Nonlinear Noise Reduction

Reducing Noise in Phase-Space

Brief Recap: Influence of Noise

- validity of embedding theorems
- invariance of characterizing measures
- reduced length scales

  (Lyapunov-exponents: 10^{-4} is too much)
- break-down of self-similarity

  (dimensions: 2% noise is too much)
- limited performance of prediction algorithms

Brief Recap: Influence of Classical Filtering

- do not use classical filter for chaotic signals!
  chaotic signal typically broad-band; filtering can destroy chaotic motion; can lead to over-estimation of dimension

- classical filtering of noise can induce structure in phase space
  spatial (long-ranged) correlations that can not be minimized with Theiler's correction scheme

- filtered noise can mimic low-dimensional nonlinear structure

what is noise? lament of a physicist

Rauschen, ein Problem - ungern gesehen.
Niemand kann etwas dabei verstehen.

Rauschen, jeder Fluß und jeder Ozean diese Töne spielen kann,
In der Technik jedoch nervt es den fleiß'gen Mann.

Rauschen, es kommt als weißes, Schrot und Dunkel,
Es gibt zudem noch Pattern, Signal und auch Funkel.

Rauschen, abhängig nicht allein von Temperatur,
Von Signal und auch von Einstreuungen elektrischer Natur.

Rauschen, auch im Alltag plagt es manchen,
Erzeugt durch Kinder-Wasserplanschen.

Rauschen, und am Tage auf Arbeit betroffen die Signale,
Eine Qual und das alle Male.

Rauschen, im Wald von Blättern und von Ästen widerhallt;
Der Wind Dir diese Melodien malt.

Rauschen, Dein Erscheinungsbild hat manche Formen,
Kunterbunt, gehorcht nur selten Normen.

Rauschen, es läßt sich filtern, auch glätten,
Schön wär's, wenn im Griff wir's hätten.

Rauschen, periodisch, auch statistisch oder beides gemischt,
Analysen nach Fourier zeigen deren Gesicht.

Rauschen, was für Sorgen,
Die Klospülung und die Dusche des Nachbarn jeden Morgen.

Rauschen, warum der Herrgott solch erfand,
Nein, es war halt da - von Anfang an.

Rauschen by Jürgen Kohrell

what is noise? some definitions

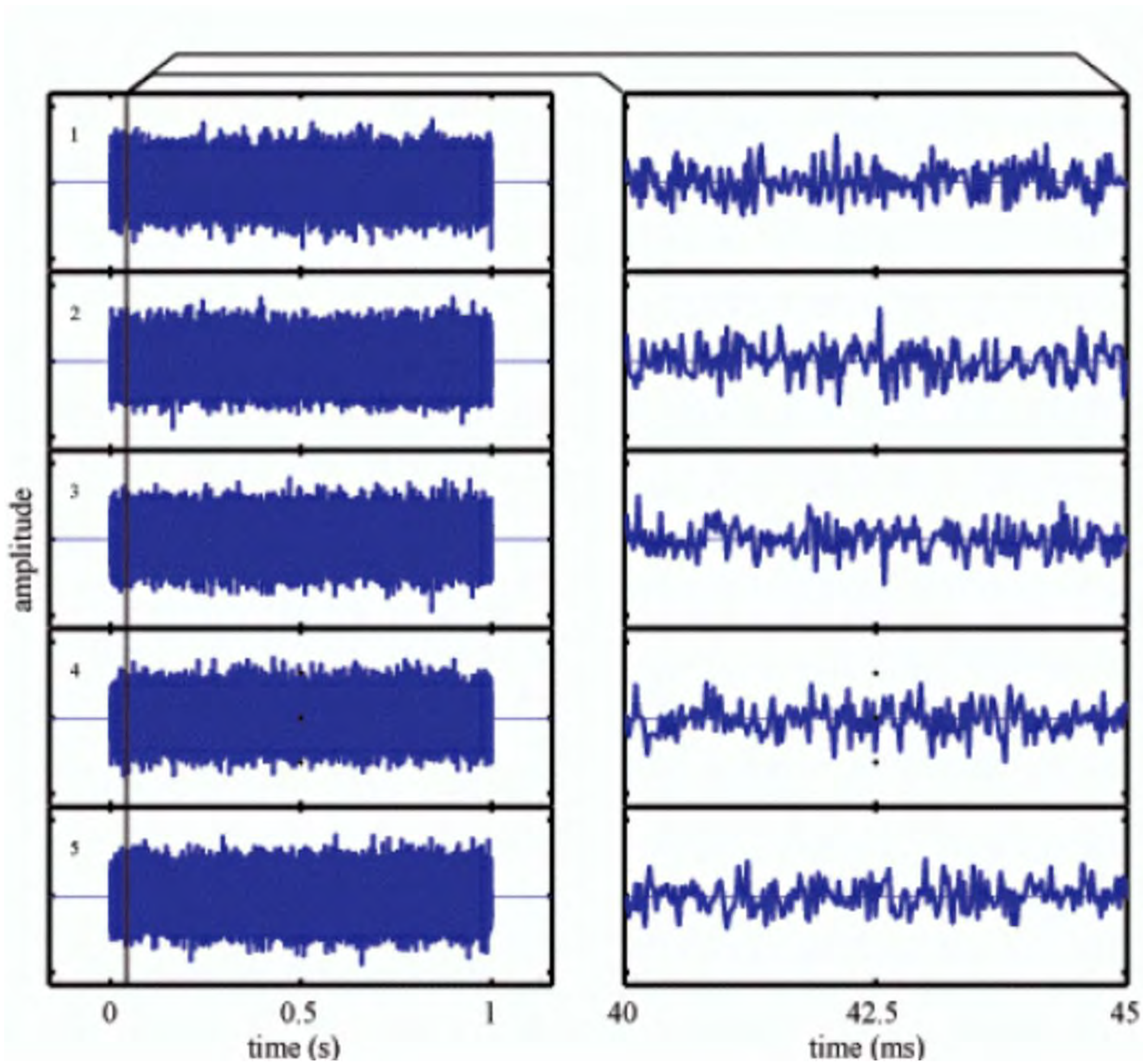
- an undesired *disturbance* within the frequency band of interest; the summation of unwanted or disturbing energy introduced into a communications system from man-made and natural sources
- a disturbance that affects a signal and that may *distort* the information carried by the signal
- *random* variations of one or more characteristics of any entity such as voltage, current, or data
- a *random* signal of known statistical properties of amplitude, distribution, and spectral density
- loosely, any *disturbance* tending to *interfere* with the normal operation of a device or system

what is noise? some definitions

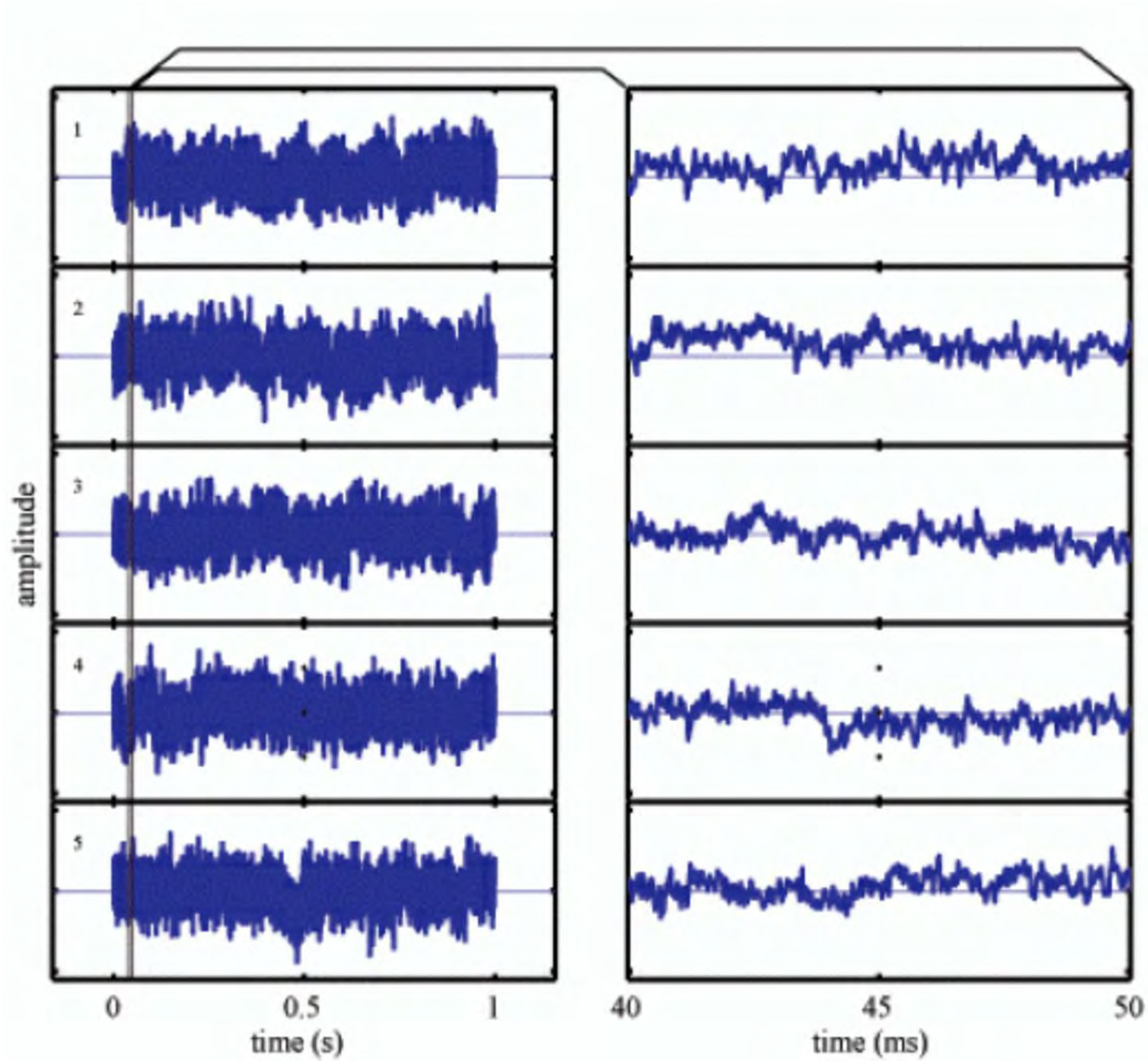
if a fluctuating voltage is amplified by a low-frequency amplifier and fed into a speaker it produces a hissing sound

physical noise is produced by stochastic processes and can be modeled mathematically as random variables

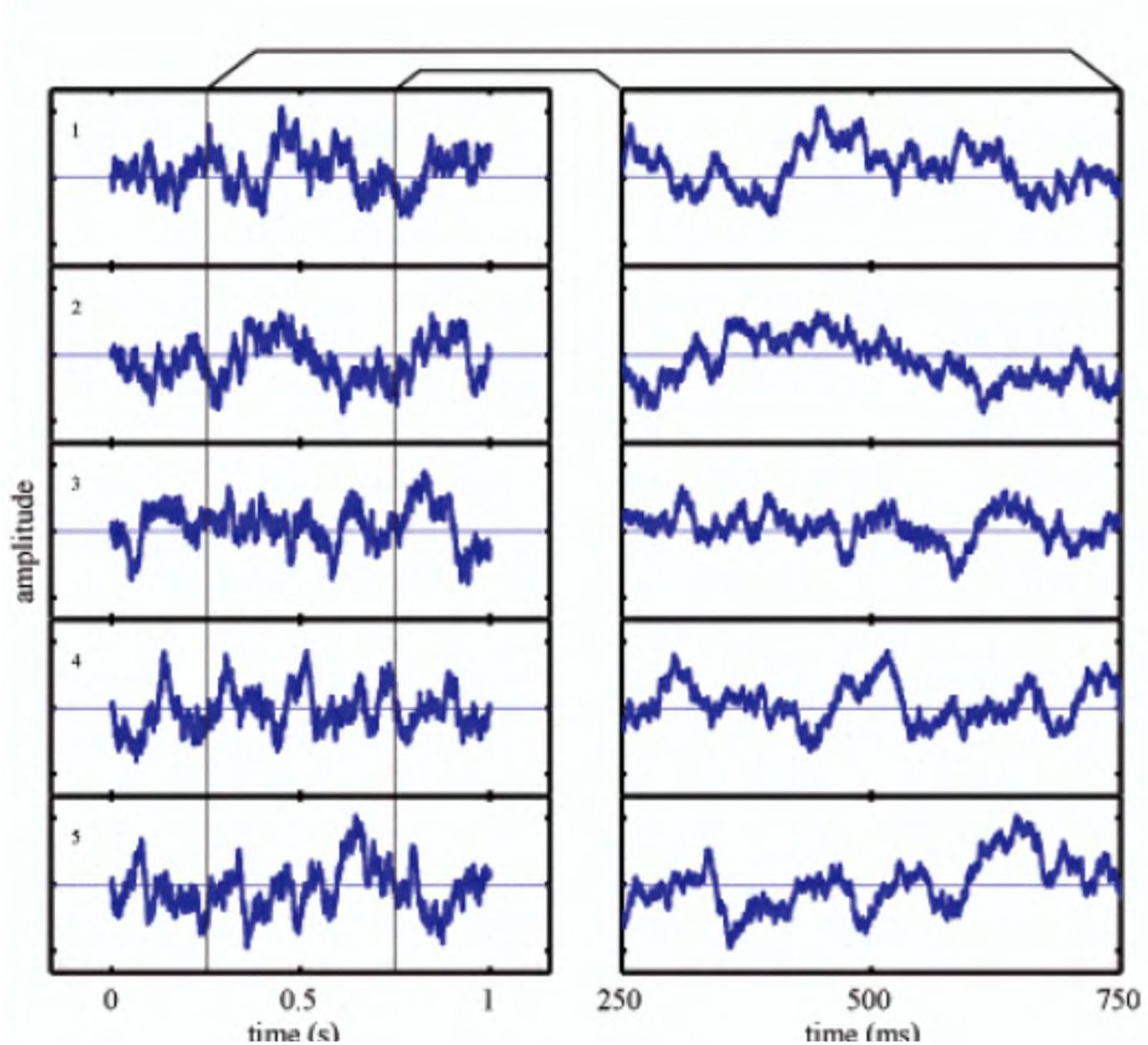
noise **some examples**



noise **some examples**



noise **some examples**



types of noise

shot noise

thermal noise

flicker noise

burst noise

avalanche noise

quantum 1/*f* noise

other?

types of noise — shot noise

short for Schottky noise (Walter Schottky, 1918)

origin: it occurs whenever a phenomenon can be considered as a series of independent events occurring at random

non-equilibrium process associated with current flow through a conductor (much more pronounced in semiconductors)

spectrally flat, uniform power density, Gaussian amplitude distribution

analogy:
stress in an earthquake fault that is suddenly released as an earthquake

types of noise **thermal noise**

also referred to as Brownian or Nyquist or Johnson noise
(R. Brown, 1827; Nyquist /Johnson, 1928)

origin: random motion of particles due to ambient heat energy
(e.g. carriers in any conductor)

equilibrium process (does not (!) require current flow)
- temperature dependent: noise energy ~ kT
- the higher the temperature the more noise
- thermal noise stops at 0 K

spectrally flat, uniform power density, Gaussian amplitude distribution

types of noise — flicker noise

also referred to as 1/*f* or excess or low-frequency noise

first seen in tubes (flickering of filament glow)

origin: *unknown* !
one of the oldest unsolved problems in physics !

widespread in nature (more examples later on)

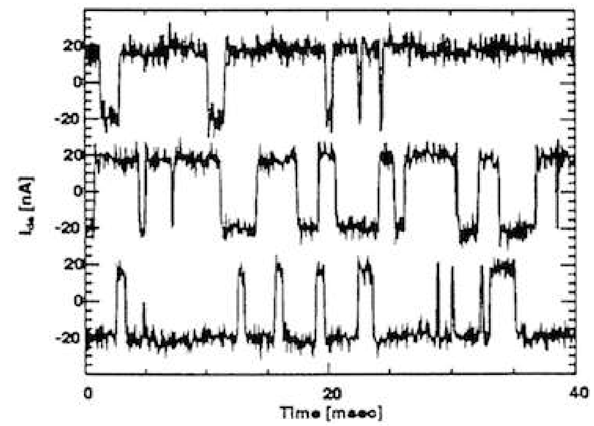
power increases as frequency decreases (P ~$1/f$)

same power content in each octave (or decade)

types of noise **burst noise**

also referred to as *popcorn* noise or Random Telegraph Noise

origin: exact mechanism not fully understood



is often related to imperfections in materials but is also seen in e.g. astrophysics (activity bursts of super-novae)

discrete high frequency pulses

low frequency noise which varies as 1 / f^2 at higher frequencies

types of noise avalanche noise

mainly seen semiconductors

origin: avalanche breakdown in *pn*-junctions (Zener effect)
(multiplicative process resulting in a random series of noise spikes

spectrally flat, uniform power density

types of noise **quantum noise**

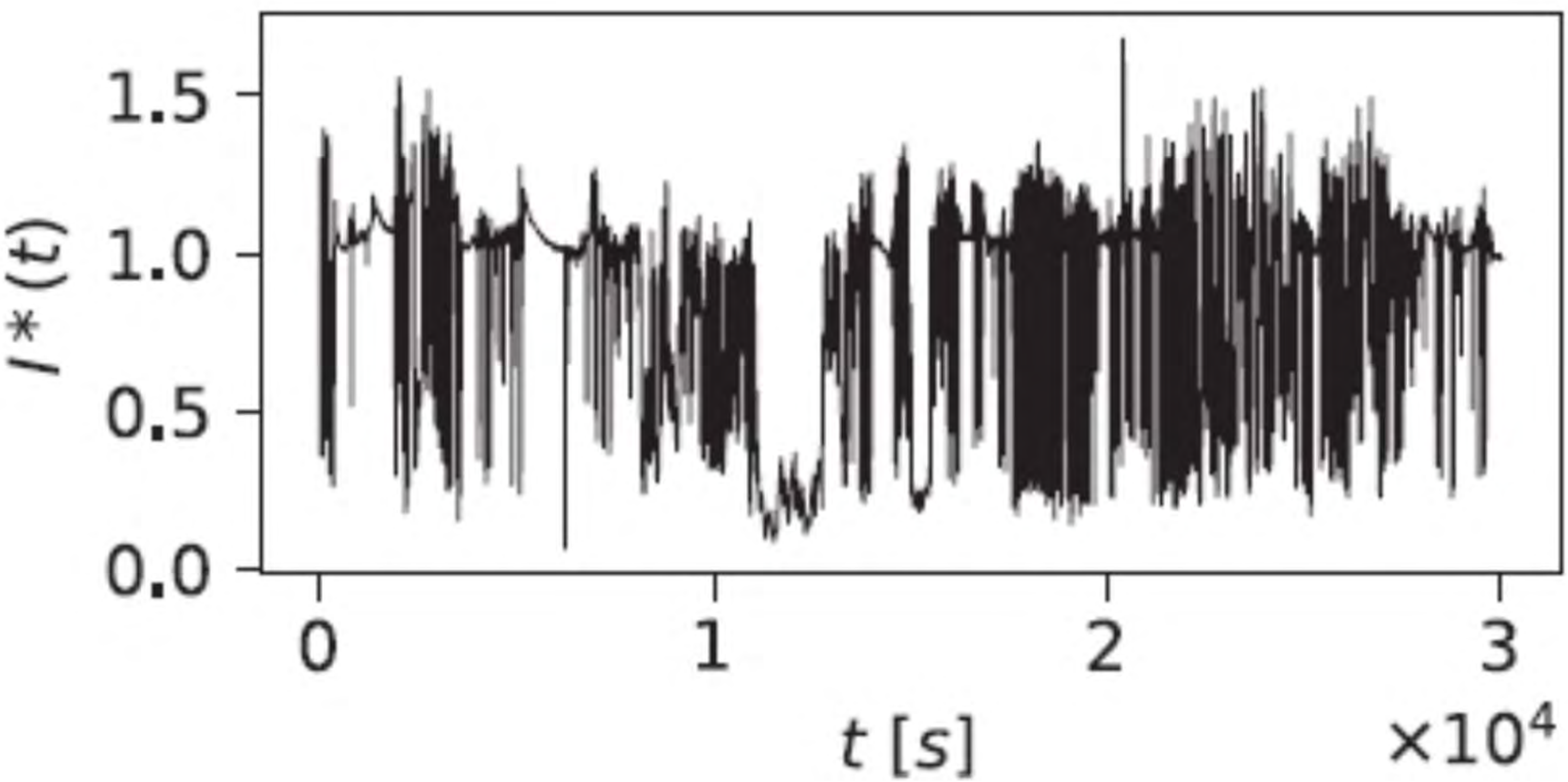
frontier of noise research

origin: largely unknown
(very small amplitudes, usually masked by other 1/f noise sources)

has been observed in pentodes

types of noise

what is this?

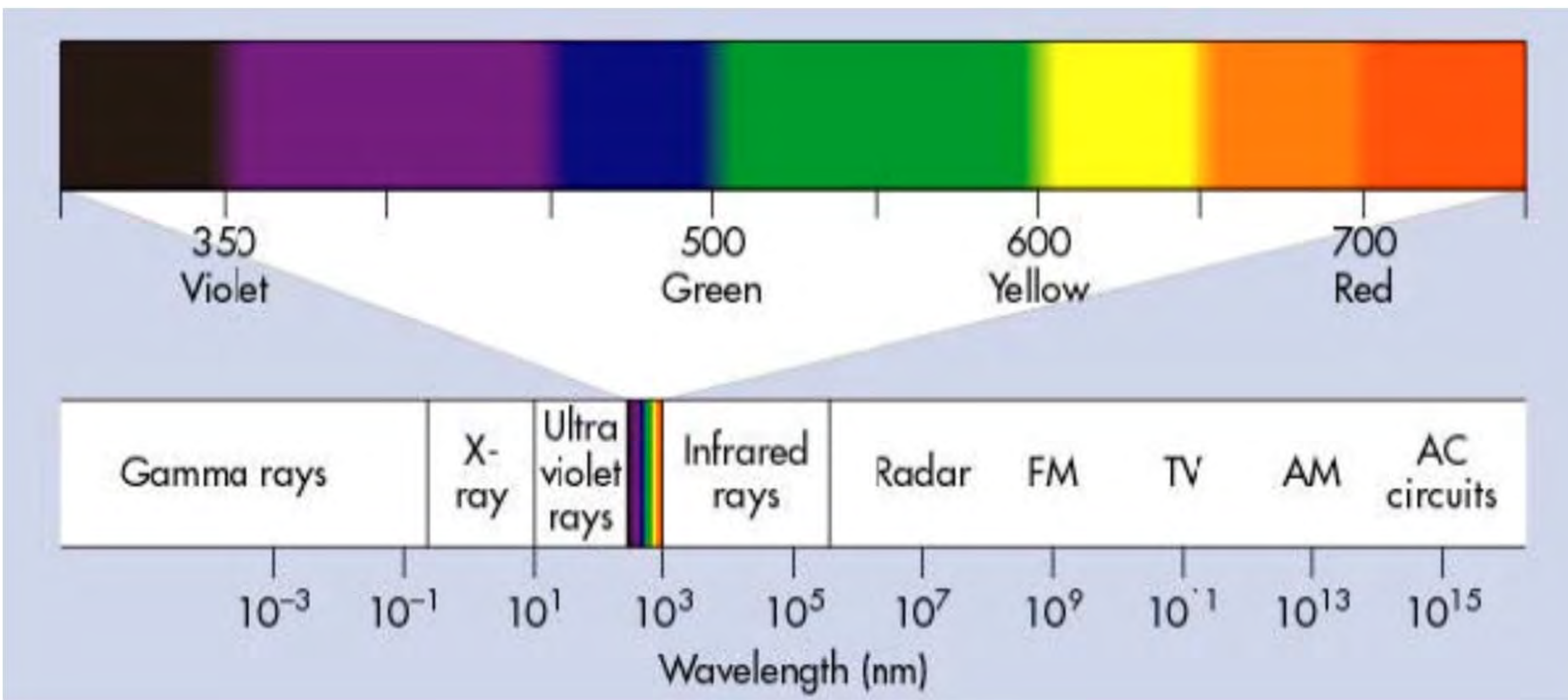


global solar irradiance on horizontal and inclined surfaces conducted by the United States' National Renewable Energy Laboratory at Kalaeloa Airport (21.312° N, -158.084°W), Hawaii, USA, from March 2010 until March 2011

K. Lehnertz, L. Zabawa, MRR Tabar. Characterizing abrupt transitions in stochastic dynamics. New J Physics 20, 113043, 2018

the colors of noise

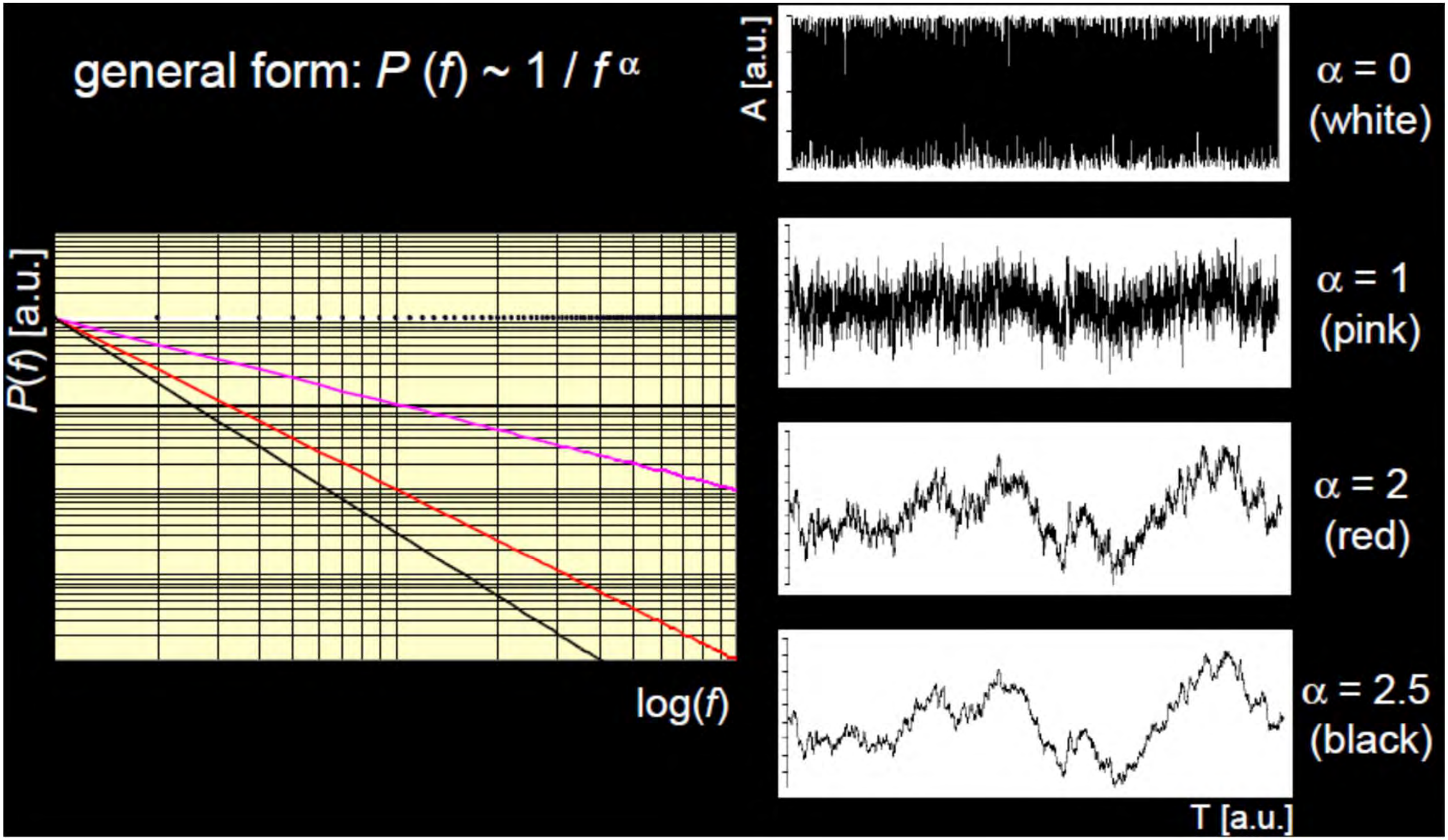
- alternative way to describe noise
- rough analogy to light
- refers to frequency content
- some colors have relationships to the real world
  some are more attuned to psycho-acoustics



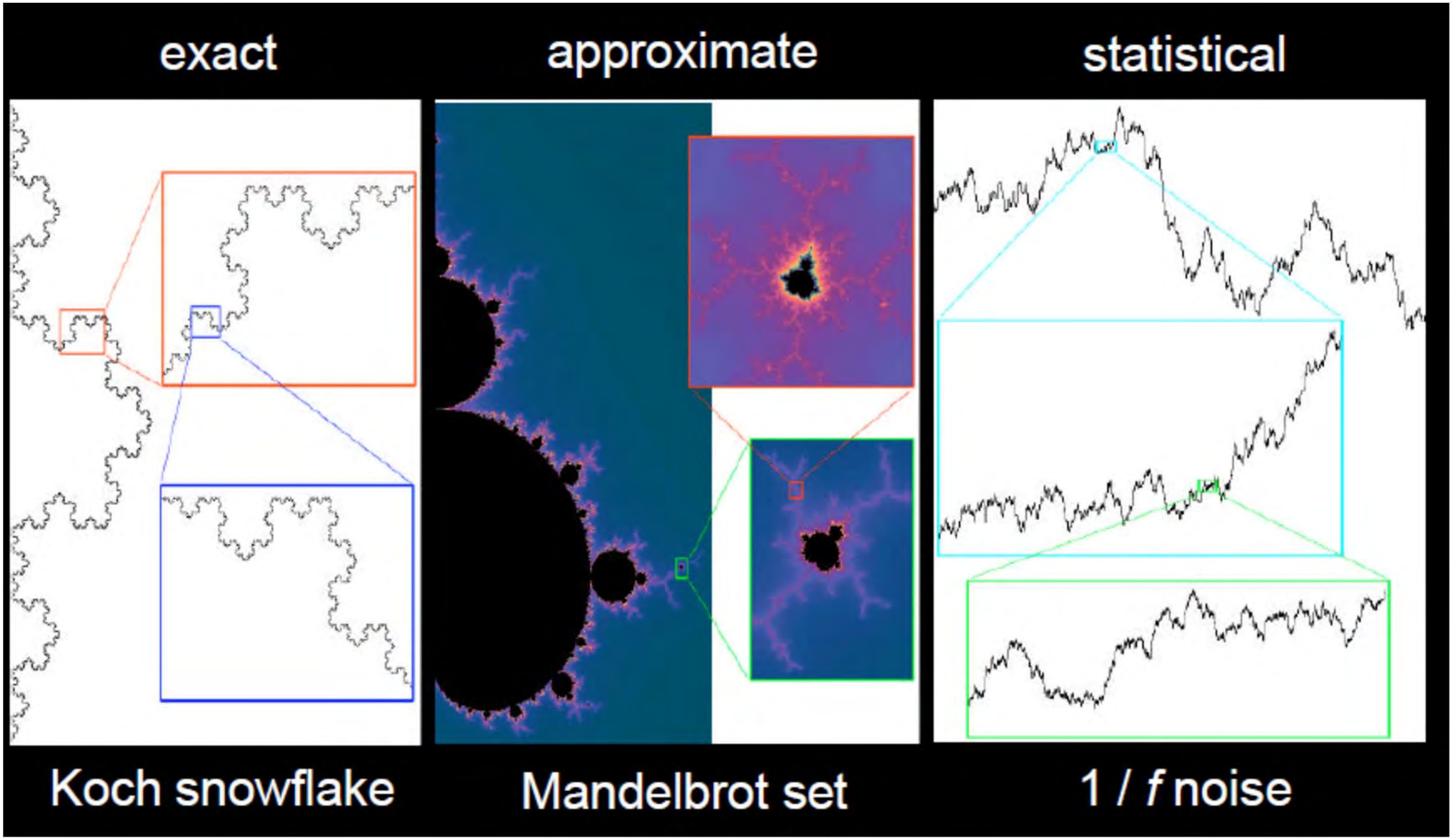
the colors of noise

color	frequency content	types of noise
white	1 (const.)	thermal, shot
purple/violet	f^2	artificial ?
blue	f	artificial ?
pink	$1/f$	flicker
red/brown	$1/f^2$	Brownian, popcorn *random walk*
black	$1/f^{\alpha}$ $(\alpha > 2)$	natural and unnatural catastrophes

the colors of noise power law spectra



noise and self-similarity



1/f noise a ubiquitous phenomenon

- current in carbon composition resistors
- current in ionic solutions
- solid-state components (e.g. Si MOSFET)
- body sway
- earth's wobble on its axis
- magnitude of ocean waves, earthquakes, thunder storms
- magnitude of tornados or hurricanes
- speech, classical and jazz music
- economic data
- neuronal activity, heart
- traffic
- ...

1/f noise music

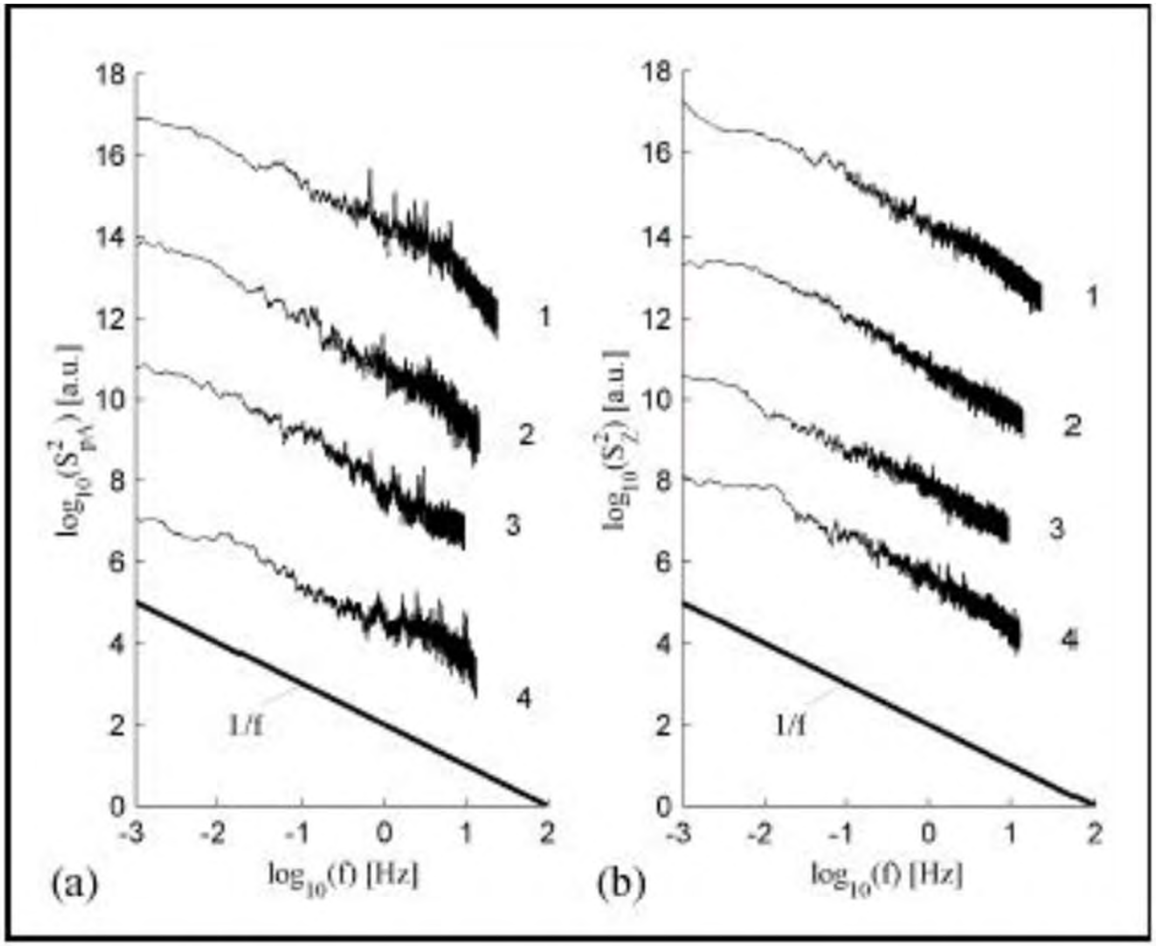


Figure 1. Spectrum of A-weighted sound pressure (a) and pitch (b) fluctuation of: 1. The 1st Brandenburgs Concerto by J.S. Bach; 2. The 2nd piano concerto by S. Rachmaninov; 3. Requiem by W.A. Mozart; 4. The 4 seasons by A. Vivaldi.

Voss and Clarke, Nature 258, 317, 1975

power-law noise and self-similarity

noise with power law spectra (e.g. 1/*f*)

↓

self-similarity

↓

scale invariance (time and space)

↓

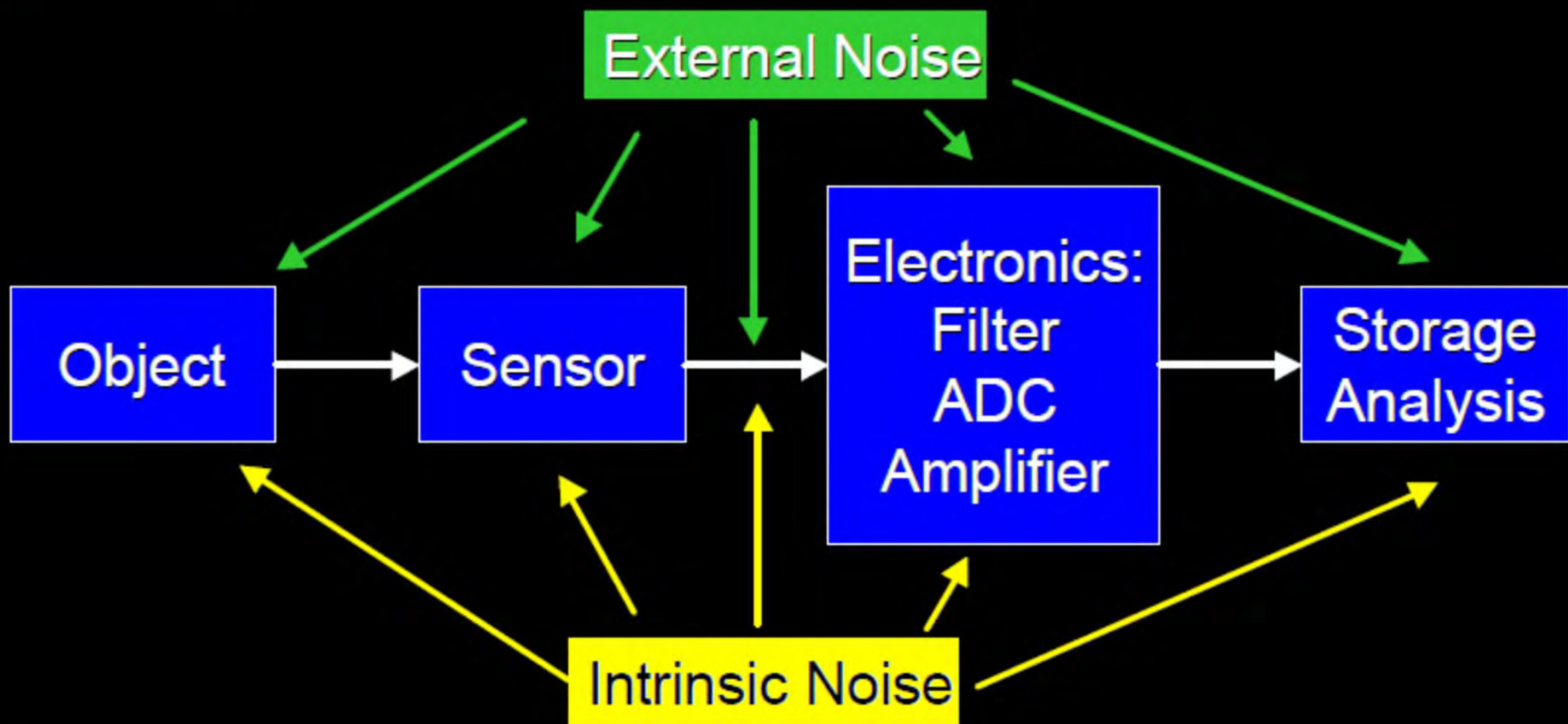
long-term correlation

↓

structures in time and space

noise and measurements the problem of interference

Interference: any kind of physical influence on a given system which reduces quality and performance of that system



noise and measurements — external noise

sources of interference

- temperature, humidity, EM-fields, radiation, mechanical shocks, digital equipment, ...

characteristics of external noise

- transient and/or constant
- periodic (e.g. power line @50 Hz) and/or random (white, pink)
- other ?

noise and measurements — external noise

guarding

measures and precautions to prevent noise entering sensitive parts of measurement system

shielding

placing electronic systems in a metal casing to prevent electrostatic and/or magnetic fields entering sensitive components

guarding and shielding

capacitive and inductive coupling, adequate grounding, analog filtering, differential amplifiers
lock-in amplifier (requires noise-reference signal)

noise and measurements **intrinsic noise**

sources of interference

- noise generated by object, sensor, electronics

characteristics of intrinsic noise

- transient and/or constant
- periodic (e.g. power line @50 Hz) and/or random (white, pink)
- other ?

noise and measurements **minimizing intrinsic noise**

object

? (depends on object)

sensor

chose adequate low-noise sensor (intrinsic limits?)

electronics

chose adequate electronics (intrinsic limits?)

measurement noise vs. dynamical noise

noise

unwanted disturbance

part of dynamics

how to differentiate?

measurement noise

$$\dot{\mathbf{x}} = f(\mathbf{x}, \beta)$$
$$s(t) = h(\mathbf{x}(t)) + \epsilon(t)$$

dynamical noise

$$\dot{\mathbf{x}} = f(\mathbf{x} + \eta, \beta)$$
$$s(t) = h(\mathbf{x}(t)) + \epsilon(t)$$

h is the measurement function

η, ϵ are random numbers drawn from some distribution

phase-space-based noise reduction general ideas

modeling the dynamics

dynamical ansatz; approximation of local dynamics; model fitting; shadowing problem

local projections

geometric ansatz; appropriate projection onto sub-manifold; shadowing problem

additionally:
filtered embeddings: restriction to some lower-dimensional manifold through singular-value decomposition; may be used as pre-processing step as it does not evaluate actual dynamics

E.J. Kostelich & T. Schreiber. Noise reduction in chaotic time-series data: A survey of common methods. Phys Rev E, 48, 1752, 1993

shadowing problem

"is there a noise-free trajectory close to the observed one?"

if so, does it hold for different initial conditions?

↓

nonlinear noise reduction:
separation of a low-dimensional dynamics from a complex (high-dimensional) signal

requirements for nonlinear noise reduction techniques

- appropriate strategy to embed a time series (dynamical and geometric ansatz)
- appropriate approximation of local dynamics in phase-space (dynamical ansatz: model class, fitting procedures)
- appropriate approximation of “de-embedded” time series (dynamical and geometric ansatz: consistency with chosen model)
- fast, efficient, easy-to-implement, easy-to-interpret

nonlinear noise reduction — dynamical ansatz

- idea: use “past” and “future values” to adjust one or more observations in the middle
- ansatz: $0 = f(v_1, \ldots, v_m, v_{m+1}) + \epsilon_{m+1}$
- choose embedding dimension m sufficiently large to reconstruct dynamics
- linear approximation of f by least-squares estimate:

$$\hat{\mathbf{v}}_{m/2} = \sum_{\substack{k=1 \\ k \neq m/2}}^{m-1} a_k \mathbf{v}_k - b$$

advantages:
- easy to implement
- fast
- reduction up to factor of 10

disadvantages:
- insufficient approx. of f
- neglecting first and last $m/2$ values

T. Schreiber & P. Grassberger, A simple noise-reduction method for real data. Phys. Lett. A, 160, 411, 1991

nonlinear noise reduction — dynamical ansatz

- idea: use "past" and "future values" to adjust one or more observations in the middle
- ansatz: $0 = f(v_1, \ldots, v_m, v_{m+1}) + \epsilon_{m+1}$
- choose embedding dimension m sufficiently large to reconstruct dynamics
- replace linear approximation of f by a constant and replace current phase-space vector by its mean value derived from its neighborhood

$$\hat{\mathbf{v}}_i = \frac{1}{|\mathcal{U}_i^{(\epsilon)}|} \sum_{\mathcal{U}_i^{(\epsilon)}} \mathbf{v}_i$$

<u>advantages:</u>
- easy to implement
- even faster
- reduction up to factor of 10

<u>disadvantages:</u>
- insufficient approx. of f

T. Schreiber, Extremely simple nonlinear noise-reduction method. Phys Rev E, 47, 2401, 1993

nonlinear noise reduction | dynamical ansatz

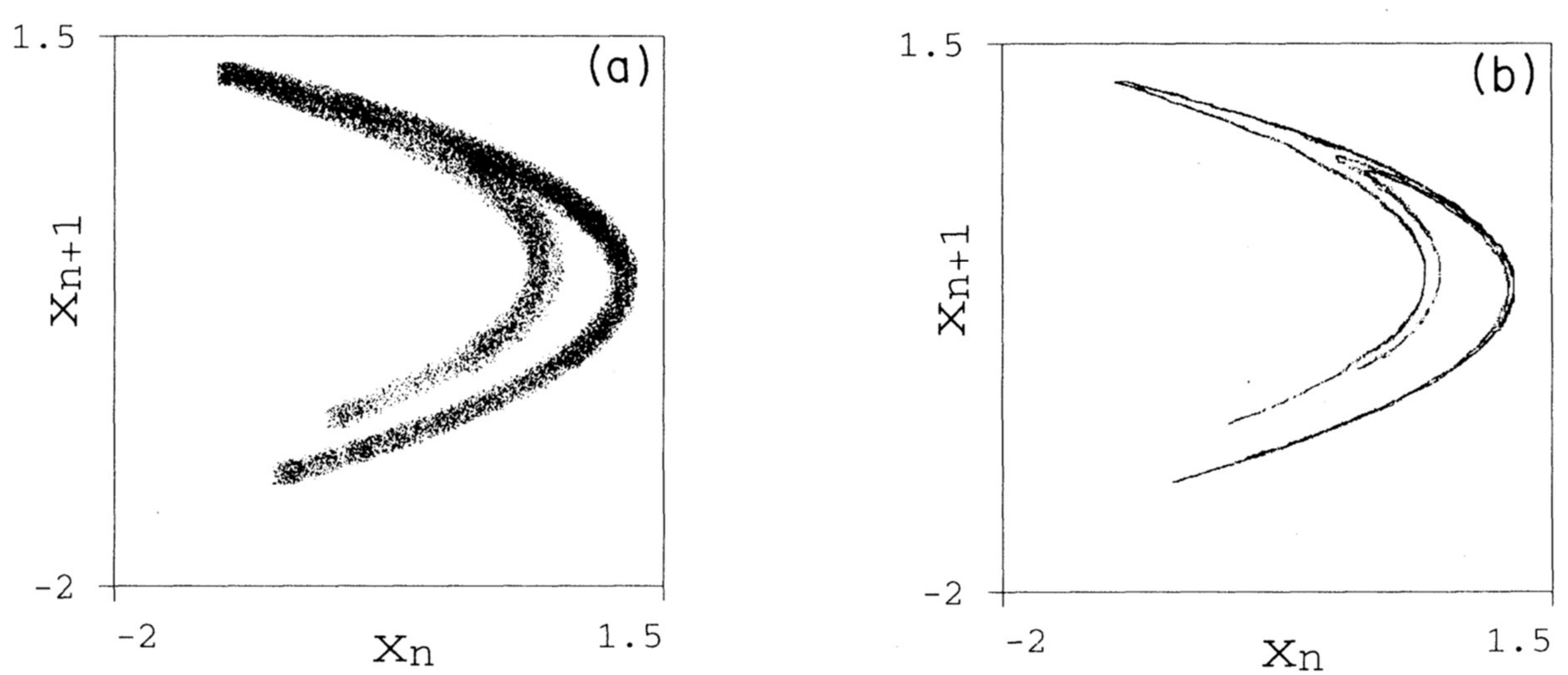


FIG. 1. Phase plots of iterates of the Hénon map. (a) A sample with 5% noise and (b) the same after noise reduction. Each panel contains 20 000 points.

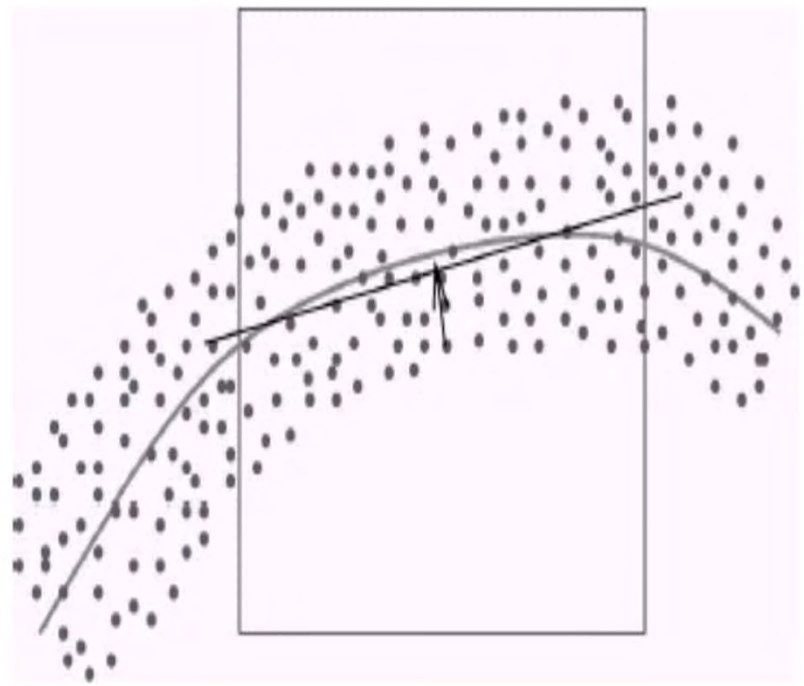
T. Schreiber, Extremely simple nonlinear noise-reduction method. Phys Rev E, 47, 2401, 1993

nonlinear noise reduction

geometric ansatz

observations and ideas:

- attractor is a subset of a smooth manifold in an *m*-dimensional phase-space
- estimate local tangent planes in each point by singular value decomposition
- noise reduction by projection onto a subspace that is spanned by appropriate eigenvectors



Cawley & Hsu, Phys Rev A, 46, 3057, 1992; Sauer, Physica D, 58, 193, 1992; Grassberger et al., Chaos, 3, 127,1993

nonlinear noise reduction geometric ansatz

how to (1):

- reconstruct dynamics in m-dimensional phase-space
- choose k nearest neighbors around some reference point $\mathbf{x}_i$ on trajectory
- represent "local" (demeaned) dynamics in k x m matrix $\mathbf{X}$
- singular value decomposition via $\mathbf{X} = \mathbf{U}^T \mathbf{\Sigma} \mathbf{V}$
- columns of $\mathbf{U}$ und $\mathbf{V}$ form an orthonormal basis for rows and columns of $\mathbf{X}$ (eigenvectors)
- diagonal matrix Σ comprises eigenvalues σ_i of $\mathbf{X}$
- total variance of $\mathbf{x}_i$ amounts to $\sigma_1^2 + \cdots + \sigma_m^2$

Cawley & Hsu, Phys Rev A, 46, 3057, 1992; Sauer, Physica D, 58, 193, 1992; Grassberger et al., Chaos, 3, 127,1993

nonlinear noise reduction geometric ansatz

how to (2):

- *assumption 1*: noise dominates in all phase-space directions;
- *assumption 2*: most components of dynamics are confined to a low-dimensional hyperplane through $\mathbf{x}_i$ (direction of largest variance);
- *assumption 3*: all other (orthogonal) components are noise

projection (example):

- let p ($p<m$) denote an integer number such that the first p eigenvalues of $\mathbf{X}$ explain 95 % of total variance
- define tangent hyperplane through $\mathbf{x}_i$ using the first p eigenvectors
- project noisy phase-space vectors onto that hyperplane

Cawley & Hsu, Phys Rev A, 46, 3057, 1992; Sauer, Physica D, 58, 193, 1992; Grassberger et al., Chaos, 3, 127,1993

nonlinear noise reduction geometric ansatz

advantages

- purely geometrical approach, no assumptions on f
- reduction up to factor of 10
- fast
- easy to implement

disadvantages

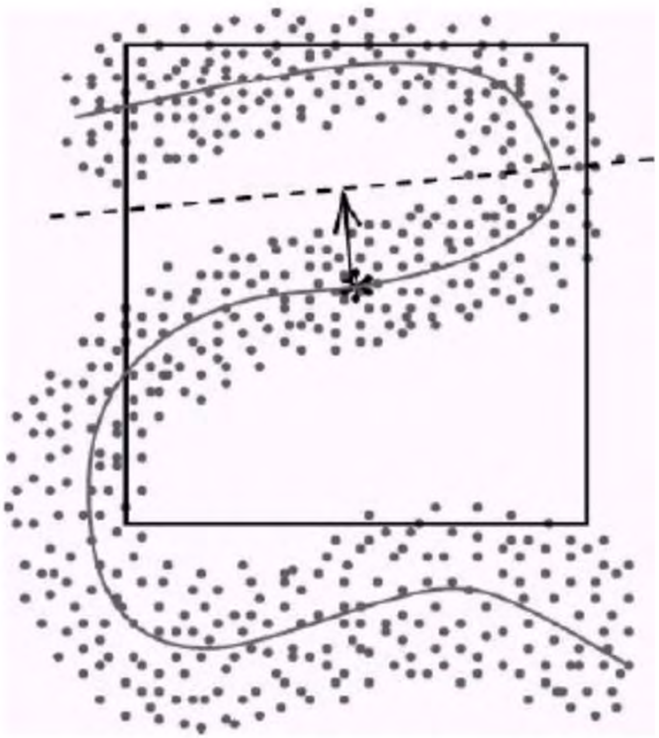
- anomalous large corrections (e.g., due to outlier)
- choice of appropriate neighborhood
- erroneous corrections in the presence of small nonlinearities
- accounts for direction of largest variance only

Cawley & Hsu, Phys Rev A, 46, 3057, 1992; Sauer, Physica D, 58, 193, 1992; Grassberger et al., Chaos, 3, 127,1993

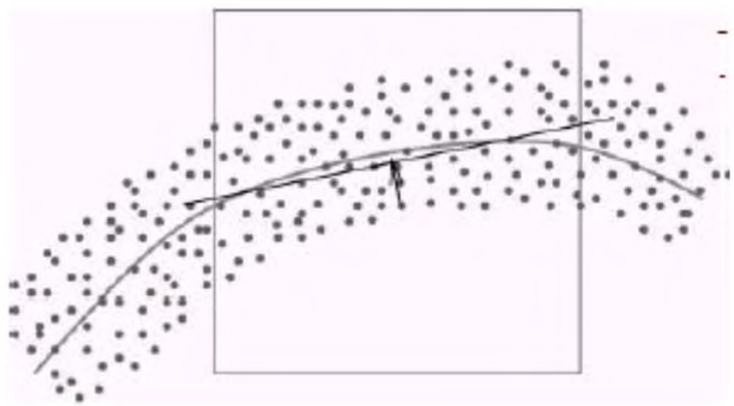
nonlinear noise reduction

geometric ansatz

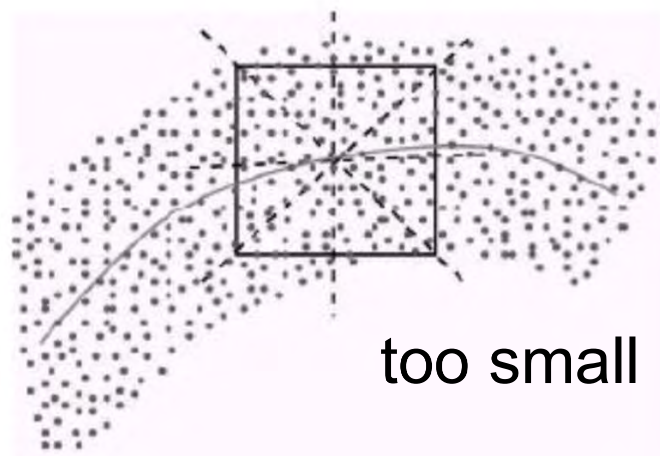
impact of choice of neighborhood



too large



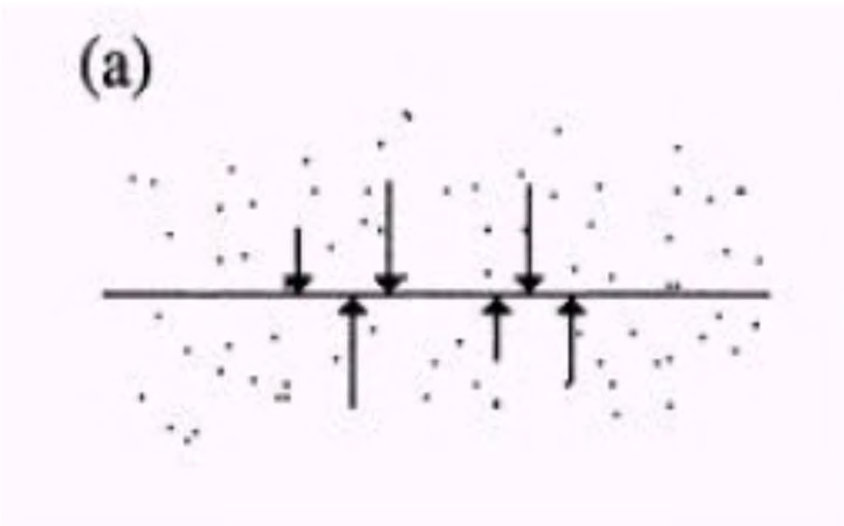
optimal

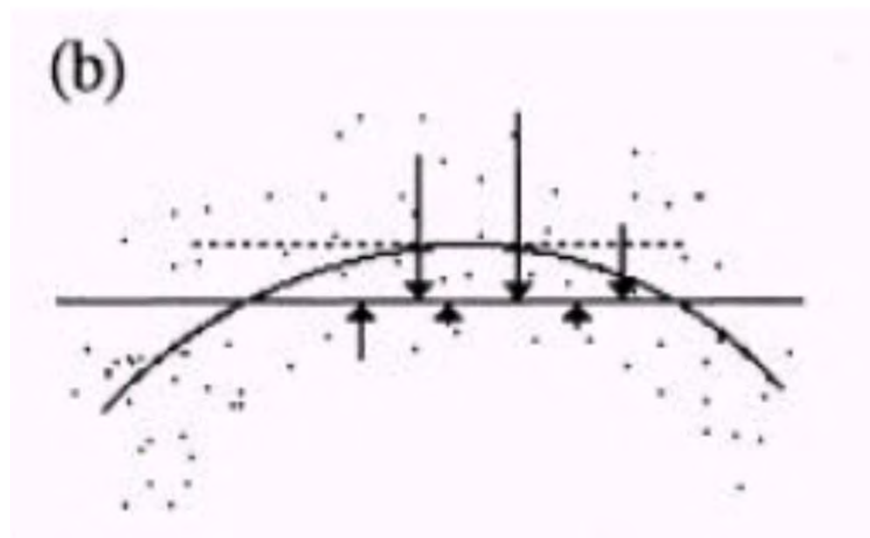


too small

nonlinear noise reduction **geometric ansatz**

impact of local nonlinearities





nonlinear noise reduction geometric ansatz

examples

PHYSICAL REVIEW E VOLUME 53, NUMBER 5 MAY 1996

Signal separation by nonlinear projections: The fetal electrocardiogram

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(Received 2 January 1996)

We apply a locally linear projection technique which has been developed for noise reduction in deterministically chaotic signals to extract the fetal component from scalar maternal electrocardiographic (ECG) recordings. Although we do not expect the maternal ECG to be deterministic chaotic, typical signals are effectively confined to a lower-dimensional manifold when embedded in delay space. The method is capable of extracting fetal heart rate even when the fetal component and the noise are of comparable amplitude. If the noise is small, more details of the fetal ECG, like *P* and *T* waves, can be recovered. [S1063-651X(96)50405-8]

nonlinear noise reduction geometric ansatz

examples

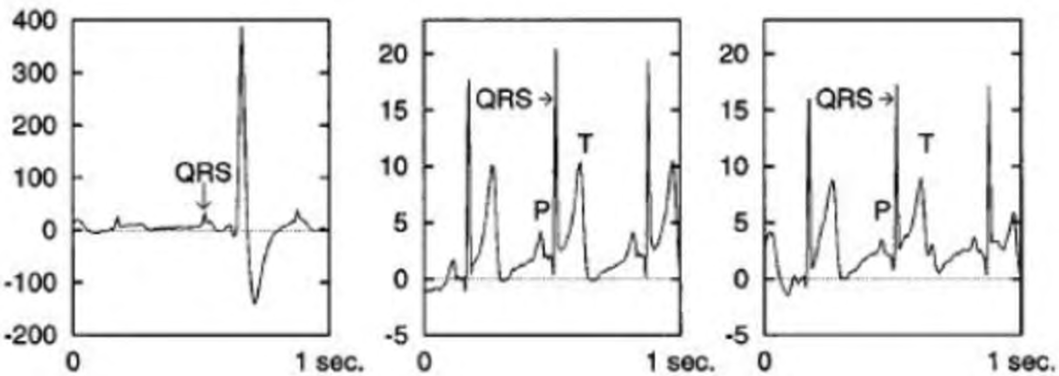


FIG. 4. Detail of the input signal (left), the fetal component (middle), and the reconstructed fetal component (right). Same series as in Fig. 3. We also identified some clinically relevant features of the fetal part: the P wave indicates the depolarization of the atrium. The QRS complex reflects the depolarization and the T wave the repolarization of the ventricle.

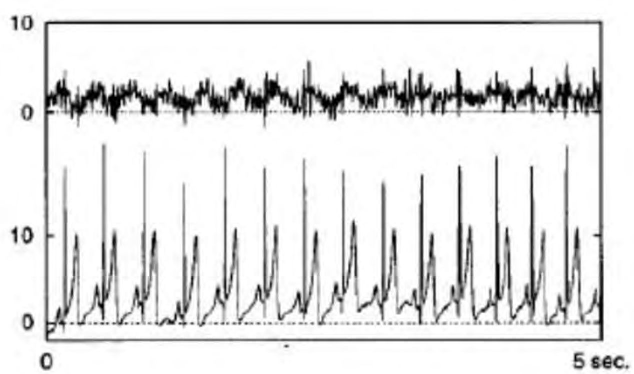


FIG. 5. The result of Wiener filtering (top) is shown in comparison with the true fetal component (bottom). Same data as in Figs. 3 and 4. Although we used the known spectrum of the fetal component to construct the filter, the optimal linear filter is essentially useless for extracting the fetal signal. The large peaks in the Wiener filter output correspond to the maternal QRS complexes.

FIG. 8. Top: original fetal component included in the series shown in Fig. 1. Bottom: reconstructed fetal component after nonlinear noise reduction. Although the amplitude of the fetal QRS complex is reduced in the reconstruction, at least the heart rate can be determined reliably.

nonlinear noise reduction **geometric ansatz**

examples ***geometric filtering with wavelets***

observations:

- singular value decomposition limited in some cases, since it only considers directions of largest variance
- singular value decomposition not well suited for transient signals

idea:
replace singular value decomposition with wavelet transform of phase-space vectors

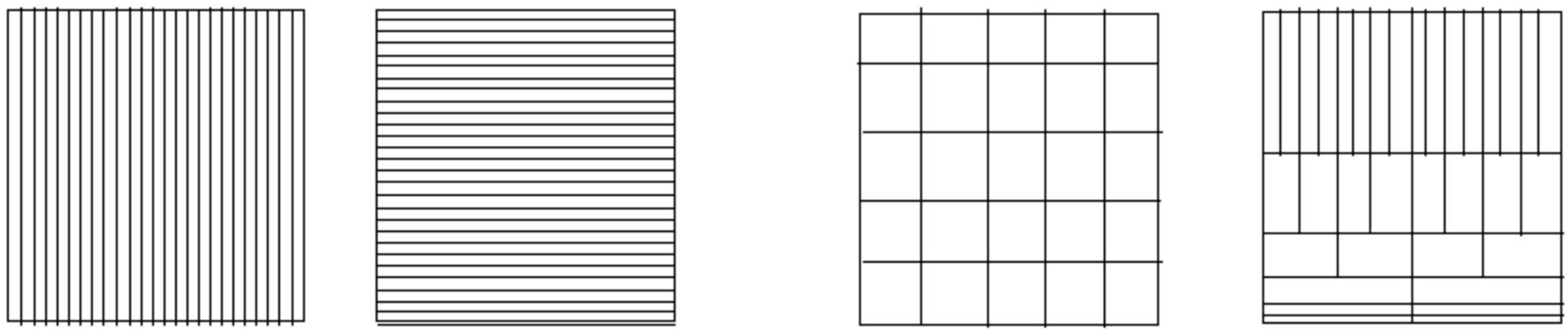
Effern A, Lehnertz K, Schreiber T, David P, Elger CE Nonlinear denoising of transient signals with application to event related potentials Physica D, 140, 257, 2000

nonlinear noise reduction geometric ansatz

brief intermezzo: wavelets

Fourier transform Gabor transform wavelet transform

frequency



time

time-frequency uncertainty

better adopted time-frequency decomposition

Ingrid Daubechies: *Ten Lectures on Wavelets*. SIAM 1992.

nonlinear noise reduction **geometric ansatz**

brief intermezzo: ***wavelets***

from waves to wave-packets

- wavelets can represent smooth functions and singularities
- wavelets are based on local and compact basis functions

  (improved adoption to inhomogeneities)
- many basis functions for a large number of signal classes
- fast wavelet-transformation ≈ [O(N)]

Ingrid Daubechies: *Ten Lectures on Wavelets*. SIAM 1992.

nonlinear noise reduction — geometric ansatz

examples — *geometric filtering with wavelets*

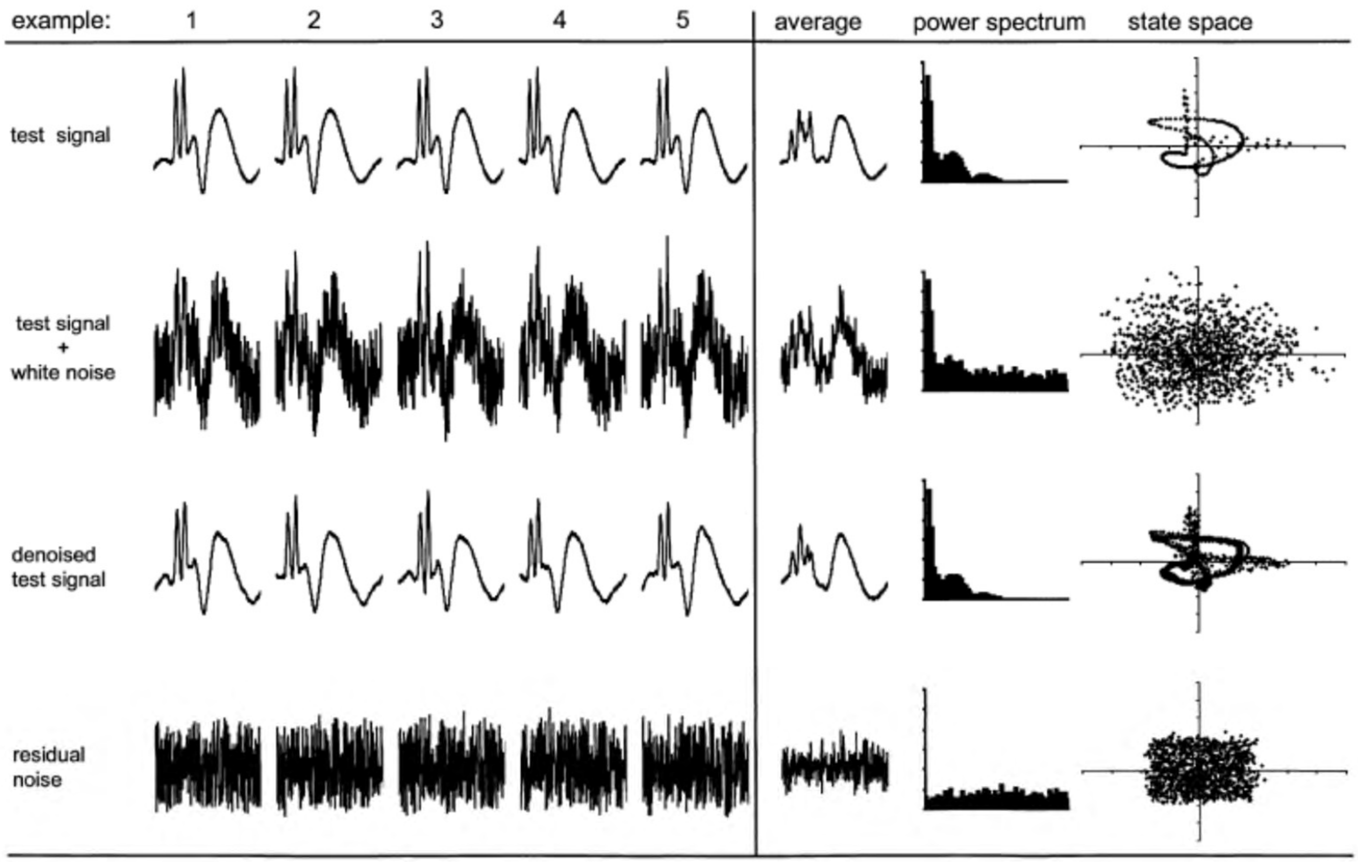


Fig. 3. Nonlinear denoising applied to white noise contaminated test signals (five sequences embedded, each 256 sample points, randomly shifted in time (S.D.: 20 sample points, max. shift: 40 sample points), noise amplitude 75%, $m = 128$, $\tau = 1$, $\lambda = 1.5$). Power spectra in arbitrary units. For state space plots we used a time delay of 25 sample points.

Effern A, Lehnertz K, Schreiber T, David P, Elger CE Nonlinear denoising of transient signals with application to event related potentials Physica D, 140, 257, 2000

nonlinear noise reduction geometric ansatz

examples *geometric filtering with wavelets*

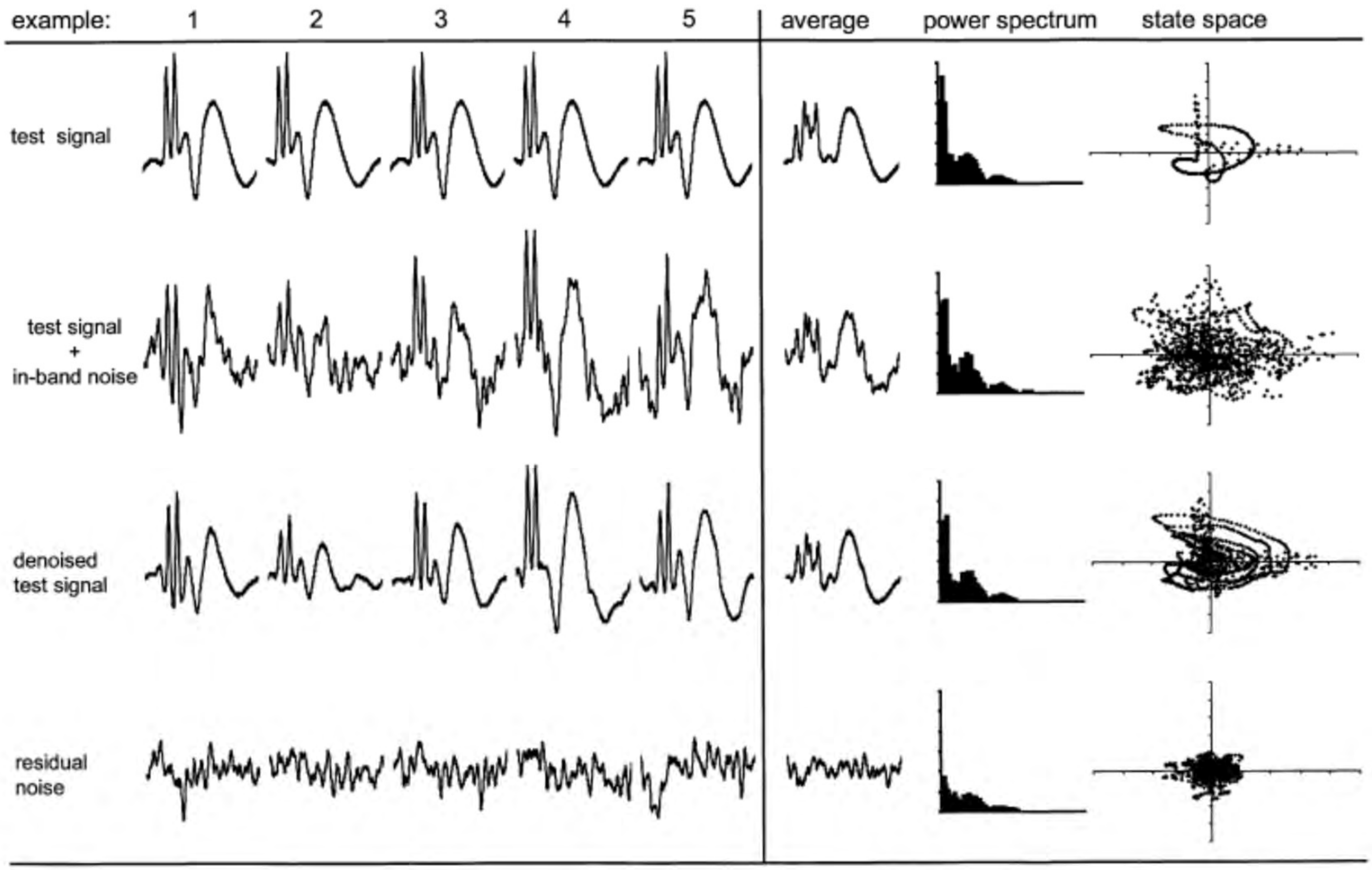


Fig. 4. Same as Fig. 3 but for in-band noise and $\lambda = 0.75$.

Effern A, Lehnertz K, Schreiber T, David P, Elger CE Nonlinear denoising of transient signals with application to event related potentials Physica D, 140, 257, 2000

nonlinear noise reduction — geometric ansatz

examples — *geometric filtering with wavelets*

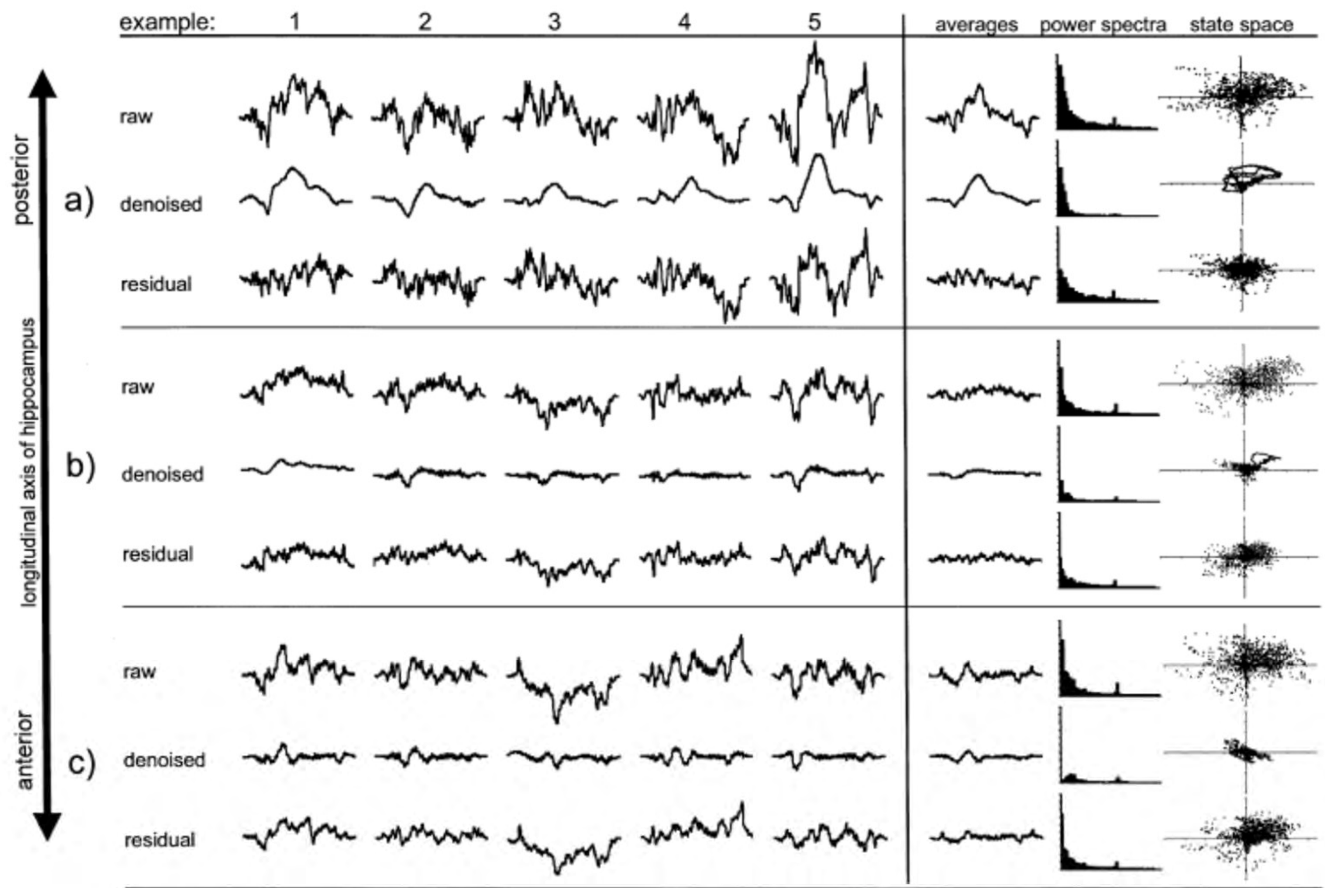


Fig. 6. Examples of denoised MTL-P300 potentials (cf. Fig. 1). Power spectra in arbitrary units. For state space plots we used a time delay of 25 sample points.

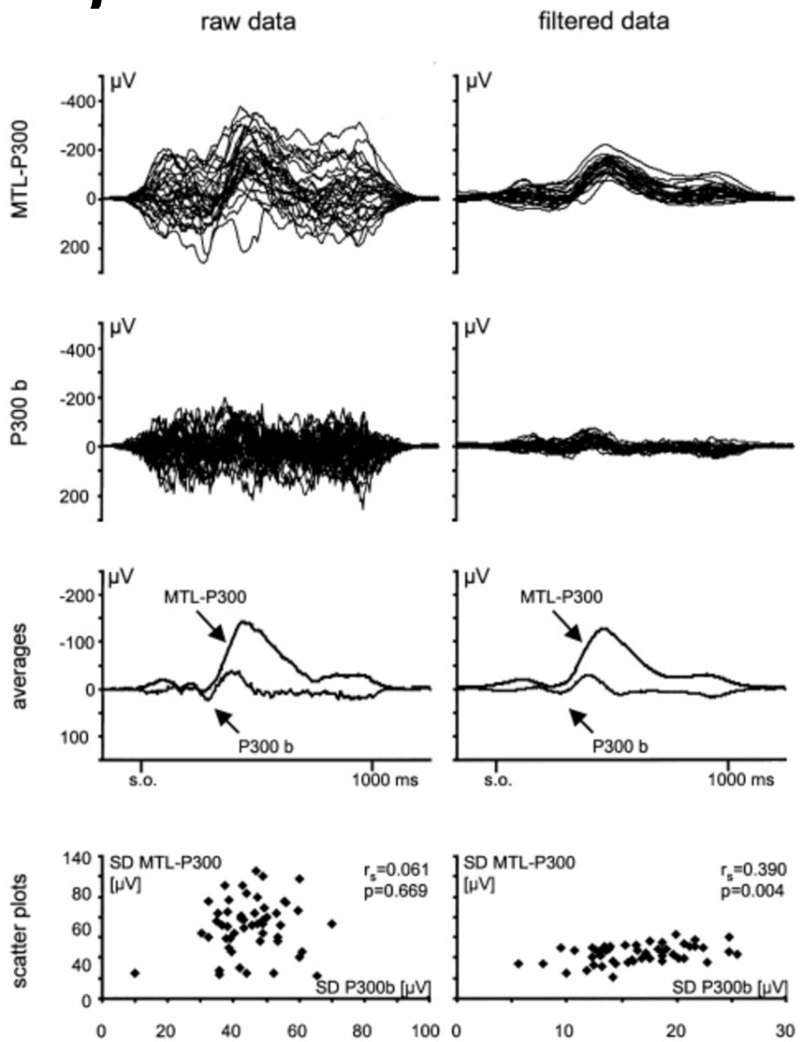
Effern A, Lehnertz K, Schreiber T, David P, Elger CE Nonlinear denoising of transient signals with application to event related potentials Physica D, 140, 257, 2000

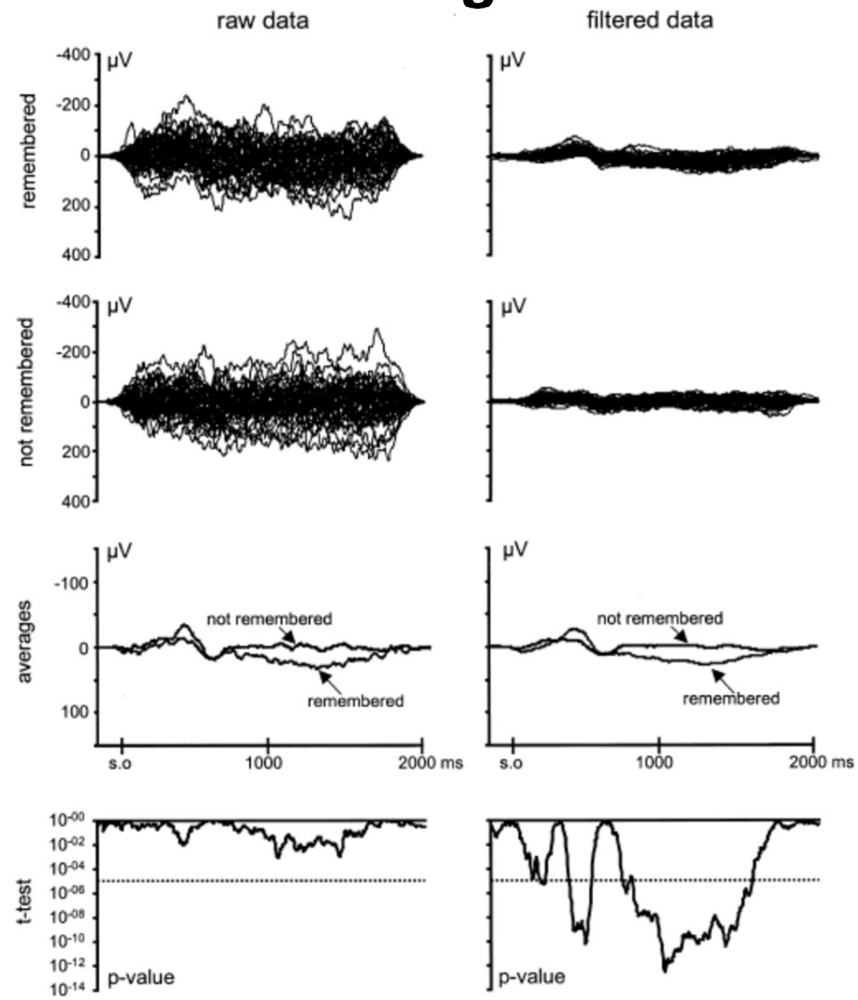
nonlinear noise reduction

geometric ansatz

examples

geometric filtering with wavelets





Effern A, Lehnertz K, Fernandez G, Grunwald T, David P, Elger CE. Single trial analysis of event related potentials: nonlinear denoising with wavelets. Clin. Neurophysiol, 111, 2255, 2000

nonlinear noise reduction geometric ansatz

examples *geometric filtering with wavelets*

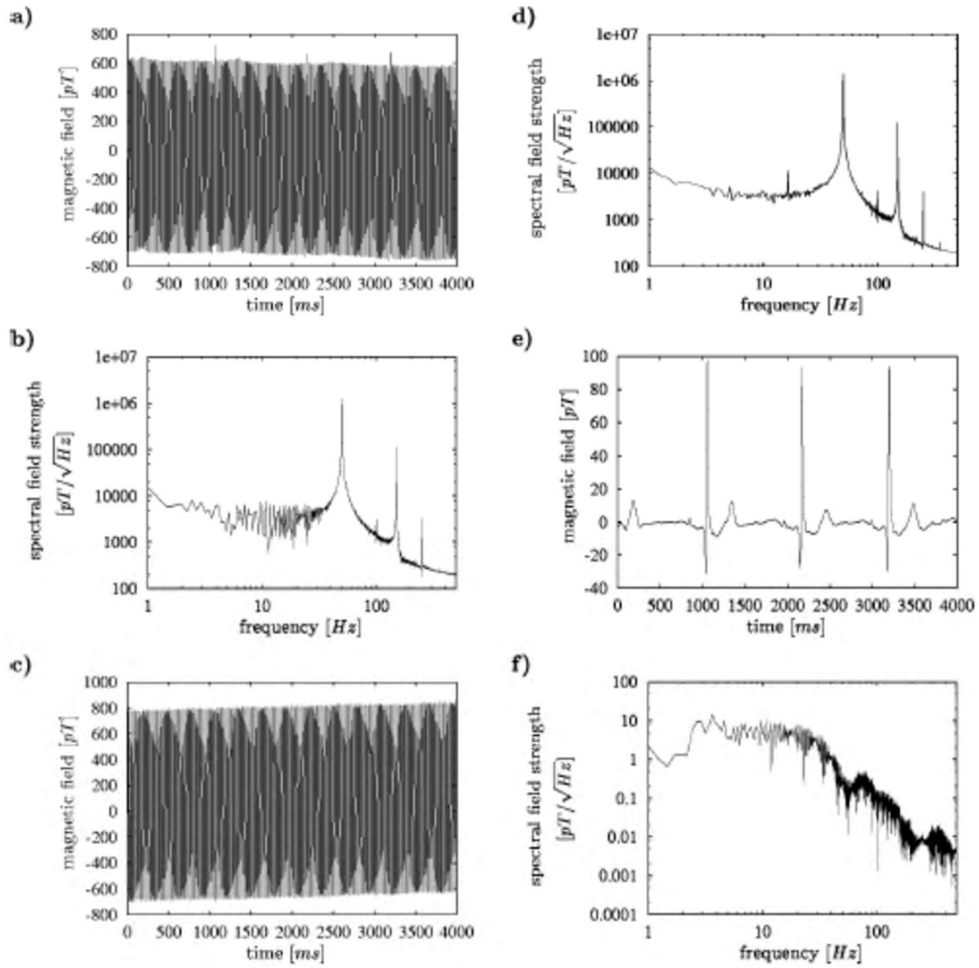


FIG. 2. (a) shows the MCG sequence recorded outside a shielding room. Only the main component of the heart signal (R wave) is visible. The width of the 50 Hz peak in the corresponding Fourier spectrum (b) actually prohibits the application of a notch filter. (c) exhibits the reference signal and (d) its Fourier spectrum. The noise reduced MCG sequence (e) even shows details of the heartbeat and an almost noise-free baseline. The corresponding Fourier spectrum is given in (f).

Sternickel K, Effern A, Lehnertz K, Schreiber T, David P.
Nonlinear denoising using reference data. Phys. Rev. E 63, 036209, 2001

nonlinear noise reduction **judging efficiency**

- judgment is application-specific and depends on assumptions about nature of noise

- if a noise-free trajectory is known a priori, estimate "distance" between that trajectory and the denoised one

- if equations of motion are known a priori, estimate "distance" between "true" time series and the denoised time series

- in case of unknown dynamics and/or system:

  visual inspection of denoised time series (looks good, more reliable)

  analyze denoised time series (e.g. Fourier spectrum, correlation sum for small ε)

  consistency checks (analyze residues (if accessible), only minor or no correlations between residues and original time series

nonlinear noise reduction what can go wrong?

field applications

- all issues related to embedding
- specific issues related to presented methodologies already discussed
- failure of noise reduction technique due to
    - false assumptions (e.g. additive vs multiplicative noise)
    - nonstationary noise amplitudes

- avoid wishful thinking!
  (sometimes it's just P2C2E*)

*process too complicated to explain: Iff in S. Rushdie: Haroun and the Sea of Stories. Granta Books, London (1990)