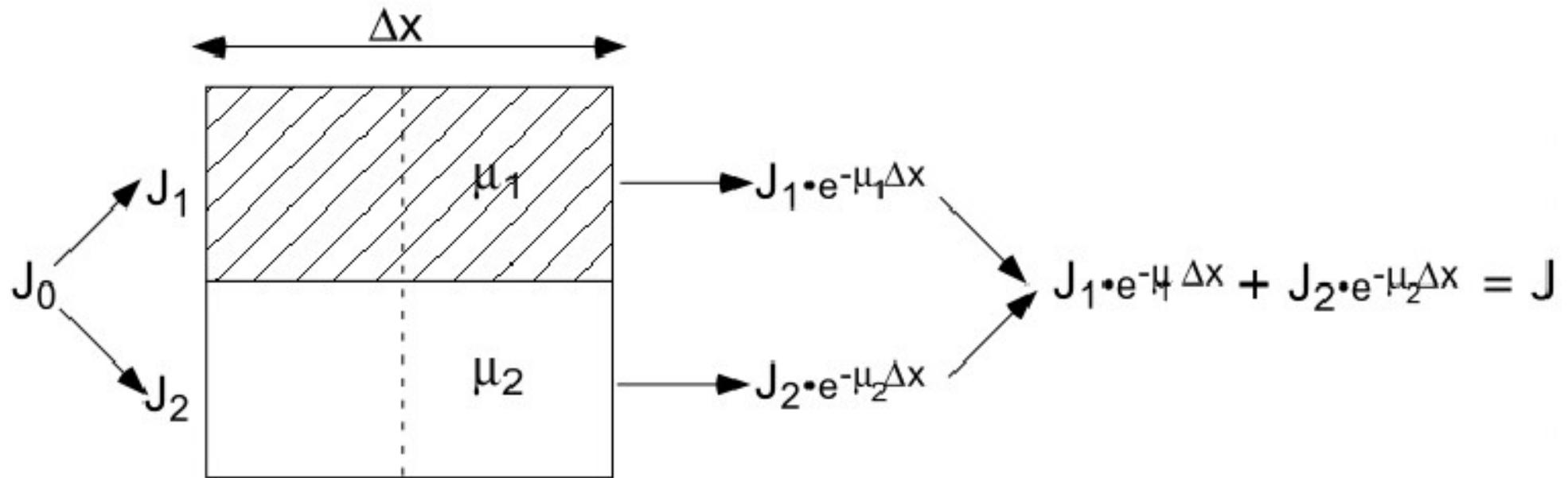
# x-ray computed tomography (CT)

## artifacts

## partial volume artifacts

areas with strongly differing  $\mu$  captured in a single pixel

case A:



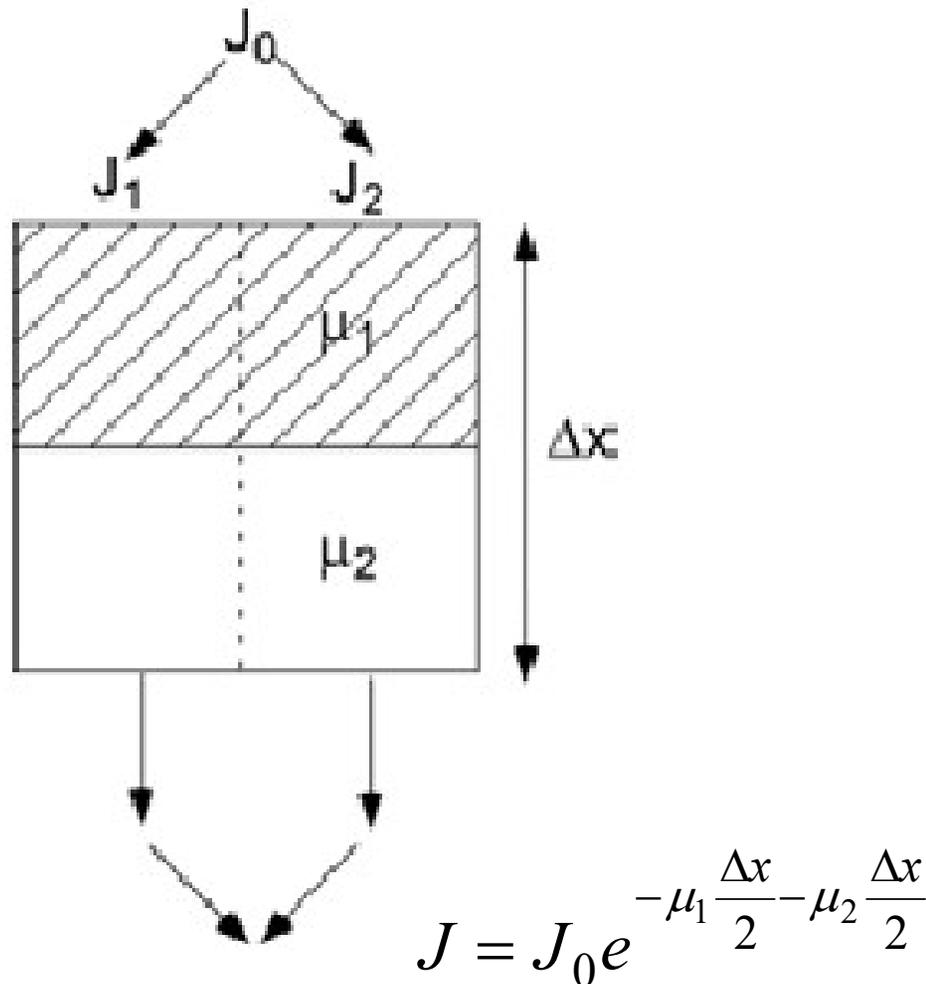
# x-ray computed tomography (CT)

## artifacts

## partial volume artifacts

areas with strongly differing  $\mu$  captured in a single pixel

case B:



## *x-ray computed tomography (CT)*

### **artifacts**

### **partial volume artifacts**

consider x-ray intensity at detector site

$$\text{case A: } J = J_1 e^{-\mu_1 \Delta x} + J_2 e^{-\mu_2 \Delta x}$$

$$\text{case B: } J = J_0 e^{-\mu_1 \frac{\Delta x}{2} - \mu_2 \frac{\Delta x}{2}}$$

in general, we do NOT have:  $\bar{\mu} = \ln \frac{J_0}{J}$

even worse: mean  $\mu$ -values from different projections do NOT match

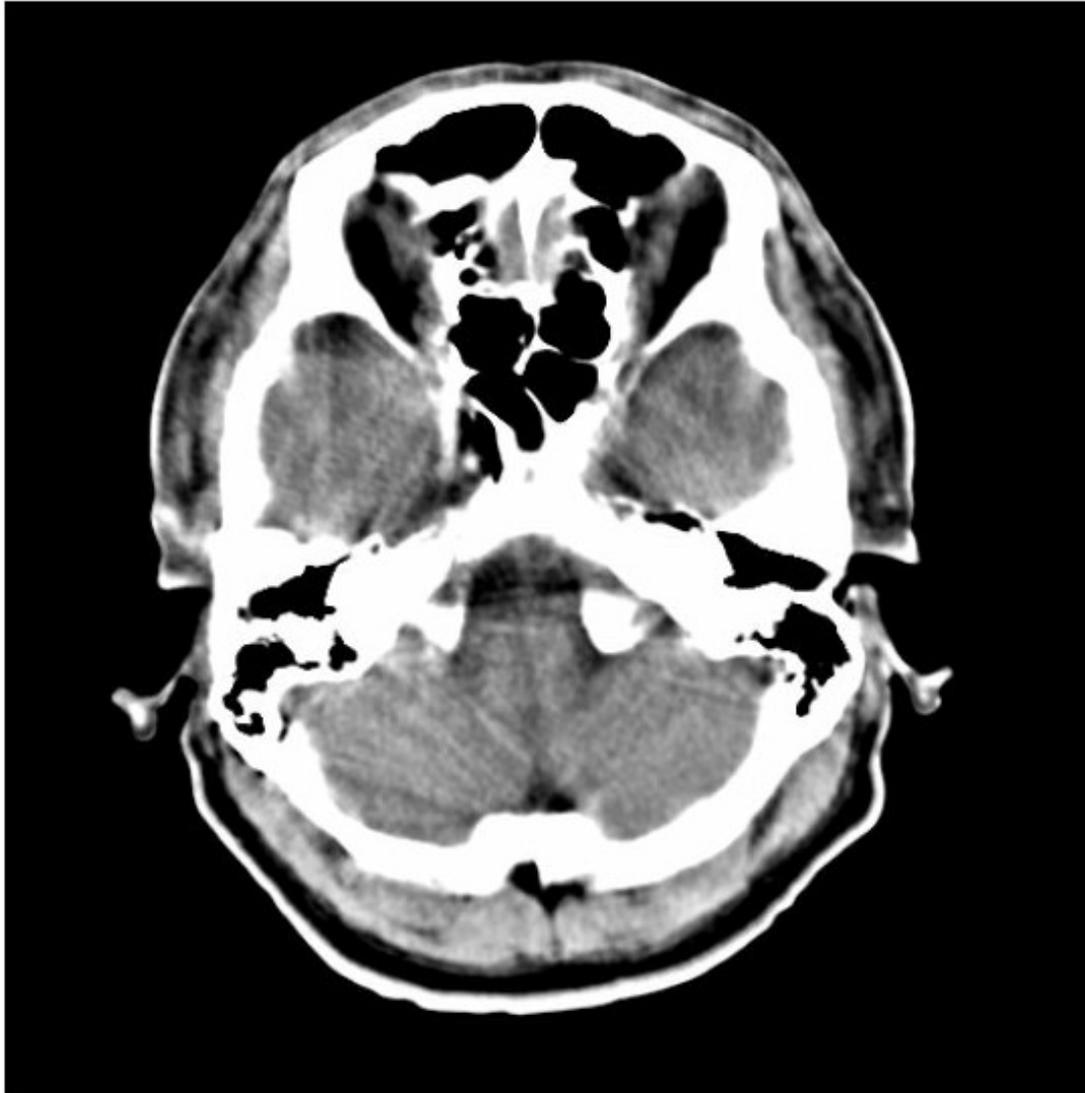
result: banding

prevention: thin slices, higher sampling

*x-ray computed tomography (CT)*

**artifacts**

**partial volume artifacts**



# x-ray computed tomography (CT)

## artifacts

recap:

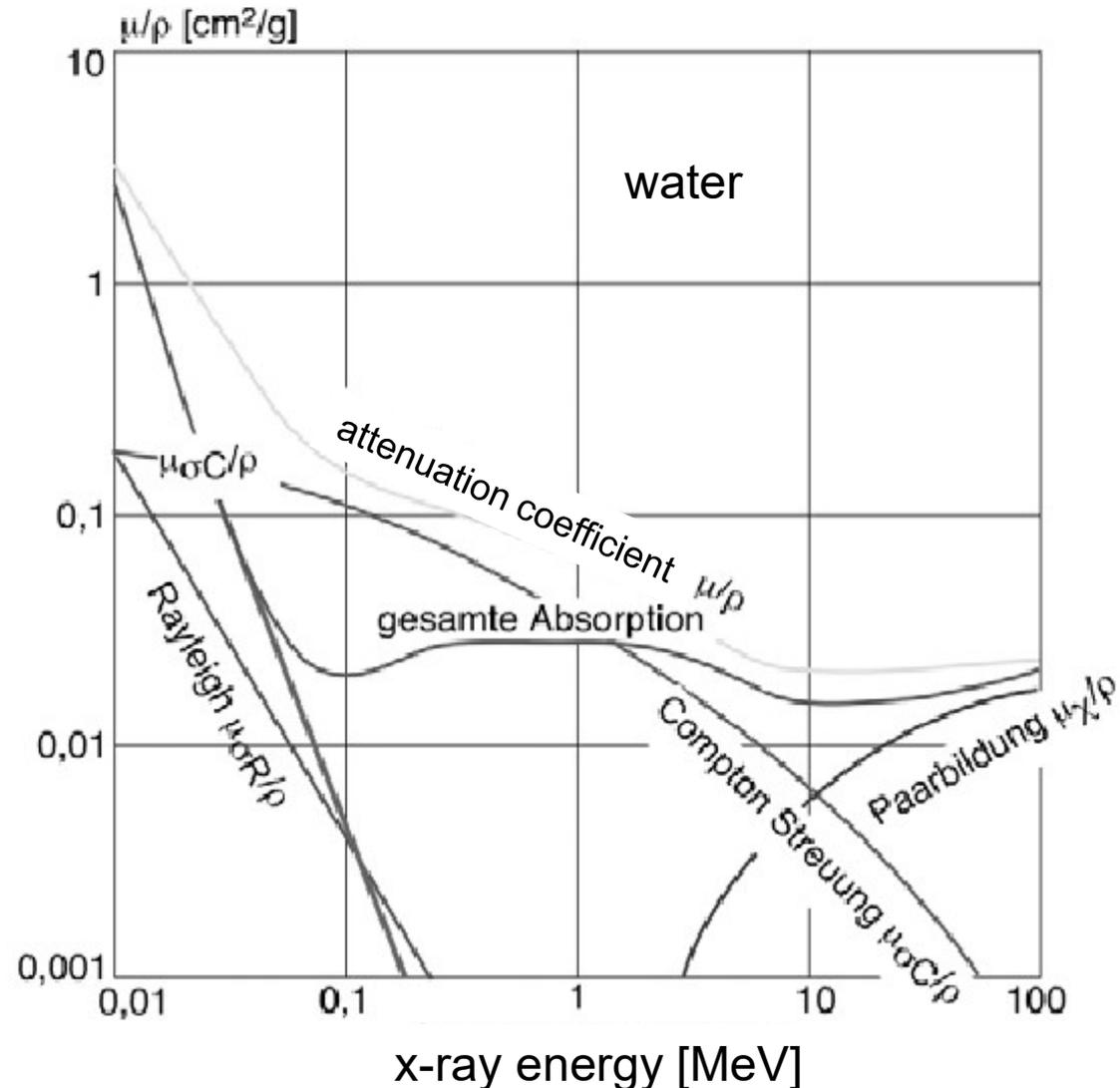
- $\mu$  depends on x-ray energy
- x-ray tube provides broadband energy spectrum

absorption:

- “soft” low-energetic radiation is strongly absorbed
- “hard” high-energetic radiation remains

⇒ **beam hardening**

## beam hardening



# *x-ray computed tomography (CT)*

## **artifacts**

## **beam hardening**

effective radiation power of x-ray tube  
(polychromatic radiation):

$$J_0 = \int_{E_{\min}}^{E_{\max}} \underbrace{\frac{dJ_0(E)}{dE} dE}_{\text{power in energy interval } dE}$$

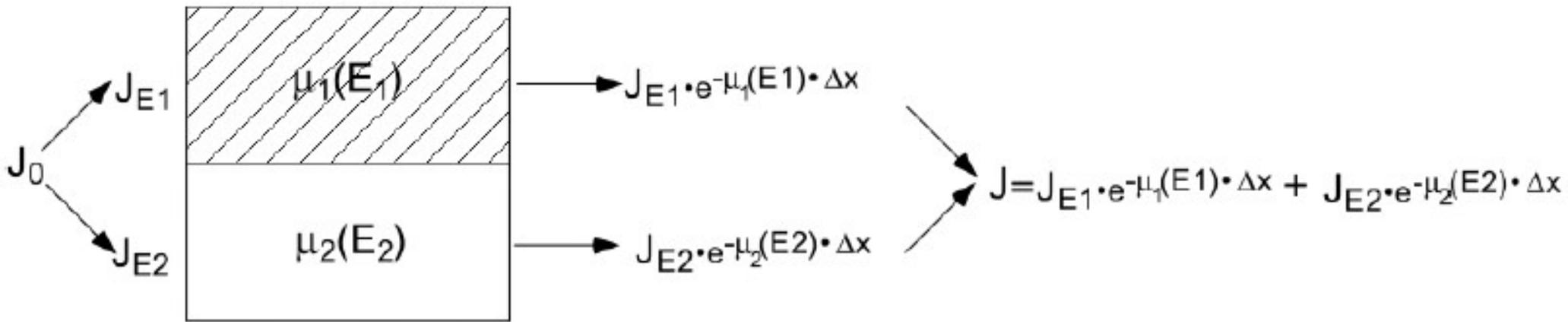
total radiation power passed through the body:

$$J = \int_{E_{\min}}^{E_{\max}} \frac{dJ_0(E)}{dE} \cdot e^{-\int \mu(x,y,E) d\ell} dE$$

# x-ray computed tomography (CT)

## artifacts

## beam hardening



in general, we do NOT have:  $\bar{\mu} = \ln \frac{J_0}{J}$

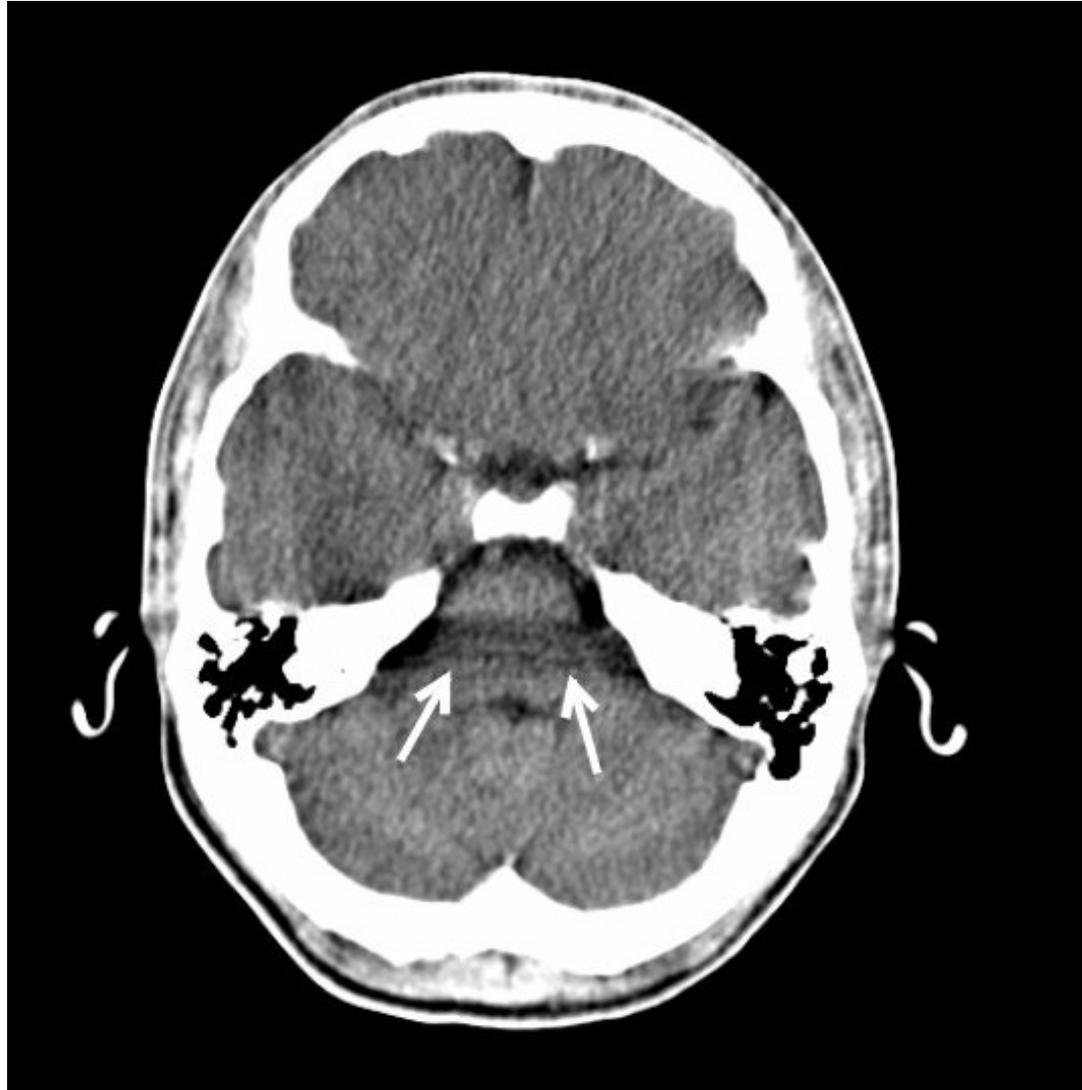
effect: banding (as with partial volume artifacts)

avoidance: - higher-energetic radiation (flat  $\mu(E)$ -dependence)  
- filtering of low-energetic part of energy spectrum (e.g. copper)

*x-ray computed tomography (CT)*

**artifacts**

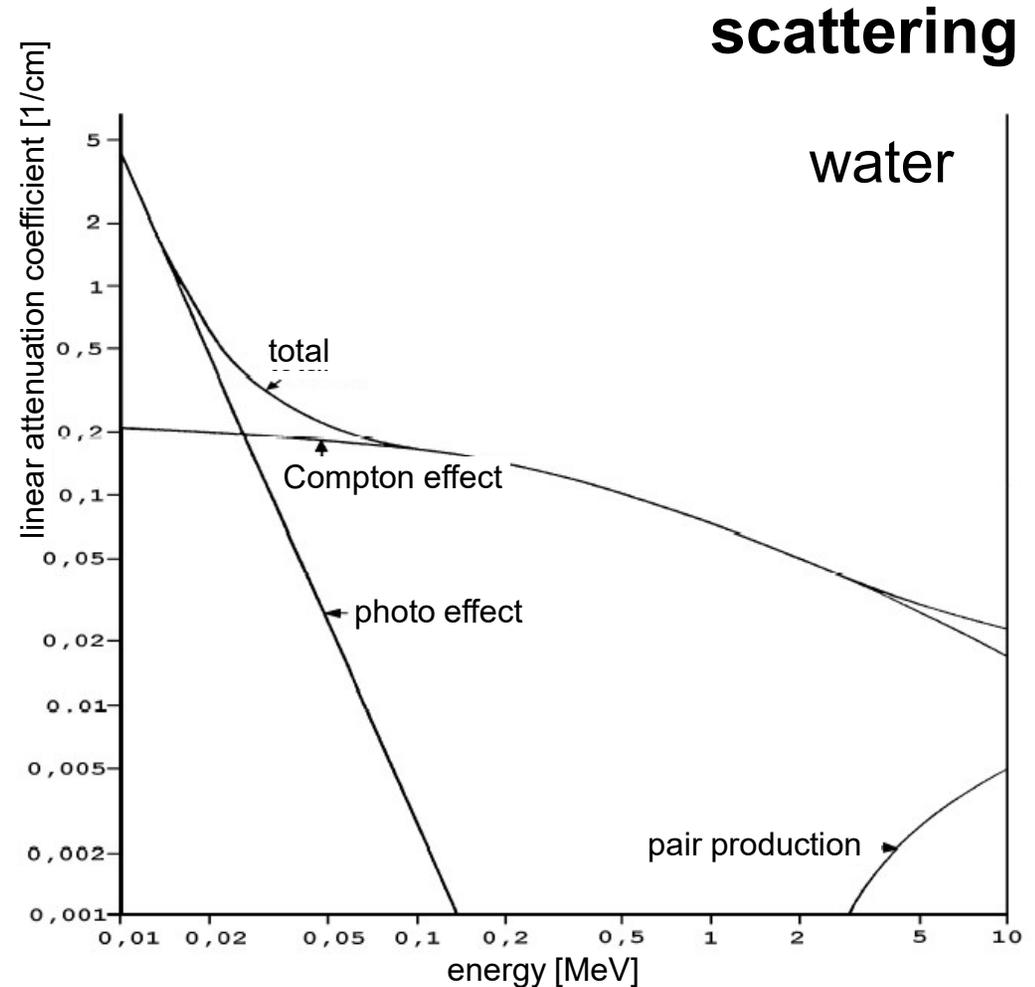
**beam hardening**



# *x-ray computed tomography (CT)*

## artifacts

- Compton scattering leads to an uniform increase of radiation power
- inconsistent data (for reconstruction)
- remedy:
  3. generation scanner:  
raster
  4. generation scanner:  
subtraction using additional detectors

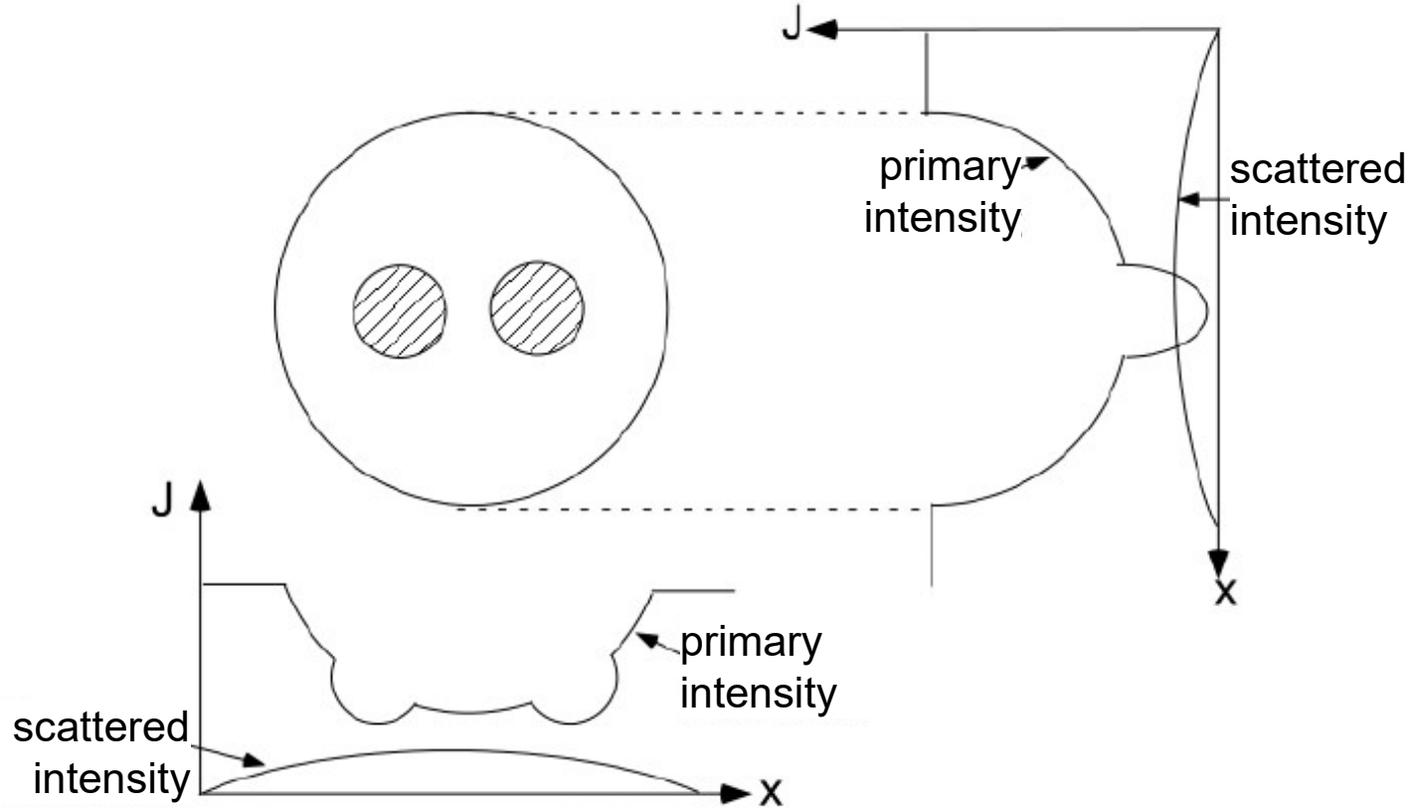


# *x-ray computed tomography (CT)*

## **artifacts**

## **scattering**

scattering can lead to false data for back projection, depending on relative orientation of detector to absorber



# x-ray computed tomography (CT)

## areas of application

Trauma	Unfalldiagnostik im gesamten Körper
Kopf-Hals	Akutes nicht-traumatisches neurologisches Defizit (Blutung, Infarkt)
Spinalkanal	Spinales Trauma
HNO	Mittelohr, Innenohr, Schädelbasis, Trauma der Schädelbasis, cranio-faziales Skelett und Nasennebenhöhlen, Hypopharynx, Larynx, Tumor
Augenheilkunde	Intra-Okulärer Fremdkörper, Ductus nasolacrimalis
Thoraxorgane	Thoraxwand: Tumor, Pleura: Entzündungen, Tumor, Lunge: Lungenstruktur, Pulmonale Embolien, Nekrose, Verkalkungen, Tumorausdehnung und Infiltration, Interstitielle Pneumonie, Bronchiektasen, Hämoptoe, Hämofusion, Lungenmetastasen, Zentrales Tracheobronchiales System, Gefäßmalformation, Sequestration, Mediastinum: Raumforderung (angeboren/entzündlich/Neoplastisch) Lokalisation, Ausdehnung, Ätiologie, Tumor-Staging
Herz-Kreislauf-System	Aorta Thoracalis: Dissektion, Aneurysma
Bewegungsapparat	Knochen: Biopsie, Hüftgelenk: Dissektat, Frakturen, Orthopädische Operationsplanung
Gastroenterologie	Pankreas: Endokrin, akute Pankreatitis, Verdauungstrakt: Tumor-Staging

## *x-ray computed tomography (CT)*

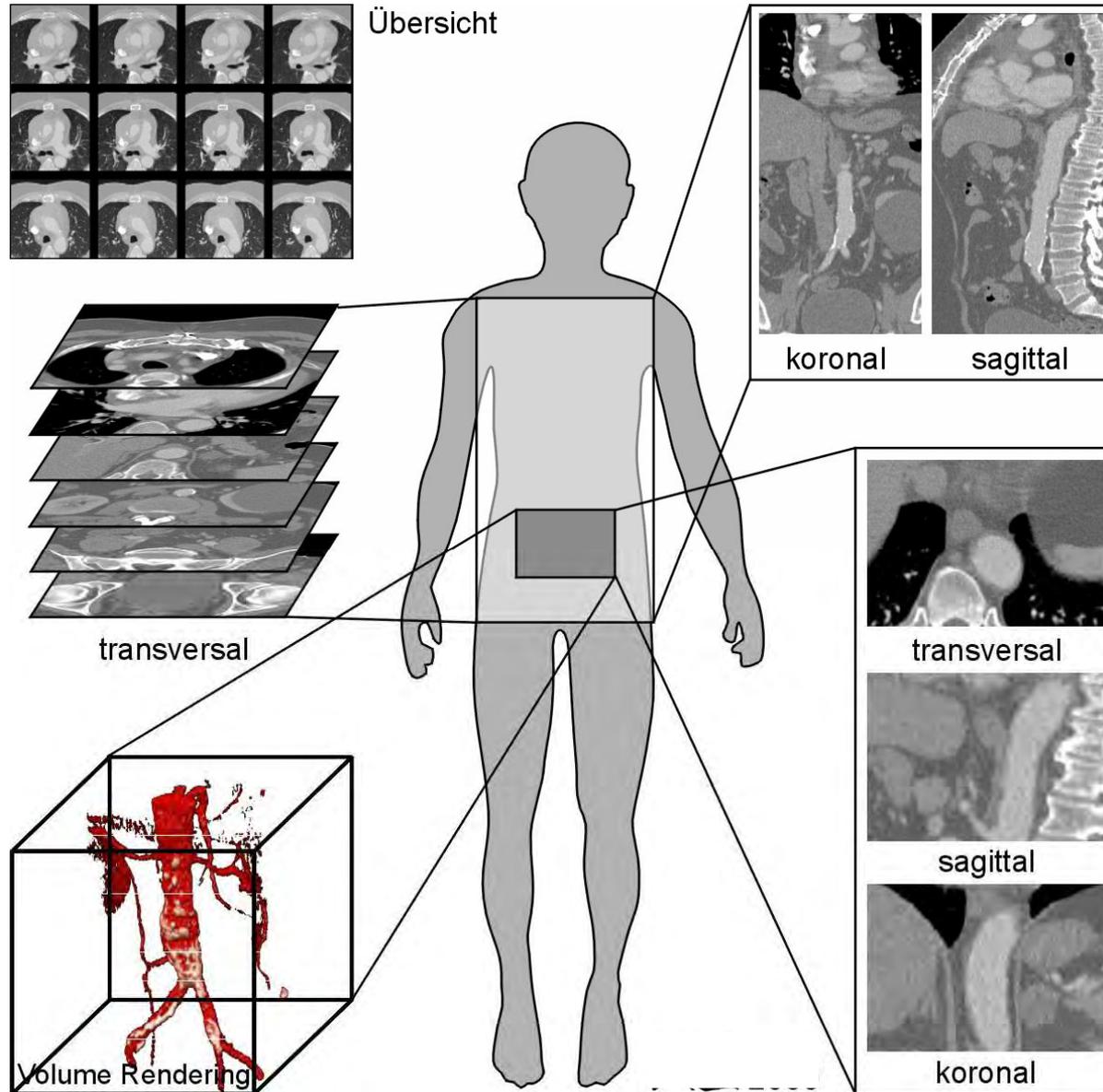
### **special applications:**

- bone density measurement (quantitative CT)  
osteoporosis treatment and monitoring
- lung density measurement (xenon inhalation)
- 3D planning (tumor irradiation / surgery)
- virtual endoscopy („fly through“)
  - colonoscopy (fistula, intestinal polyp, tumor)
  - bronchoscopy (fistula, ruptures)
  - angioscopy (aneurysm, plaque diagnostics)
- tissue perfusion measurement (contrast agents)
- real time CT
- cardio CT (EKG triggered)

# x-ray computed tomography (CT)

## areas of application

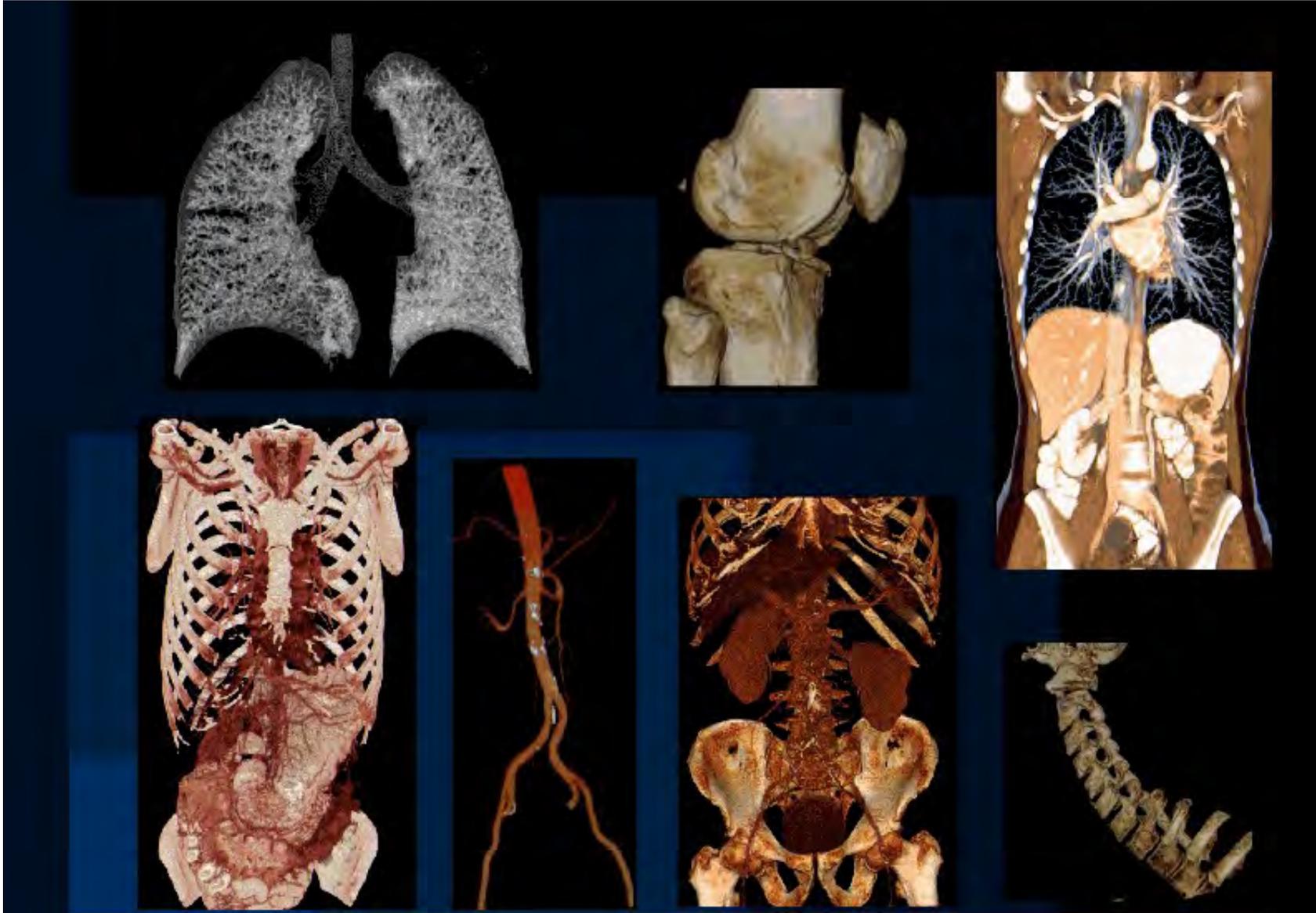
## visualization



*x-ray computed tomography (CT)*

**areas of application**

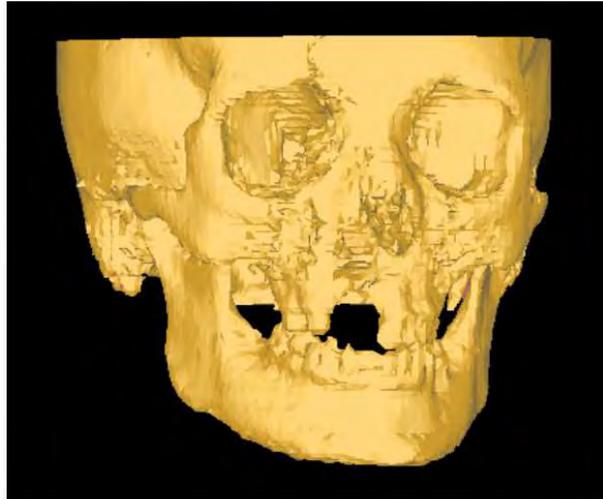
**3D visualization**



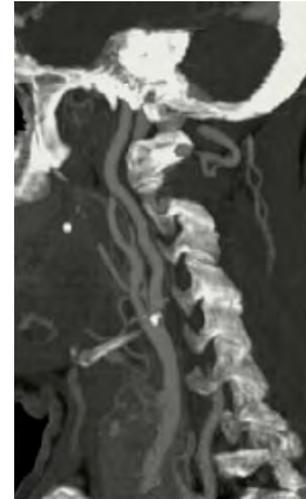
# *x-ray computed tomography (CT)*

## areas of application

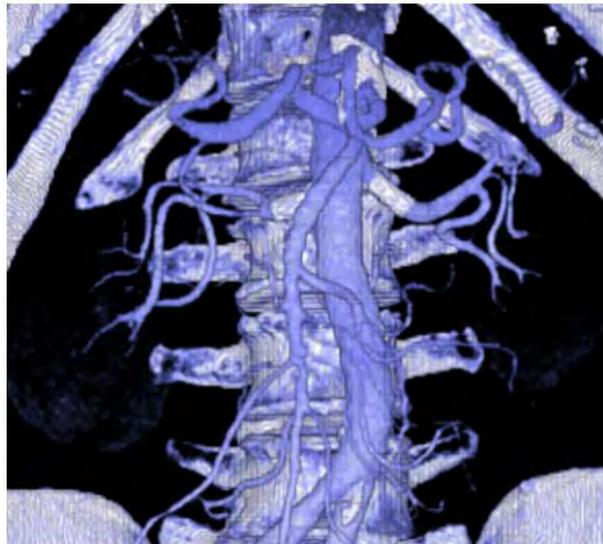
## 3D visualization



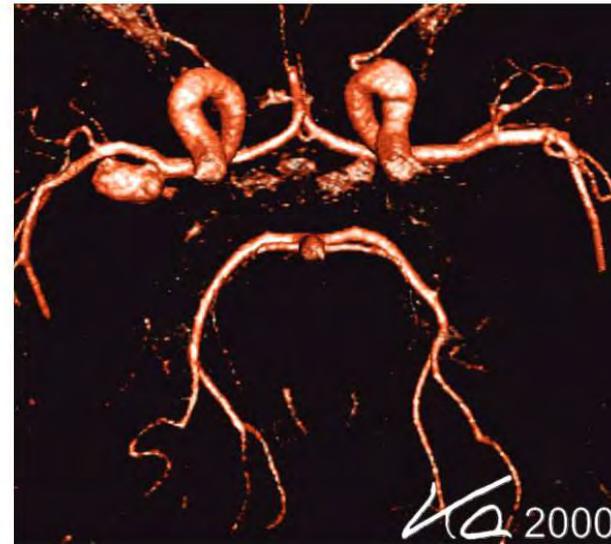
a)



b)



c)



d)

*x-ray computed tomography (CT)*

**areas of application**

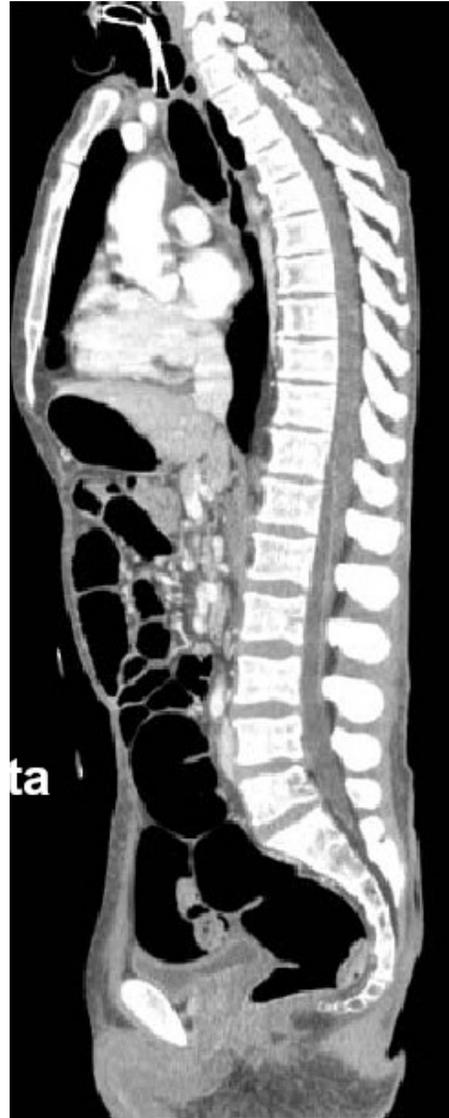
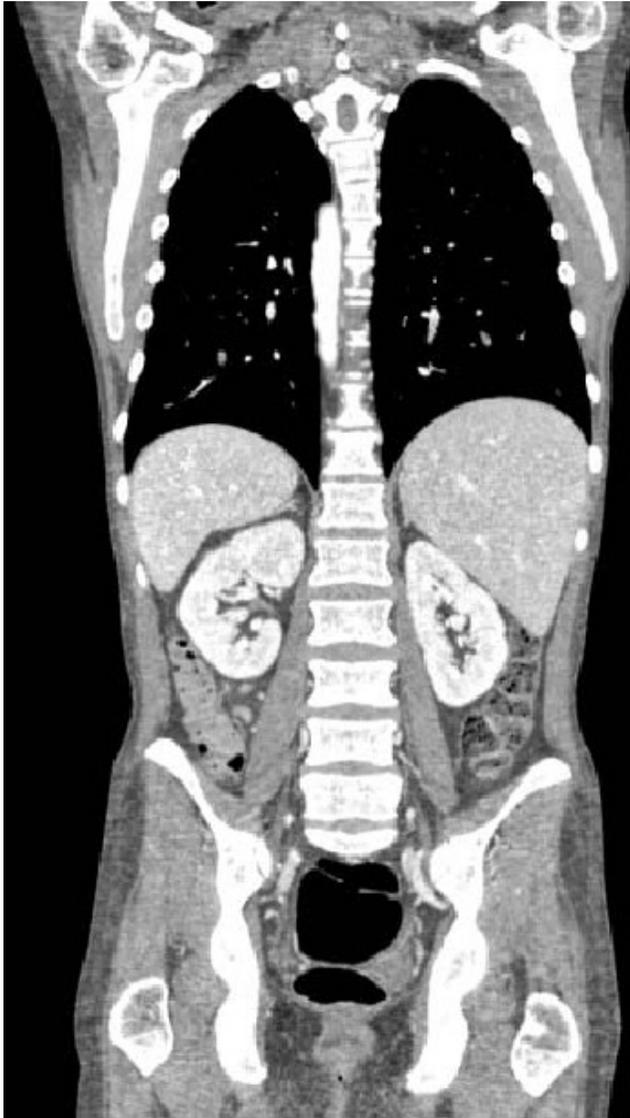
**3D visualization**

By Courtesy of University of Erlangen/Germany



# *x-ray computed tomography (CT)*

## areas of application



## 3D visualization

multi-slice CT:

- 16 detectors
- rotation time: 0.5 s
- resolution: 16 x 0.75 mm
- advance: 25 mm/s
- 70 cm in 28 s
  
- raw data: 1.4 GB
- 1400 images

*x-ray computed tomography (CT)*

**areas of application**

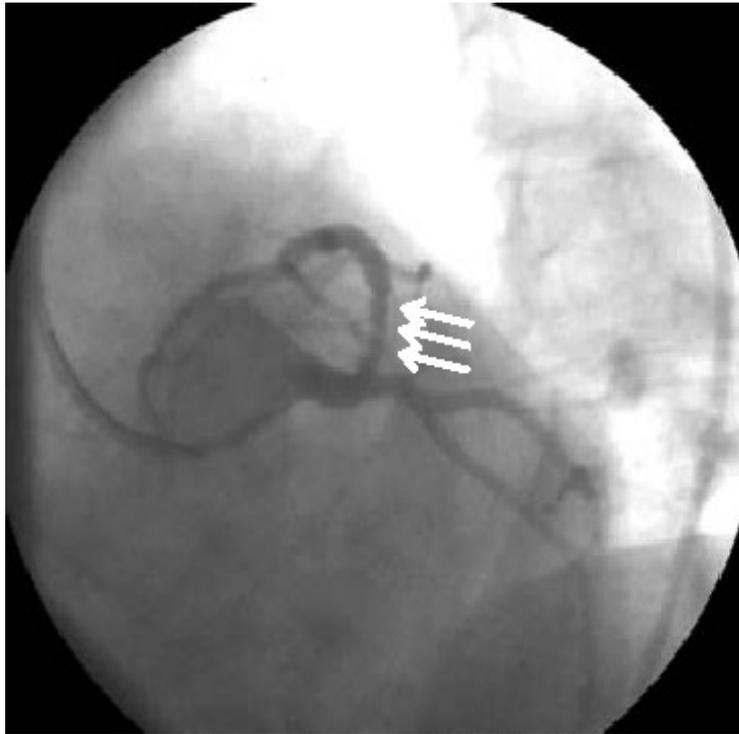
**3D visualization**



*x-ray computed tomography (CT)*

**areas of application**

**angiography**



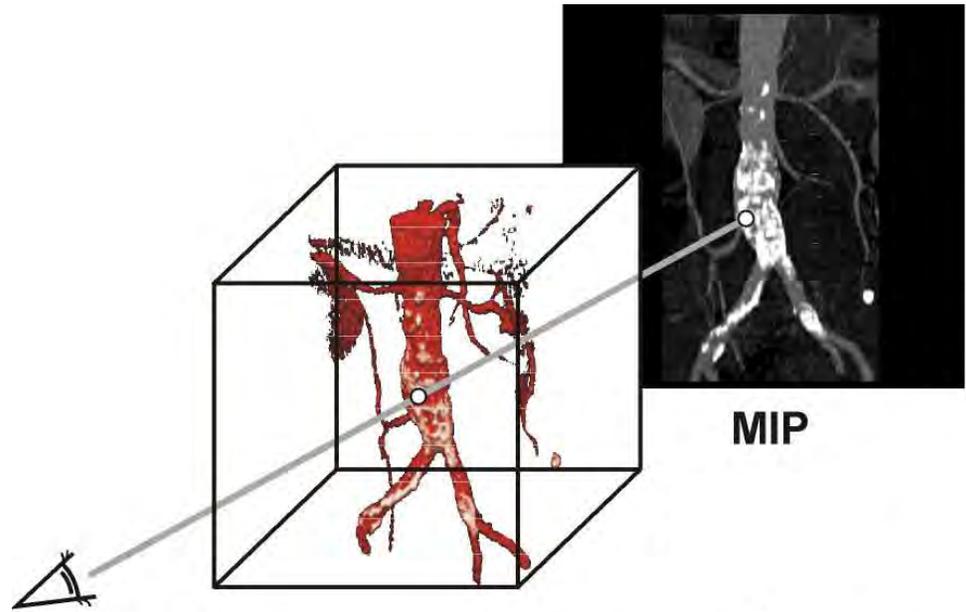
conventional  
angiography



CT angiography

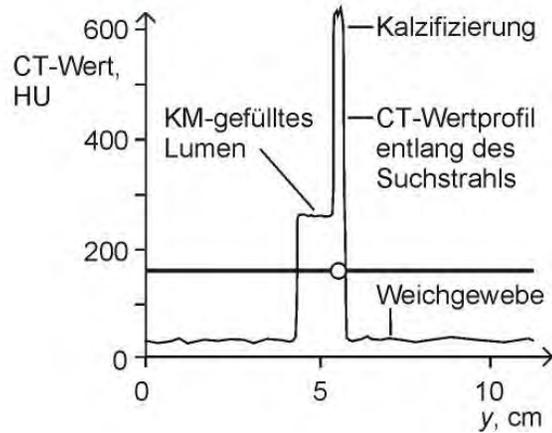
# x-ray computed tomography (CT)

## special applications

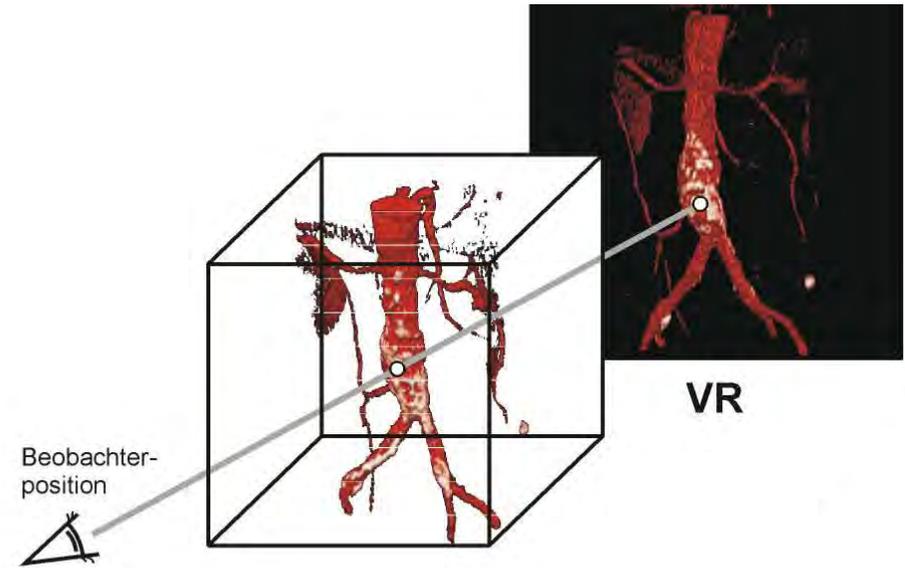


MIP

Beobachterposition

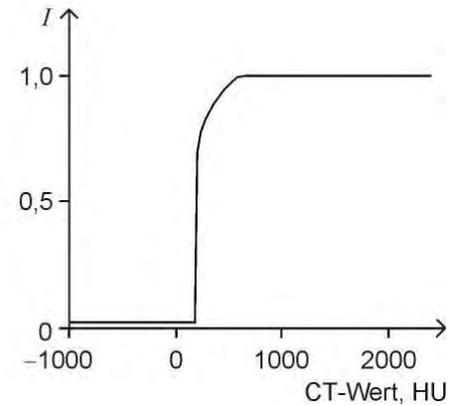
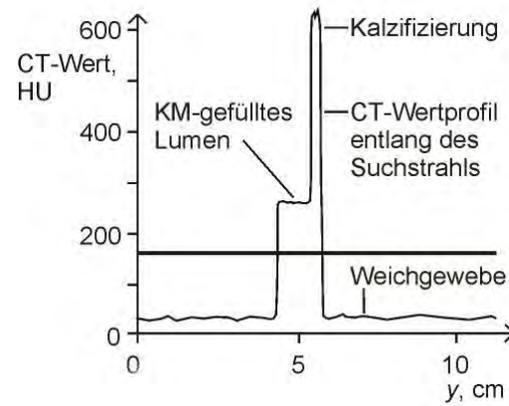


## visualization of volumes



VR

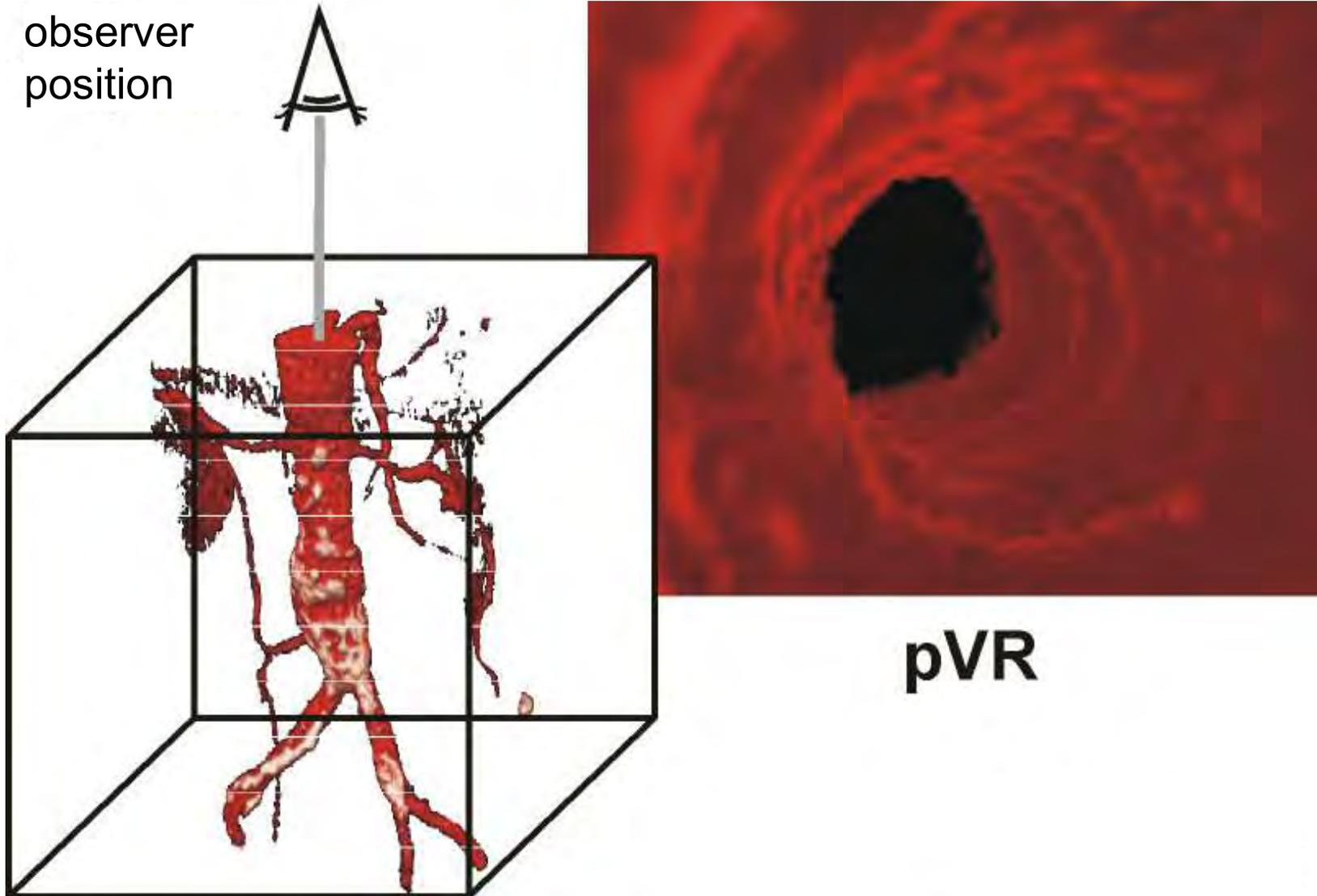
Beobachterposition



*x-ray computed tomography (CT)*

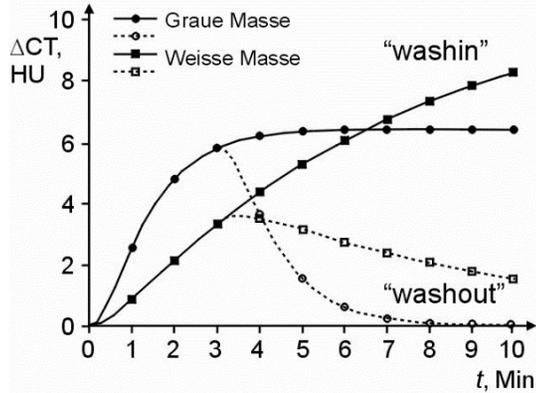
**special applications**

**virtual endoscopy**

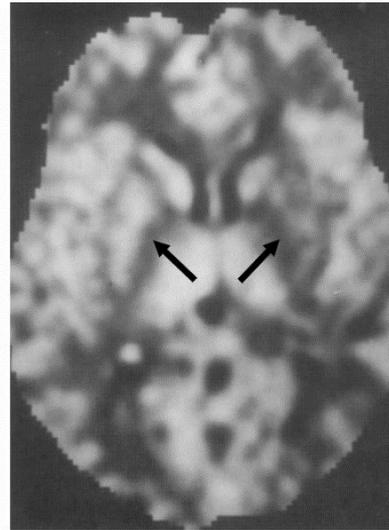


# x-ray computed tomography (CT)

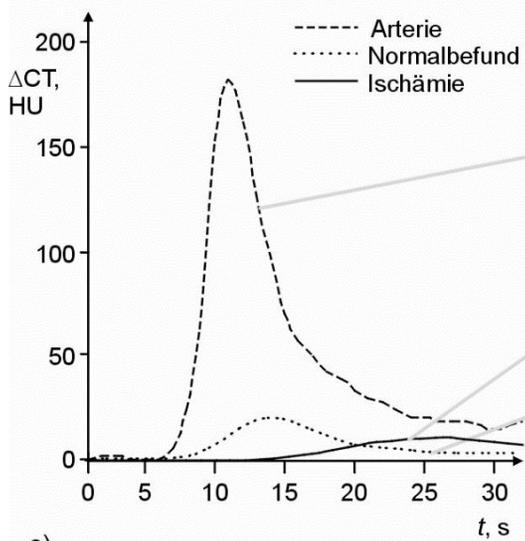
## special applications



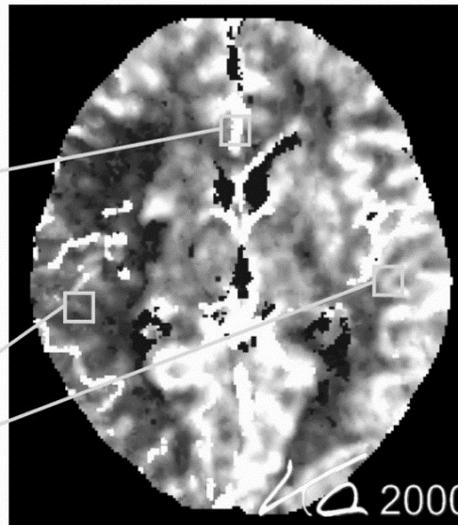
a)



b)



c)



d)

## dynamic CT

recording of  
brain tissue perfusion

with xenon inhalation  
(minute range) (a+b)

with contrast agent  
(second range) (c+d)

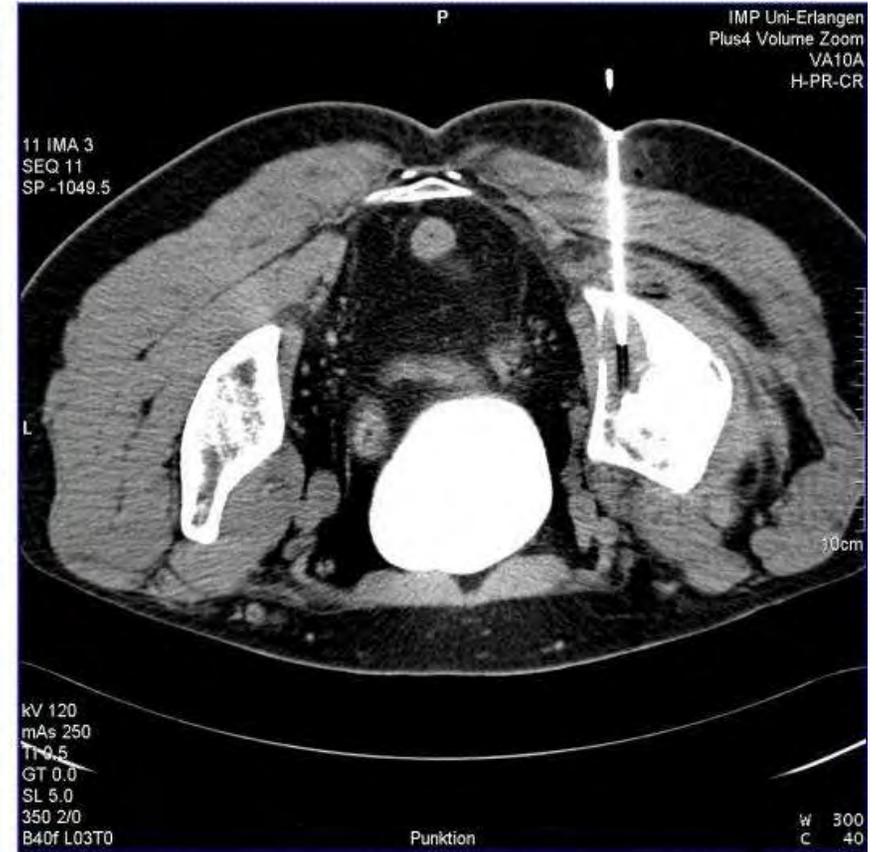
# *x-ray computed tomography (CT)*

## special applications

## CT-controlled biopsy



a)



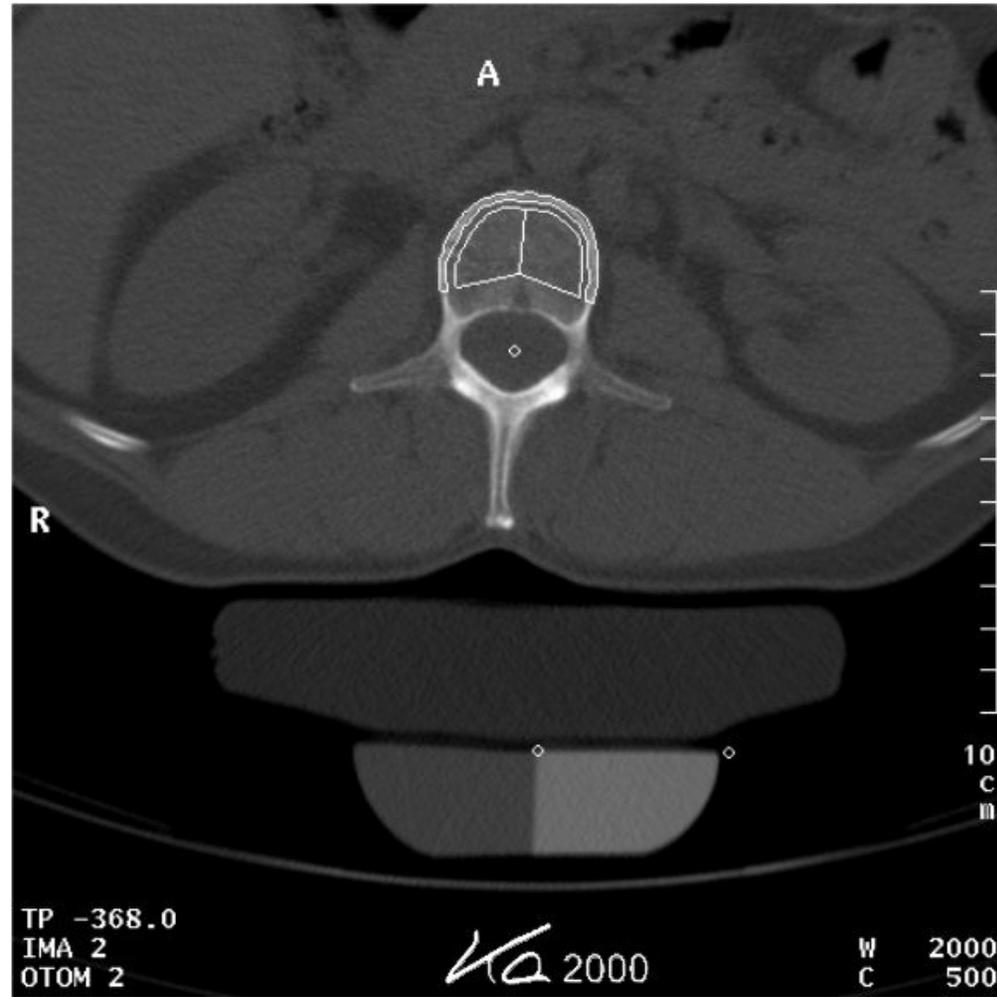
b)

# x-ray computed tomography (CT)

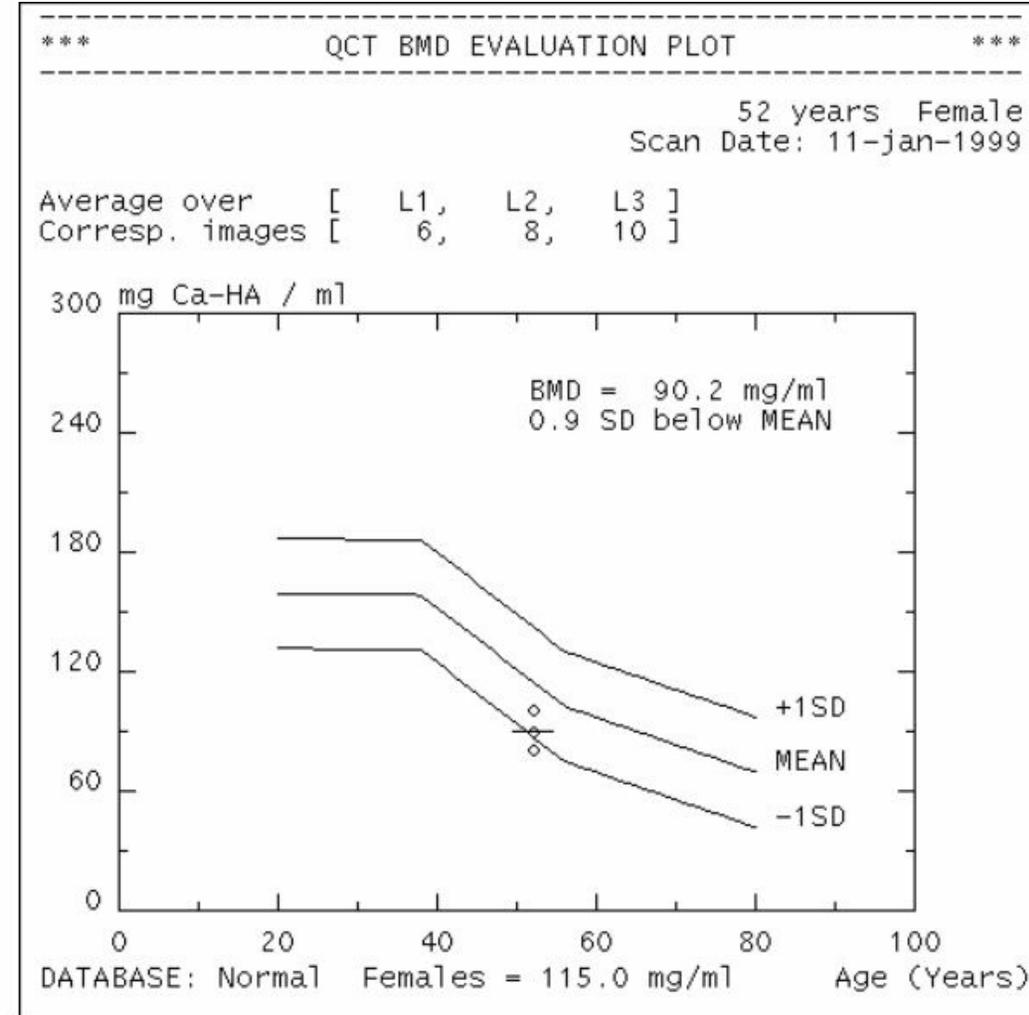
## special applications

bone density measurement (lumbar spine)

## quantitative CT



a)



b)

## *x-ray computed tomography (CT)*

### **comparison projection radiography / CT**

	Röntgen	CT
imaging of bones	+++	+++
soft tissue	-/+	-
vessels	++	++
functions	-	-
volumes	-	++
real time	fluoroscopy only	+
image quality	excellent	good
psychic burden	low	medium
somatic burden	high	high
examination time [min]	10	25
costs/examination [€]	ca. 40	ca.100